

PREDICTION OF CREEP CRACK INITIATION UNDER THE INTERACTION BETWEEN LONG RANGE RESIDUAL STRESS AND APPLIED LOAD

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Abstract In this paper, a three bar structure is examined to simulate the behaviour of a component subjected to combined applied and residual stresses. This structure (or system) permits long-range residual stress to be created in a compact tension (CT) specimen through the introduction of a misfit. The magnitudes of the residual and applied stresses in the CT specimen are a function of the initial misfit displacements, applied load and the relative stiffness of the components of the system. The prediction of cracking initiation under combinations of residual and apply loads are investigated when the compact tension specimen creep according to a power law. We find that the creep crack initiation time is sensitive to the assumed creep constants and is significantly different under different loading conditions. The effect of residual stress on the crack initiation time is dependent on the ratio of the residual stress to the total stress. Overall, this study provides important insights into the assumptions adopted in structural analysis for creep crack initiation.

Keywords Crack initiation, Residual stress , elastic follow-up, 316H stainless steel

1. Introduction

Many components operating at high temperature are subjected to combinations of applied and residual stresses, especially welded steel sections. Evidence from industry is that the presence of residual stress is a contributing factor for initiation and growth of creep cracks, which has important consequences for the lifetime of components at high temperature. A typical practical example is reheat crack initiation observed in stainless steel welded components where the presence of residual stress is seen as a major factor [1]. In this paper, the purpose is to better understand whether the existence of residual stress plays an important role in contributing the crack initiation in components driven by a combination of applied and residual stress at elevated temperature. Previous work involved generating residual stresses directly into specimens using a variety of methods. A recent review of this work [2] concluded that in order to improve our understanding of the effects of residual stress on fracture new methods should be sought that do not introduce microstructural changes during the generation of residual stress. Therefore, in this paper we develop a simple method of introducing long range residual stress through strain incompatibility in a classical three bar model. An additional force can applied to the three bar structure system to simulate the behaviour of a component subjected to combined applied and residual stresses. The subsequent creep behaviour is governed by the materials properties and elastic follow-up provided by the system. Elastic follow-up is a consequence of the presence of a region of differing stiffness relative to the remainder of the structure. [3].

In parallel to a series of laboratory tests [4], an analysis is presented to predict creep crack initiation for different levels of elastic follow-up. We find is that the crack initiation time is sensitive to the assumed creep constants and is significantly different under different loading conditions. The effect of residual stress on the crack initiation time is dependent on ratio of the residual stress to the total stress. Overall, this study provides important insights into the assumptions adopted in structural analysis for creep crack initiation.

2. Response of the three bar system

In this analysis, the response of the three bar system will be determined for a system containing an initial misfit, (state 0). The system consists of 2 outer bars (Bar 3) and an inner bar (Bar 2) connected in series to a CT specimen. A long range residual stress is introduced in state 1. State 2 corresponds to the system subjected to external loading as shown in Fig. 1.

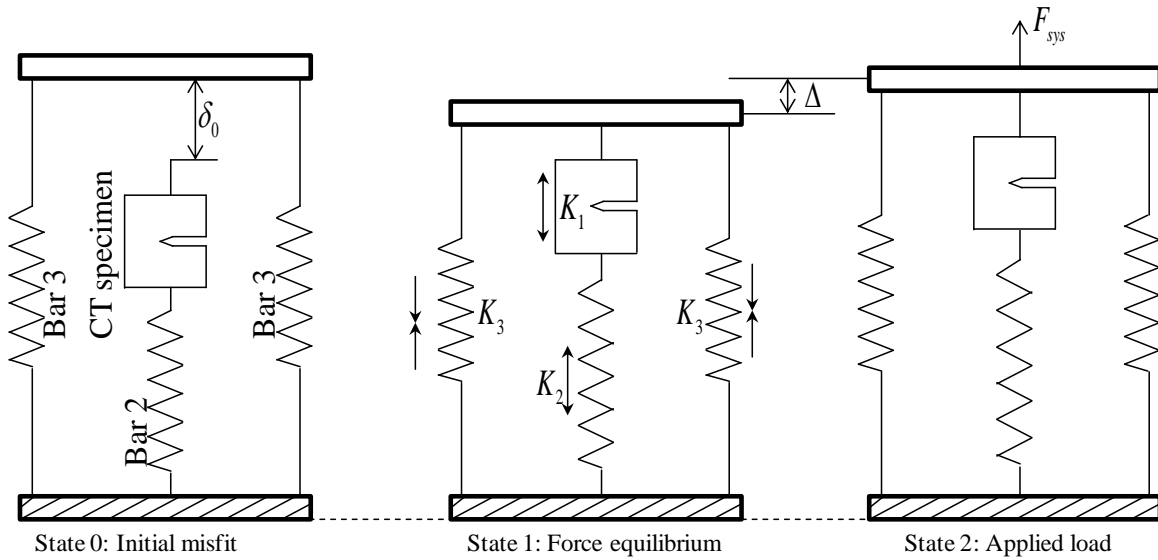


Figure 1. Parallel bars with an initial misfit subjected to an applied load.

In state 1, an initial residual stress field is introduced into system through the introduction of an incompatibility misfit. The displacements in the bars due to the misfit are:

$$\delta_1 = \left[\gamma / (\alpha_{eff} + 1) \right] \delta_0 \quad \delta_1 + \delta_2 = \left[\alpha_{eff} / (\alpha_{eff} + 1) \right] \delta_0 \quad \delta_3 = - \left[1 / (\alpha_{eff} + 1) \right] \delta_0 \quad (1)$$

where δ_0 is the initial misfit, δ_1 , δ_2 and δ_3 are the displacements in the CT specimen, and bars 2 and 3 respectively, K_1 , K_2 and K_3 are the stiffness for the CT specimen, and bars 2 and 3 respectively. Various ratios for the stiffness of the components are given by

$$\beta = K_2 / K_1 \quad \gamma = 2K_3 / K_1 \quad \alpha_{eff} = 2K_3 / K_{eff} \quad 1 / K_{eff} = 1 / K_1 + 1 / K_2 \quad (2)$$

The structure is then loaded through a rigid block (state 2) such that the displacement Δ for the whole system is given by:

$$\Delta = F_{sys} / (K_{eff} + 2K_3) \quad (3)$$

and the displacements in the bars are given by

$$\delta_1 = \left(\frac{\gamma}{\alpha_{eff} + 1} \right) \delta_0 + \left(\frac{\beta}{\beta + 1} \right) \Delta \quad \delta_2 = \left(\frac{\alpha_{eff}}{\alpha_{eff} + 1} \right) \delta_0 + \left(\frac{1}{\beta + 1} \right) \Delta \quad \delta_3 = - \left(\frac{1}{\alpha_{eff} + 1} \right) \delta_0 + \Delta \quad (4)$$

The presence of elastic follow-up in the system results in a slower stress relaxation rate when compared to classical stress relaxation. There is also additional strain accumulation in the CT specimen. As an approximation, the creep strains accumulated in the CT specimen are considered to be a scalar factor Z times the creep strain which would be accumulated in the corresponding laboratory relaxation test (at the same initial stress, dwell time and temperature). Z is called the elastic follow-up factor and is given by [5].

$$Z = (\Delta \varepsilon_{inc} + \Delta \varepsilon_{el}) / \Delta \varepsilon_{el} \quad \text{where} \quad \Delta \varepsilon_{el} = \Delta \sigma_{ref}^{Total} / E_1 = \Delta F_1 / L_{ref} K_1 \quad (5)$$

where $\Delta \varepsilon_{inc}$ is the incremental strain accumulation in specimen during creep stress relaxation; and $\Delta \sigma_{ref}^{Total}$ is the change of total reference stress on the CT specimen; ΔF_1 is the change in load on the CT specimen and L_{ref} is the reference length of the CT specimen.

For state 1 the incremental strain accumulation is given by

$$\Delta \varepsilon_{inc}^{state-1} = [\Delta F_1 / 2K_3 + \Delta F_1 / K_2] / L_1 \quad (6)$$

In state 2, the strain accumulation in specimen has two different solutions. The first solution corresponds to the total reference stress on the CT specimen at any time being greater than the initial residual stress

$$\sigma_{ref-t}^{Total} \geq \sigma_{ref-0}^{Rs}, \text{ and } \Delta \varepsilon_{inc}^{state-2} = \Delta \varepsilon_{inc}^{state-1}; \quad (7)$$

The second solution corresponds to when the total reference stress at a given time is less than the initial reference stress from the residual stress, ie:

$$\sigma_{ref-t}^{Total} < \sigma_{ref-0}^{Rs} \quad \Delta \varepsilon_{inc}^{state-2} = [\Delta F_1 / (2K_3 + K_{eff})] / L_1 \quad (8)$$

Combining Eqs. 5-8, the solutions for Z are as follows

State 1 :

$$Z_1 = 1 + 1/\gamma + 1/\beta = \left[(1 + \alpha_{eff}) / \alpha_{eff} \right] \left[(1 + \beta) / \beta \right] \quad (9)$$

State 2:

$$\sigma_{ref-t}^{Total} \geq \sigma_{ref-0}^{Rs} \quad Z_2 = Z_1 \quad (10a)$$

$$\sigma_{ref-t}^{Total} < \sigma_{ref-0}^{Rs} \quad Z_2 = 1 + \left[1 / (\gamma + \beta / (\beta + 1)) \right] \quad (10b)$$

Equations 9 and 10 show that Z for states 1 and 2 are the same if the current total reference stresses in the CT specimen is larger than the initial reference residual stress. When the current total reference stress is smaller than the initial reference residual stress, a new solution for Z for state 2 is given by Eq. 10. Those results are based on the assumption that only the CT specimen creeps while the remainder of the structure remains elastic. Z_1 , Eq. 9, is independent of the initial stress and the creep deformation behavior of the CT specimen, i.e., Z is a constant geometrical value

dependent on the stiffness of the system. However, Z_2 is influenced by the stress in the CT specimen.

3. Creep of a three bar system

We now consider the behavior of states 1 and 2 with the aim of determining how the stress relaxes in the three bar system due to creep. The stress-strain and creep properties of the materials are required and hence the first part of this section presents relevant material properties for a Type 316H stainless steel. The solution for the stress relaxation of states 1 and 2 are given in the second part of this section.

3.1 Creep rupture and creep crack growth properties

Tensile, steady state creep rate and creep crack initiation properties of Type 316H Austenitic stainless steel at 550 °C are summarized in Table. 1. Tensile properties of Type 316H Austenitic stainless steel were measured at a strain rate of 1.5% per min [6]. The relevant creep data are taken from Douglas [7] and fitted using a simple power law as shown in Fig. 2a. The steady state creep rate is given by

$$\dot{\epsilon} = D\sigma^m \quad (11)$$

where $\dot{\epsilon}$ is the creep rate in 1/h, σ is the applied stress in MPa and D and m are material constants. A regression fit to data shown in Fig 2a provided values for the constants D and m . Values are shown in Table 1 for mean, upper and lower bound fits. The upper (UB) and lower bound (LB) curves correspond to ± 1 standard deviations on the mean, assuming the slope is constant.

Seven creep crack tests using CT specimens were completed with test times ranging from about 60 to 16600hrs [8, 9]. The relationship between the applied reference stresses and time t_i (h) to initiate a crack equal to 0.2mm is shown in Fig. 2b. The initiation time t_i is given by

$$t_i = C\sigma_{ref}^f \quad (12)$$

where σ_{ref} is the plane stress reference stress in MPa for pre-cracked CT specimens, and C and f are the corresponding material constants. A regression fit to data shown in Fig. 2b provided values for the constants C and f . These are shown in Table 1.

Table 1. Tensile, steady state creep rate and time corresponding to crack extension of 0.2mm properties of Type 316H Austenitic stainless steel at 550 °C [6].

Material properties (MPa)	Steady state creep rate properties	Creep crack initiation properties
Young's Modulus: 160000	n: 10.154	f: - 5.45
Yield stress: 145	D: $10^{-29.54}$	C: 5×10^{15}
0.2% proof stress: 194	Upper bound (UB) and lower bound (LB) factor on D: 2.72	
UTS: 648		

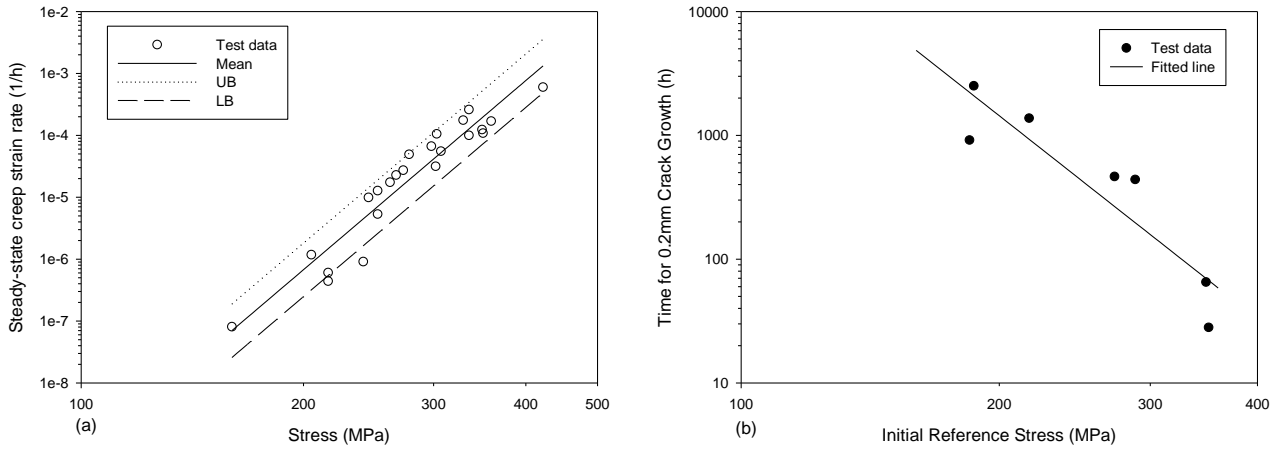


Figure 2. Material properties of Type 316H Austenitic stainless steel at 550 °C (a) Minimum creep strain rate, illustrating the mean, upper and lower bound regression fits ; (b) Time to crack initiation with respect to the reference stress.

3.2. Determination of stress relaxation in the three bar system

In order to simplify the analysis, the following three assumptions were made. a) The misfit and the applied load may be sufficient to cause plasticity in the CT specimen, but the surrounding structure remained elastic. The CT specimen was always at the creep temperature and creep only occurred in the specimen. b) For state 2, the applied force on the assembly was constant during the test period. Finally, c) the forces between the specimen and the elastic elements were always in equilibrium.

3.2.1 Relaxation of residual stress alone

Stress relaxation is a reduction of stresses with time due to the conversion of elastic strain to inelastic strain under constant total strain. For a material that follows a power creep law the reference strain rate in CT specimen can be expressed in terms of a reference stress, where

$$\dot{\epsilon}_{ref} = D\sigma_{ref}^n \quad (13)$$

In a CT specimen the relationship between force, displacement, reference stress and strain, and stress intensity factor are given by [10]

$$\sigma_{ref} = F_1/B_n W m_L \quad \epsilon_{ref} = \delta/L_{ref} \quad L_{ref} = E_1 B_n W m_L / K_1 \quad F_1 = K_1 \delta \quad K_1 = B E_1 / f(\alpha/W) \quad (14)$$

$$\text{where } f(\alpha/W) = \left(\frac{1+\alpha/W}{1-\alpha/W} \right)^2 \left[\begin{array}{l} 2.163 + 12.219(\alpha/W) - 20.065(\alpha/W)^2 \\ -0.9925(\alpha/W)^3 + 20.609(\alpha/W)^4 - 9.9314(\alpha/W)^5 \end{array} \right] \quad (15)$$

$$\text{and } m_L = \sqrt{(1+\varphi)(1+\varphi(\alpha_0/w)^2)} - (1+\varphi(\alpha_0/w)) \quad 0 \leq \alpha_0/W \leq 1 \quad (16)$$

where σ_{ref} is the plane stress reference stress, ϵ_{ref} L_{ref} are the reference strain and length respectively of a CT specimen, E_1 is the elastic modulus, δ is the load line displacement; α_0 , B_n

and W are the initial crack depth, width, net thickness of a CT specimen respectively. The constant m_L given by Eq. 16 is for plane stress and a von-Mises yield criterion with $\varphi = 2/\sqrt{3}$. It can be shown that the change in reference stress with time for the three bar system in state 1 is given by

$$\tau^{Rs} = \left[1 / \left(1 + (1/Z_1) A E_1 (n-1) (\sigma_{ref-0}^{Rs})^{n-1} t \right) \right]^{1/n-1} \quad \text{where} \quad \tau^{Rs} = \sigma_{ref-t}^{Rs} / \sigma_{ref-0}^{Rs} \quad (17)$$

where σ_{ref-t}^{Rs} and σ_{ref-0}^{Rs} are the current and initial reference residual stresses respectively in the CT specimen.

For a time much greater than

$$t \gg 1 / \left((1/Z) A E_1 (n-1) (\sigma_{ref-0}^{Rs})^{n-1} \right) \quad (18)$$

Equation 17 is approximately independent of the initial reference stress. The influence of the magnitude of initial reference stress is greatest during the early stages of the relaxation process.

As an example; take an initial residual reference stress of 200MPa. Using equation 17 together with creep behavior described by equation 13, and the material constants in Table 1, the predicted residual stress relaxation using mean, upper and lower bound creep constants and different elastic follow-up factors are shown in Fig. 3. The results show that the stress relaxation behavior is sensitive to the creep constants and also significantly affected by the elastic follow-up. For $Z=1$, there is no strain accumulated in the specimen; therefore the total strain across the entire cross-section of specimen remains zero during the relaxation process. This is equivalent to the stress relaxation in a bolt that holds two rigid flanges together. For $Z>1$, there is slower stress relaxation and extra strain is accumulated in the CT specimen.

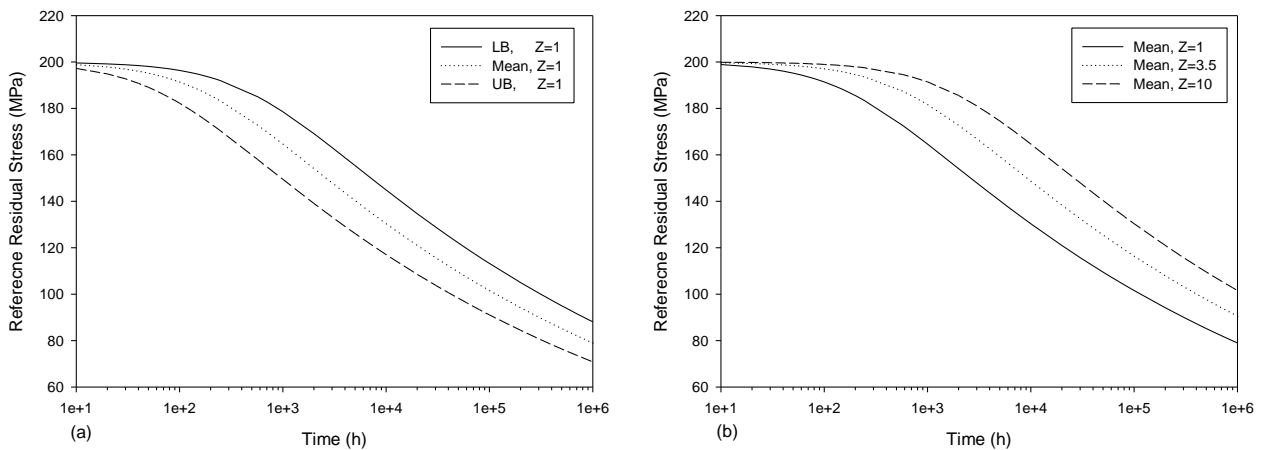


Figure 3. Prediction of reference residual stress relaxation from an initial value of 200MPa (a) using mean, and upper and lower bound creep constants and $Z=1$, (b) with mean creep data and different values of Z .

3.2.2 Relaxation of combined residual and applied stress

In state 2 the initial total reference stress (equal to residual plus applied stresses), and the elastic

follow-up factor change during total stress relaxation. The relaxation of the total reference stress for combinations of residual and applied loading is similar to Eq. 17, where

$$\tau^{Total} = \left[1 / \left(1 + (1/Z_2) AE_1 (n-1) (\sigma_{ref-0}^{Total})^{n-1} t \right) \right]^{1/n-1} \quad \text{where} \quad \tau^{Total} = \sigma_{ref-t}^{Total} / \sigma_{ref-0}^{Total} \quad (19)$$

$$\eta = \sigma_{ref-0}^{Rs} / \sigma_{ref-0}^{Total} \quad \sigma_{ref-0}^{Total} = \sigma_{ref-0}^{Rs} + \sigma_{ref-0}^{Applied} \quad \sigma_{ref-t}^{Rs} = \sigma_{ref-0}^{Rs} - (\sigma_{ref-0}^{Total} - \sigma_{ref-t}^{Total}) \quad (20)$$

where τ^{Total} is the normalized total reference stress, η is the ratio of the initial residual to the total stress in the CT specimen. σ_{ref-t}^{Total} , σ_{ref-0}^{Total} , σ_{ref-0}^{Rs} and σ_{ref-t}^{Rs} are the reference total stress and residual stresses at time t and 0 respectively. From Eqs. 7 to 10, the change from Z_1 to Z_2 occurs when $\tau^{Total} = 1 - \eta$. This corresponds to when the residual stress is relaxed to zero.

Therefore

$$t \leq \frac{1 - (1 - \eta)^{n-1}}{(1/Z) AE_1 (n-1) (\sigma_{ref-0}^{Total})^{n-1} (1 - \eta)^{n-1}}, Z=Z_1; \quad (21a)$$

$$t > \frac{1 - (1 - \eta)^{n-1}}{(1/Z) AE_1 (n-1) (\sigma_{ref-0}^{Total})^{n-1} (1 - \eta)^{n-1}}, Z=Z_2 \quad (21b)$$

Now consider a three bar structure system with properties so that the relative stiffness ratios are $\beta = 0.25$, and $\gamma = 0.2$. From Eqs 9 and 10 $Z_1 = 10$ and $Z_2 = 3.5$. Assume an initial residual reference stress of 200MPa and subject the CT specimen to additional applied stress to give a total reference stress equal to 330MPa. For this case $\eta = 0.606$. The relaxation of the total and residual stresses is shown in Figure 4, again using the creep law given by Eqn 11 with creep constants given in Table 1. The change from Z_1 to Z_2 occurs when the total stresses relaxes to $(1 - \eta) 330MPa = 130MPa$. This corresponds to a time where the residual stress relaxes to zero as shown in Figure 4a. Also Figure 4a shows that the residual stress decreases significantly due to the presence of the applied load.

Now consider the case when the total reference stress is 330MPa and residual stress is such that the ratio $\eta = 0.5$ and 0.75 . The stress relaxation behaviour for these two different values of η is shown in Figure 4b. The solid line represents the stress relaxation path for $Z_1 = 10$ and the dashed line represents the stress relaxation for $Z_2 = 3.5$.

When $\eta = 1$ the residual stress accounts for 100% of total stress and stress relaxation corresponds to $Z_1 = 10$. When $\eta = 0$ this represents the case when there is no residual stress and the applied stress accounts for 100% of the total stress. Stress relaxation corresponds to $Z_2 = 3.5$.

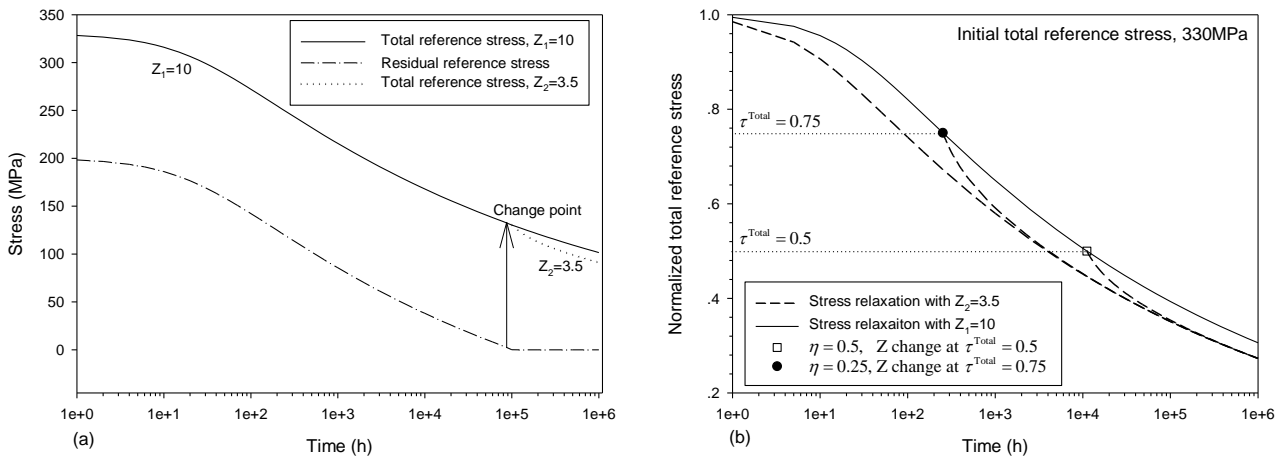


Figure 4. Initial total reference stress 330MPa (a) $\eta = 0.606$, the changing of residual reference stress with total reference stress (b) The relationship between total reference stress relaxation with different η .

4. Prediction of crack initiation

In this section crack initiation will be examined for two different cases. (1) Relaxation of a residual stress in the absence of any external stress. This represents stress relief cracking situations. (2) Relaxation in the presence of a superimposed applied stress. This simulates crack initiation for combined residual and applied stresses.

4.1 Prediction of crack initiation for residual stress alone

By combining Eqs 12 and 17 together with the creep deformation and crack initiation properties listed in Table. 1, the predicted time to crack initiation for a three bar system for state 1 is given by

$$\int_0^{t_i} \frac{1}{C} \left[\left(\frac{1}{Z_1} \right) (n-1) DE_1 t + (\sigma_{Spec-0}^{Rs})^{1-n} \right]^{f/(n-1)} dt = 1 \quad (22)$$

Equation 22 is solved numerically to obtain the initiation time. Predicted creep crack initiation times are shown in Fig 5, for initial residual stresses ranging from 75MPa to 400MPa.

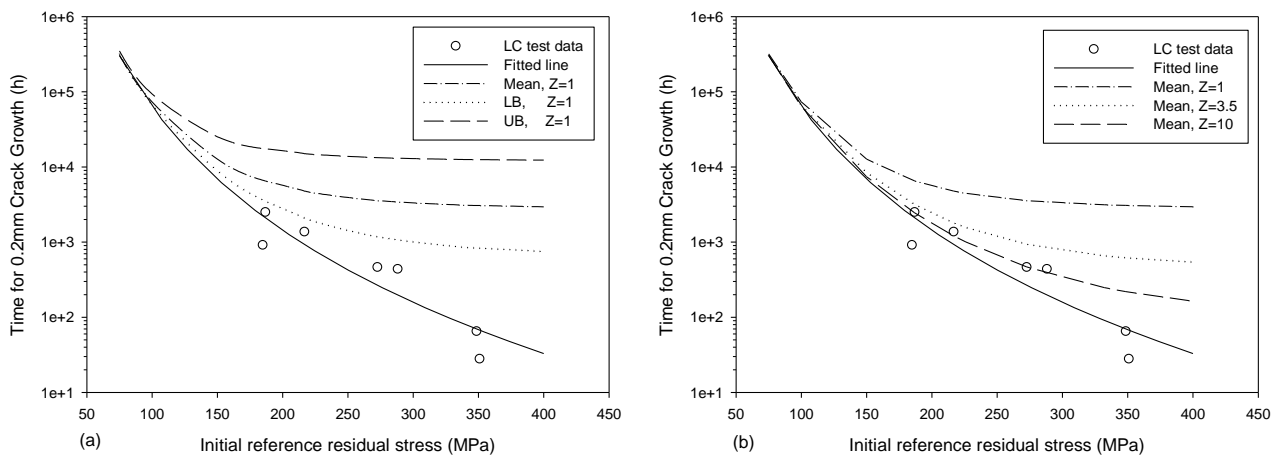


Figure 5. The prediction of crack initiation time with different initial reference residual stress, (a) illustrating the mean, upper and lower bound value (b) with different EFU factor with mean value.

Mean, lower and upper bound creep properties together with $Z=1$ were assumed for Fig 4a and mean creep properties and different elastic follow-up factors in Fig 4b. The results illustrate that the predictions are sensitive to the creep constants and elastic follow-up factors respectively. However, the initiation times for different constants and different Z tend to converge to similar initiation times for low initial reference stresses.

4.2 Prediction of crack initiation for combined residual and applied stress

By combining Eqs. 12 and 19 with the creep properties listed in table 1, the time for crack initiation can be obtained. If the initiation time is less than t_1 given by

$$t_1 \leq \frac{1 - (1 - \eta)^{n-1}}{(1/Z) AE_1 (n-1) (\sigma_{ref-0}^{Total})^{n-1} (1 - \eta)^{n-1}} \quad (23)$$

Then the initiation time is solved from Eqn. 22.

If, however, the initiation time is greater than t_1 then the initiation time is obtained by solving

$$\int_0^{t_i} \frac{1}{C} [(1/Z_1)(n-1)DE_1 t + (\sigma_{Spec-0}^{Rs})^{1-n}]^{f/(n-1)} dt + \int_{t_i}^{t_1} \frac{1}{C} [(1/Z_2)(n-1)DE_1 t + (\sigma_{Spec-t_i}^{Rs})^{1-n}]^{f/(n-1)} dt = 1 \quad (24)$$

Solutions obtained by numerically solving equations 22 and 24 are shown in Figure 6. When the system is under load control the initiation time is 91 hours. If the same system is under displacement control (ie the initial stress is entirely residual and $Z_2 \sim 1$) the initiation time is 3132 hours. If was assumed that $\beta = 0.25$, and $\gamma = 0.2$ and therefore $Z_1 = 10$ and $Z_2 = 3.5$. When the stress (of 330MPa) on the CT specimen arises from applied loading along then $Z=3.5$ and the initiation time is 659 hours. In contrast if the CT specimen is subjected to a residual stress equal to 330MPa, the $Z_1=10$. And the initiation time is 248 hours.

The influence of different ratios of residual to total stress (η) are shown in Fig 6b. Applied stress conditions lie to the left hand side while conditions where residual stress conditions dominate lie on the right hand side of the horizontal scale.

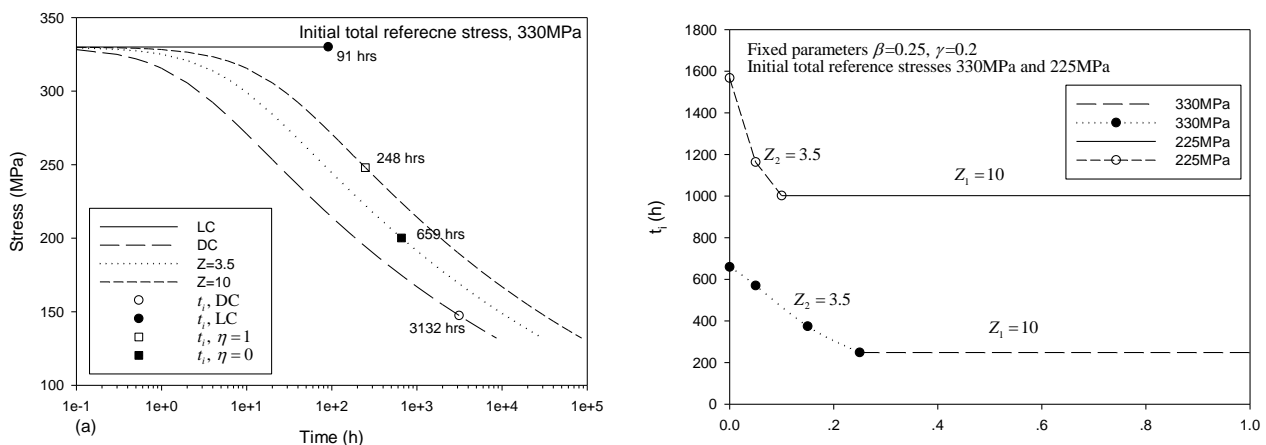


Figure 6. (a) The crack initiation time for initial total reference stress 330MPa under the displacement control, load control and mixed boundary control (b) the crack initiation time with different η and two different reference stresses.

4. Discussion and concluding remarks

A three bar structure has been designed to simulate the behaviour of a component subjected to combined applied and residual stresses. The properties of the three bar system are determined depending when the long range residual stress (state 1) is induced and following application of an external load (state 2). Closed form solutions of the elastic follow-up factor for states 1 and 2 are given and we find that the expression for elastic follow-up factor in state 2 changes when corresponding residual stress relaxes to zero.

Calculations performed to predict creep crack initiation in the presence of long range residual stress in Type 316H austenitic stainless steel provide results that yield significant insight to the behaviour of the system. Sensitivity studies have been included to determine the influence of changes in the material creep properties and the effects of elastic follow-up factor on stress relaxation. We find that the predictions of crack initiation are sensitive to both the creep constants and elastic follow-up factors respectively. However, the initiation times for different constants and different Z tend to converge to similar values for low initial reference stresses.

In state 2, the effect of residual stress on crack initiation is found to depend on the value of total stress (residual stress plus stress created by applied load) and the ratio of the residual stress to total stress. The elastic follow-up factor decreases significantly in state 2 when all of the residual stress has relaxed to zero.

Acknowledgements

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