Modeling of Inclined Crack Growth under Creep Conditions

Vladimir I. Astafiev[1*], **Alexey N. Krutov**[2]

[1] Samara State Aerospace University, Samara 443086, Russia
[2] Samara State University, Samara 443011, Russia
* Corresponding author: vlast@ssu.samara.ru

Abstract The modeling of subcritical growth of inclined crack under creep condition is considered. In the first part of this paper the stress state near the tip of inclined crack for power creep law in the cases of plane stress and plane strain is calculated. To calculate the stress state near the tip of an inclined crack the Airy's stress function is used. The resulting nonlinear fourth order differential equation is formulated as two-point boundary value problem and is solved by shooting and Newton's methods.

The modeling of creep crack growth is based on Rabotnov-Kachanov damage theory and the criterion of crack growth ω=1 at the distance d from the crack tip, calculated for equivalent or maximum stress. The crack growth rate and the crack trajectory are calculated both for plane stress and plane strain and for n = 1, 3, 5, 7 and considered in the second part of this paper.

Keywords inclined crack, creep, stress distribution.

1. Introduction

Let us consider an infinite plate of a nonlinear elastic-creep material with a crack of the length 2a, located at the angle α to the axis *x* and loaded by the stress σ_∞ along the axis *y* (Figure 1). It is required to determine the stress state near the tip of inclined crack for the plane stress and the plane strain conditions. It should be noted that the problem of uniaxial tension of inclined crack is statically equivalent to the mixed tensile and shear loading by the stresses $\sigma_\infty \cos^2\alpha$ and $\sigma_\infty \cos\alpha \sin\alpha$, respectively (Fig. 1).

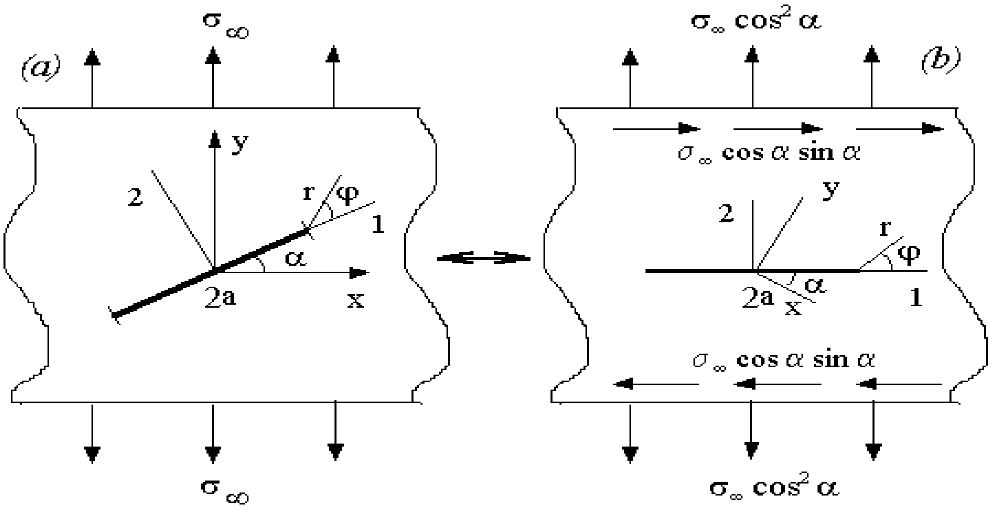

Figure 1. The geometry of the inclined crack.

2. Statement of the problem

2.1. Main equations

Let us consider the polar coordinate system (r, φ) associated with the tip of inclined crack (Fig. 1). The equilibrium equations and the Cauchy relations in the polar coordinate system for the plane strain or stress conditions are [1-3]:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\varphi}}{\partial \varphi} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = 0, \quad \frac{1}{r}\frac{\partial \sigma_{\varphi\varphi}}{\partial \varphi} + \frac{\partial \sigma_{r\varphi}}{\partial r} + 2\frac{\sigma_{r\varphi}}{r} = 0. \tag{1}$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\varphi\varphi} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi}, \quad 2\varepsilon_{r\varphi} = \frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}, \tag{2}$$

The constitutive equations for an incompressible material with a power creep law have the following form [1-2]:

$$\varepsilon_{ij} = (3/2)B\sigma_e^{n-1}s_{ij} \tag{3}$$

where ε_{ij} is the strain rate tensor, $s_{ij} = \sigma_{ij} - (1/3)\sigma_{kk}\delta_{ij}$ is the deviator of σ_{ij} stress tensor, $\sigma_e = \sqrt{(3/2)s_{ij}s_{ij}}$ is the equivalent stress, B is the material constant, n is the index of nonlinearity.

The equivalent stress σ_e for the plane stress and the plane strain is calculated as

$$\sigma_e = \sqrt{\sigma_{rr}^2 - \sigma_{rr}\sigma_{\varphi\varphi} + \sigma_{\varphi\varphi}^2 + 3\sigma_{r\varphi}^2}, \quad \sigma_e = \sqrt{(3/4)(\sigma_{rr} - \sigma_{\varphi\varphi})^2 + 3\sigma_{r\varphi}^2} \tag{4}$$

The strain compatibility equation in polar coordinates, resulting from Eq. (2), has the following form [1-3]:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\varepsilon_{\varphi\varphi}) + \frac{1}{r^2}\frac{\partial^2 \varepsilon_{rr}}{\partial \varphi^2} - \frac{1}{r}\frac{\partial \varepsilon_{rr}}{\partial r} - \frac{2}{r^2}\frac{\partial}{\partial r}(r\frac{\partial \varepsilon_{r\varphi}}{\partial \varphi}) = 0. \tag{5}$$

Taking into account Eq. (1), Eq. (3) and Eq. (4), the strain compatibility Eq. (5) can be rewritten in the stress terms for the plane stress (a) and the plane stain (b) conditions as the follows:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}\left[\sigma_e^{n-1}r\left(\sigma_{\varphi\varphi} - \frac{1}{2}\sigma_{rr}\right)\right] + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\left[\sigma_e^{n-1}\left(\sigma_{rr} - \frac{1}{2}\sigma_{\varphi\varphi}\right)\right] -$$

$$- \frac{1}{r}\frac{\partial}{\partial r}\left[\sigma_e^{n-1}\left(\sigma_{rr} - \frac{1}{2}\sigma_{\varphi\varphi}\right)\right] - \frac{3}{r^2}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial \varphi}\left(\sigma_e^{n-1}\sigma_{r\varphi}\right)\right] = 0, \tag{6a}$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}\left[\sigma_e^{n-1}r\left(\sigma_{\varphi\varphi} - \sigma_{rr}\right)\right] + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\left[\sigma_e^{n-1}\left(\sigma_{rr} - \sigma_{\varphi\varphi}\right)\right] -$$

$$- \frac{1}{r}\frac{\partial}{\partial r}\left[\sigma_e^{n-1}\left(\sigma_{rr} - \sigma_{\varphi\varphi}\right)\right] - \frac{4}{r^2}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial \varphi}\left(\sigma_e^{n-1}\sigma_{r\varphi}\right)\right] = 0, \tag{6b}$$

Thus, the main equations in stress terms for the power law constitutive equation (Eq. 4) are the equilibrium equations (Eq. 1) and the strain compatibility condition for the plane stress (Eq. 6a) or the plane stain (Eq. 6b), accordingly.

2.2. Airy's stress function

To solve the system of Eq. (1) and Eq. (6a) or Eq. (1) and Eq. (6b) it is often used the Airy's stress function $F(r, \varphi)$, defined as follows [1-3]:

$$\sigma_{rr} = \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2}, \quad \sigma_{\varphi\varphi} = \frac{\partial^2 F}{\partial r^2}, \quad \sigma_{r\varphi} = \frac{1}{r^2}\frac{\partial F}{\partial \varphi} - \frac{1}{r}\frac{\partial^2 F}{\partial r \partial \varphi}. \tag{7}$$

Substituting Eq. (7) in the strain compatibility equation for plane stress and plane strain conditions (Eq. 6a or Eq. 6b, accordingly), we obtain the following nonlinear differential equation for the Airy's stress function $F(r, \varphi)$:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}\left[\sigma_e^{n-1}\left(r\frac{\partial^2 F}{\partial r^2} - \frac{1}{2}\left(\frac{\partial F}{\partial r} + \frac{1}{r}\frac{\partial^2 F}{\partial \varphi^2}\right)\right)\right] + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\left[\sigma_e^{n-1}\left(\frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2} - \frac{1}{2}\frac{\partial^2 F}{\partial r^2}\right)\right] -$$
$$-\frac{1}{r}\frac{\partial}{\partial r}\left[\sigma_e^{n-1}\left(\frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2} - \frac{1}{2}\frac{\partial^2 F}{\partial r^2}\right)\right] - \frac{3}{r^2}\frac{\partial}{\partial r}\left[\frac{\partial}{\partial \varphi}\left(\sigma_e^{n-1}\left(\frac{1}{r}\frac{\partial F}{\partial \varphi} - \frac{\partial^2 F}{\partial r \partial \varphi}\right)\right)\right] = 0, \tag{8a}$$

where

$$\sigma_e^2 = \frac{\partial^2 F}{\partial \varphi^2}\left(\frac{2}{r^3}\frac{\partial F}{\partial r} + \frac{1}{r^4}\frac{\partial^2 F}{\partial \varphi^2} - \frac{1}{r^2}\frac{\partial^2 F}{\partial r^2}\right) + \frac{\partial F}{\partial r}\left(\frac{1}{r^2}\frac{\partial F}{\partial r} - \frac{1}{r}\frac{\partial^2 F}{\partial r^2}\right) -$$
$$-3\frac{\partial^2 F}{\partial r \partial \varphi}\left(\frac{2}{r^3}\frac{\partial F}{\partial \varphi} - \frac{1}{r^2}\frac{\partial^2 F}{\partial r \partial \varphi}\right) + \left(\frac{\partial^2 F}{\partial r^2}\right)^2 + \frac{3}{r^4}\left(\frac{\partial F}{\partial \varphi}\right)^2,$$

or

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}\left[\sigma_e^{n-1}\left(r\frac{\partial^2 F}{\partial r^2} - \left(\frac{\partial F}{\partial r} + \frac{1}{r}\frac{\partial^2 F}{\partial \varphi^2}\right)\right)\right] + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\left[\sigma_e^{n-1}\left(\frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2} - \frac{\partial^2 F}{\partial r^2}\right)\right] -$$
$$-\frac{1}{r}\frac{\partial}{\partial r}\left[\sigma_e^{n-1}\left(\frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2} - \frac{\partial^2 F}{\partial r^2}\right)\right] - \frac{4}{r^2}\frac{\partial}{\partial r}\left[\frac{\partial}{\partial \varphi}\left(\sigma_e^{n-1}\left(\frac{1}{r}\frac{\partial F}{\partial \varphi} - \frac{\partial^2 F}{\partial r \partial \varphi}\right)\right)\right] = 0, \tag{8b}$$

where

$$4\sigma_e^2 = \frac{\partial^2 F}{\partial \varphi^2}\left(\frac{6}{r^3}\frac{\partial F}{\partial r} + \frac{3}{r^4}\frac{\partial^2 F}{\partial \varphi^2} - \frac{6}{r^2}\frac{\partial^2 F}{\partial r^2}\right) + \frac{\partial F}{\partial r}\left(\frac{3}{r^2}\frac{\partial F}{\partial r} - \frac{6}{r}\frac{\partial^2 F}{\partial r^2}\right) -$$
$$-12\frac{\partial^2 F}{\partial r \partial \varphi}\left(\frac{2}{r^3}\frac{\partial F}{\partial \varphi} - \frac{1}{r^2}\frac{\partial^2 F}{\partial r \partial \varphi}\right) + 3\left(\frac{\partial^2 F}{\partial r^2}\right)^2 + \frac{12}{r^4}\left(\frac{\partial F}{\partial \varphi}\right)^2.$$

2.3. Near crack tip asymptotic

In the polar coordinate system the Airy's stress function $F(r, \varphi)$ near a crack tip has the following asymptotic representation [4-6]:

$$F(r,\varphi) = Kr^{\lambda} f(\varphi), \tag{9}$$

where $\lambda = (2n+1)/(n+1)$, $K = (J/BI_n)^{1/(n+1)}$, $J = \oint\left(\frac{n}{n+1} B\sigma_e^{n+1}\cos\varphi - \sigma_{ij} n_j \frac{\partial u_i}{\partial x}\right)ds$ is the path-independent contour integral (usually named as Cherepanov-Rice-integral or J-integral) and

$$I_n = \int_{-\pi}^{\pi}\left[\cos\varphi\left(\frac{n}{n+1}s_e^{n+1} - \frac{1}{n+1}\left(s_{rr}u_r + s_{r\varphi}u_\varphi\right)\right) - \sin\varphi\left(s_{rr}\left(u_\varphi - \frac{\partial u_r}{\partial\varphi}\right) - s_{r\varphi}\left(u_r + \frac{\partial u_\varphi}{\partial\varphi}\right)\right)\right]d\varphi$$

is the dimensionless constant.

Taking into account Eq. (7) and the asymptotic behavior (Eq. 9) of Airy's stress function $F(r, \varphi)$, the stress tensor σ_{ij} and the equivalent stress σ_e near a crack tip can be written as follows:

$$\sigma_{rr} = Kr^{\lambda-2}(\lambda f(\varphi) + d^2 f/d\varphi^2) = Kr^{\lambda-2}s_{rr}(\varphi),$$
$$\sigma_{\varphi\varphi} = Kr^{\lambda-2}(\lambda(\lambda-1)f(\varphi)) = Kr^{\lambda-2}s_{\varphi\varphi}(\varphi),$$
$$\sigma_{r\varphi} = Kr^{\lambda-2}(1-\lambda)(df/d\varphi) = Kr^{\lambda-2}s_{r\varphi}(\varphi),$$
$$\sigma_e = Kr^{\lambda-2}s_e(\varphi),$$

where $s_e(\varphi)$ is the dimensionless function, written for the plane stress and the plane strain conditions, respectively, as the follows:

$$s_e^2 = \lambda^2(\lambda^2 - 3\lambda + 3)f^2(\varphi) + 3(\lambda^2 - 2\lambda + 1)(df/d\varphi)^2 + \lambda(3-\lambda)f(\varphi)(d^2 f/d\varphi^2) + (d^2 f/d\varphi^2)^2,$$

$$s_e^2 = \lambda^2(\lambda^2 - 4\lambda + 4)f^2(\varphi) + 4(\lambda^2 - 2\lambda + 1)(df/d\varphi)^2 + 2\lambda(2-\lambda)f(\varphi)(d^2 f/d\varphi^2) + (d^2 f/d\varphi^2)^2$$

Compatibility equations (8a) and (8b) for the asymptotic of Airy's stress function $F(r, \varphi)$ (Eq. 9) can be rewritten as non-linear ordinary differential equation for the unknown function $f(\varphi)$ [7-8]:

$$n(\lambda-2)s_e^{n-1}\left[((2-\lambda)n-3)(d^2 f/d\varphi^2) + \lambda\left(2n(\lambda^2+3) + \lambda(3-7n) - 6\right)f(\varphi)\right] +$$
$$+\frac{d^2}{d\varphi^2}\left[s_e^{n-1}\left(2d^2 f/d\varphi^2 - \lambda(\lambda-3)f(\varphi)\right)\right] + 6(\lambda-1)((\lambda-2)(n-1)+\lambda-1)\frac{d}{d\varphi}\left[s_e^{n-1}df/d\varphi\right] = 0, \tag{10a}$$

$$n(\lambda-2)s_e^{n-1}\left[((2-\lambda)n-2)(d^2 f/d\varphi^2) + \lambda(\lambda-2)(n(\lambda-2)+2)f(\varphi)\right] +$$
$$+\frac{d^2}{d\varphi^2}\left[s_e^{n-1}\left(d^2 f/d\varphi^2 - \lambda(\lambda+2)f(\varphi)\right)\right] + 4(\lambda-1)((\lambda-2)(n-1)+\lambda-1)\frac{d}{d\varphi}\left[s_e^{n-1}df/d\varphi\right] = 0. \tag{10b}$$

Thus, the equation (10a) or (10b) is a non-linear differential equation of fourth order for unknown asymptotic Airy's stress function $f(\varphi)$. To solve this equation and to determine the stress field near a crack tip it is necessary to add four boundary conditions.

2.4. Boundary conditions

The first two boundary conditions are the conditions of the free crack surfaces, i.e. $\sigma_{\varphi\varphi} = \sigma_{r\varphi} = 0$ at $\varphi=\pi$. Hence, two boundary conditions for asymptotic Airy's stress function $f(\varphi)$ are the follows:

$$f = df/d\varphi = 0,\ \varphi=\pi. \tag{11}$$

For mode I or mode II crack tip two additional boundary conditions are the symmetry conditions, i.e. $\partial\sigma_{rr}/\partial\varphi = \sigma_{r\varphi} = 0$ (mode I) or $\sigma_{\varphi\varphi} = \partial\sigma_{r\varphi}/\partial\varphi = 0$ (mode II) at $\varphi=0$. In this case two additional boundary conditions for asymptotic Airy's stress function $f(\varphi)$ are the follows:

$$df/d\varphi = d^3 f/d\varphi^3 = 0,\ \varphi=0 \text{ (mode I) or } f = d^2 f/d\varphi^2 = 0,\ \varphi=0 \text{ (mode II)}. \tag{12}$$

Thus, for Eq. (10a) or Eq. (10b) with boundary conditions (Eq. 11) and (Eq. 12) it is obtained two-point boundary value problem for asymptotic Airy's stress function $f(\varphi)$. This problem can be solved by shooting method [9], when adding to symmetry conditions (Eq. 12) two additional boundary conditions

$$f = c_1, d^2 f/d\varphi^2 = c_2 \text{ (mode I) or } df/d\varphi = c_1, d^3 f/d\varphi^3 = c_2 \text{ (mode II) at } \varphi=0$$

the above-mentioned two-point boundary value problem reduces to the Cauchy problem. The Cauchy problem is usually solved by using the Runge-Kutta method [9], when choosing the values of additional boundary constants c_1, c_2 in such way as to satisfy the main boundary conditions at $\varphi=\pi$ (Eq. 11).

The boundary conditions (Eq. 12), followed from symmetry conditions, does not valid for mixed mode of loading or for inclined crack under tension (Fig. 1). In this case the boundary conditions at $\varphi=0$ can be represented as the follows [7-8]:

$$f = c_0, df/d\varphi = -\lambda c_0 tg\alpha, d^2 f/d\varphi^2 = c_1, d^3 f/d\varphi^3 = c_2,\ \varphi=0. \tag{13}$$

The constant c_0 is determined from some additional condition (named as normalization condition). Traditionally the normalization condition is selected in the form of $\max_{\phi} \sigma_e(\varphi) = 1$ [5-6].

The last two constants c_1, c_2 are sought by shooting method - to choose these constants, so that at $\varphi=\pi$ the boundary conditions (Eq. 11) are valid.

3. Method of solution

To solve this problem let us denote $f(\pi) = f_1(c_1, c_2)$, and $df(\pi)/d\varphi = f_2(c_1, c_2)$. In order to satisfy the main boundary conditions (Eq. 11) it is necessary to find the solution of the following system of two nonlinear algebraic equations:

$$f_1(c_1, c_2) = 0, \quad f_2(c_1, c_2) = 0. \tag{14}$$

It should be noted that the functions $f_1(c_1, c_2)$ and $f_2(c_1, c_2)$ are not given in analytical form, but are found numerically by solving the above-mentioned Cauchy problem for different values of c_1 and c_2. The system of nonlinear equations (Eq. 14) can be solved by the Newton's method. Newton's method is an iterative method for solving the system of nonlinear algebraic equations [9]. Rewrite the general formula of Newton's method for the system (Eq. 14) as the follows:

$$f_{1,1}^k c_1^{k+1} + f_{1,2}^k c_2^{k+1} = f_{1,1}^k c_1^k + f_{1,2}^k c_2^k - f_1^k,$$

$$f_{2,1}^k c_1^{k+1} + f_{2,2}^k c_2^{k+1} = f_{2,1}^k c_1^k + f_{2,2}^k c_2^k - f_2^k,$$

where c_i^k is the value of the parameter c_i at the *k*-th step of iteration, $f_{i,j}^k$ is the value of derivative of function f_i with respect to c_j, computed at the *k*-th step of iteration, *i,j=1,2; k= 1,2,3,…*

To calculate numerically the values of $f_{i,j}^k$ it is used the right finite difference scheme:

$$\begin{aligned} f_{1,1}^k &= (f_1^k(c_1^k + \delta, c_2^k) - f_1^k(c_1^k, c_2^k))/\delta, \\ f_{1,2}^k &= (f_1^k(c_1^k, c_2^k + \delta) - f_1^k(c_1^k, c_2^k))/\delta, \\ f_{2,1}^k &= (f_2^k(c_1^k + \delta, c_2^k) - f_2^k(c_1^k, c_2^k))/\delta, \\ f_{2,2}^k &= (f_2^k(c_1^k, c_2^k + \delta) - f_2^k(c_1^k, c_2^k))/\delta, \end{aligned}$$

where δ *is* a small number.

Thus, to find the next approximation (c_1^{k+1}, c_2^{k+1}) of the parameters c_1 and c_2 it is necessary to integrate three time the Cauchy problem by Runge-Kutta method for the values of parameters

(c_1^k, c_2^k), ($c_1^k + \delta, c_2^k$) and ($c_1^k, c_2^k + \delta$).

The above-described method of solving the nonlinear differential equation (Eq. 10a) or (Eq. 10b) has been tested for correctness and accuracy and compared with some known analytical or numerical results. Thus, in the case $n = 1$ there exist analytical expressions for the stress tensor components for any inclined cracks [1-3]. For mode I ($\alpha = 0$) it is known the analytical result for the case $n = \infty$ and the numerical results for $n > 1$ [5-6]. Testing has shown that for $n = 1$ the value of the relative error does not exceed 10^{-6}. The results of calculations based on above-described method as for the case $n = \infty$ as for $n > 1$ practically coincided with the numerical results from [5-6].

4. Results of calculations

Some results of calculations for stress distribution around the crack tip for plane stress and plane strain conditions are shown on the Figures 2-5.

4.1. Plane stress

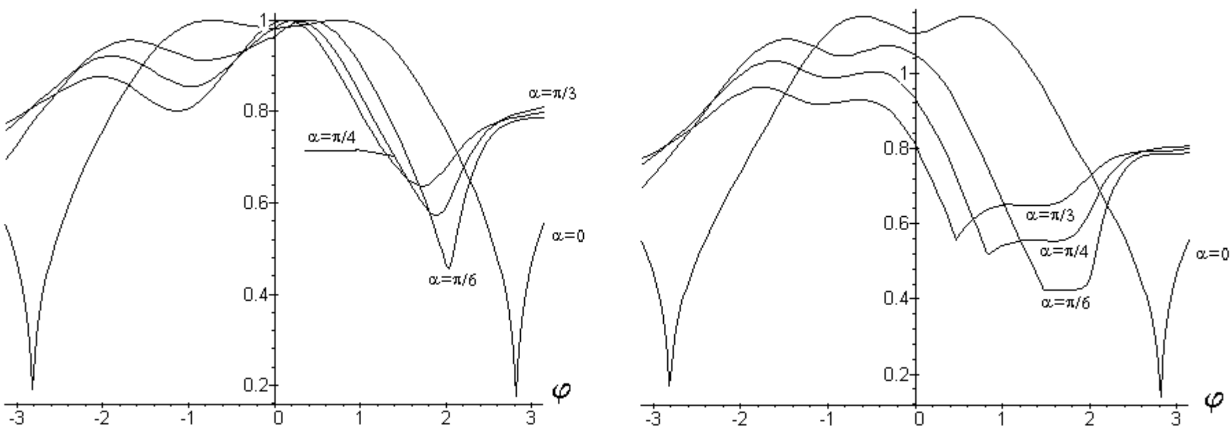

Figure 2. The distribution of the equivalent stress σ_e and the maximum stress $\sigma_{\max}$ around the crack tip for *n=3* (plane stress).

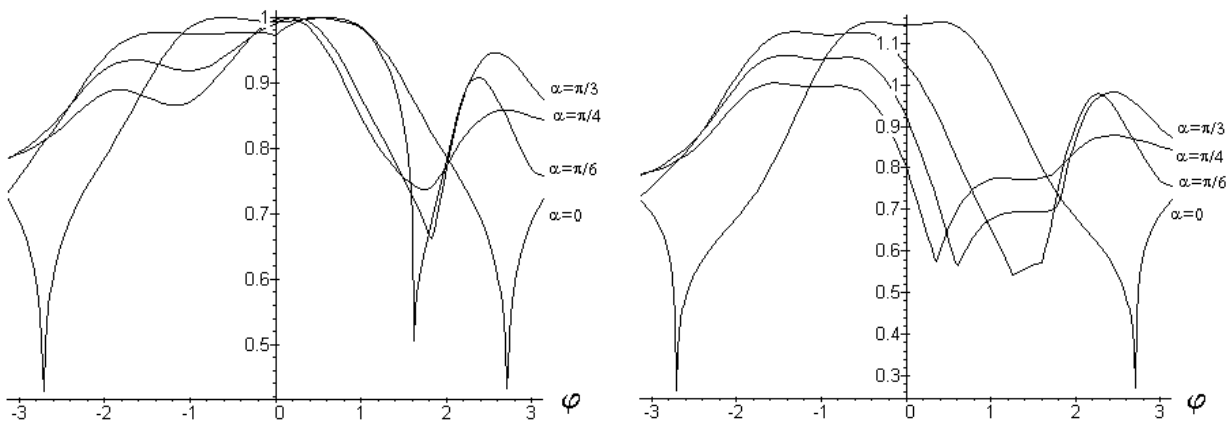

Figure 3. The distribution of the equivalent stress σ_e and the maximum stress $\sigma_{\max}$ around the crack tip for *n=7* (plane stress).

4.2. Plane strain

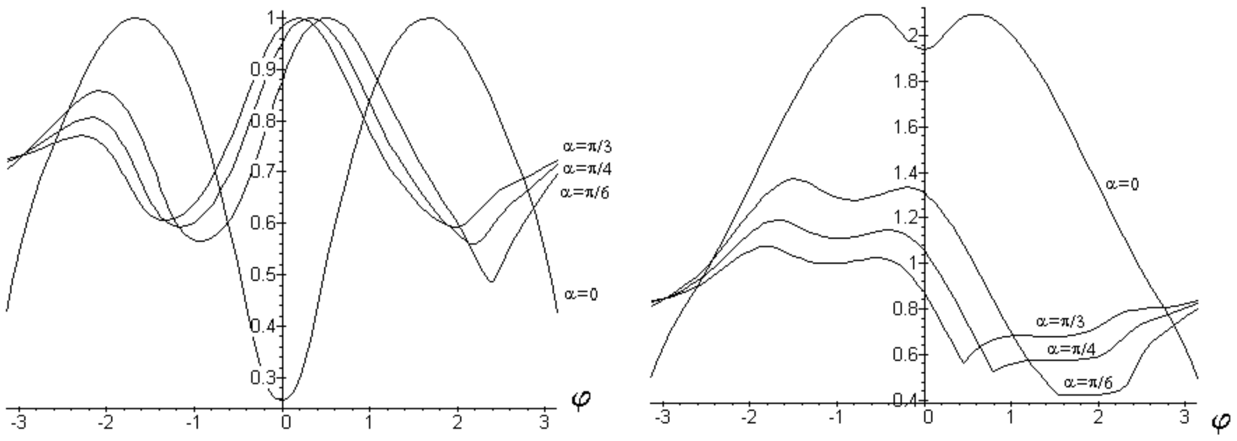

Figure 4. The distribution of the equivalent stress σ_e and the maximum stress $\sigma_{\max}$

around the crack tip for *n=3* (plane strain).

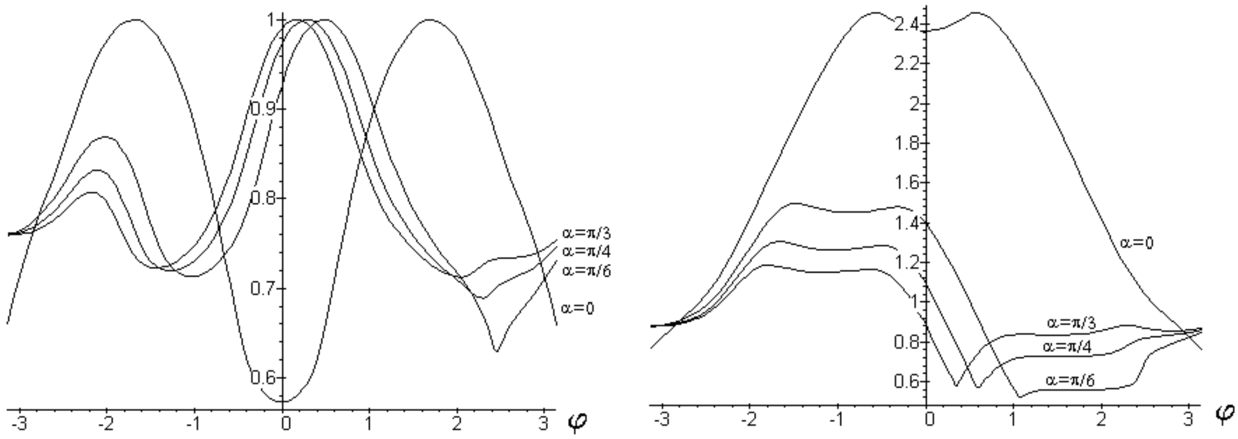

Figure 5. The distribution of the equivalent stress σ_e and the maximum stress $\sigma_{\max}$

around the crack tip for *n=7* (plane strain).

Acknowledgment

Authors are grateful the Russian Foundation for Basic Research for the financial support under Grant 12-08-00390-a.

References

[1] V.I. Astafiev, Yu.N. Radaev, L.V. Stepanova, Non-Linear Fracture Mechanics, Samara State University, Samara, 2001 (in Russian).

[2] L.V. Stepanova, Mathematical Methods in Fracture Mechanics, Fizmatlit, Moscow, 2009 (in Russian).
[3] V.M. Pestrikov, E.M. Morozov, Fracture Mechanics. Course of Lectures, EPS Professia, St. Petersburg, 2012 (in Russian).
[4] G.P. Cherepanov, Mechanics of brittle fracture, Nauka, Moscow, 1974 (in Russian).
[5] J.W. Hutchinson, Singular behavior at the end of tensile crack in a hardening material tip, J. Mech. Phys. Solids, 16, (1968) 13-31.
[6] J.R. Rice, G.F. Rosengren, Plane strain deformation near a crack tip in a power-law hardening material, J. Mech. Phys. Solids, 16, (1968) 32-48.
[7] V.I. Astafiev, A.N. Krutov, Stress distribution near a tip of inclined crack in nonlinear fracture mechanics, Vestnik SamGU, 14, (1999) 56-69 (in Russian).
[8] V.I. Astafiev, A.N. Krutov, Stress distribution near a tip of inclined crack in nonlinear fracture mechanics, Mechanics of Solids, 36, (2001) 101-108.
[9] N.S. Bakhvalov, N.P. Zhidkov, G.M. Kobelkov, Numerical Methods, Nauka, Moscow, 1987 (in Russian).