Fracture damage identification of pile using element strain energy method based on sensitive modals

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Abstract A sensitive identification method for fracture damage identification of pile was established using the rate of change of natural frequency, element modal strain energy method and fracture mechanics theory. According to the structural vibration equation of damaged pile, a structural damage identification equation was deduced which containing modal damage sensitivity factor, fracture injury factor of pile and element modal strain energy change ratio. Sensitive damage identification modals were selected applying modal damage sensitivity factor, then damage warning indicators of pile were established according to the strain energy changes ratio, and the fracture damage location identification of pile was implemented using wavelet analysis techniques. On the basis of the damage location identification, the pile fracture damage size was identified using structural damage identification equation. Pile damages of Wenzhou City Stadium were identified applying the proposed method. The comparation of the results using proposed method and low strain test results verified the correctness and practicability of the proposed method. This study provided a new and efficient method for structural damage identification for pile.

Keywords sensitive modals; element modal strain energy method; Fracture damage identification of pile; low strain test

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1. Introduction

In service process, pile foundation’s quality is influenced by factors related to the conditions of geotechnical engineering, structural design, construction quality, and the external environment, it is difficulty to found quality problems and hard to deal with the accident. For pile foundation, the characteristics of hidden and complexity, the limitations of the existing identification methods, and some other uncertainty factors make its damage detected in both theory and practice to face many problems[1-4]. How to early detect the crack damage of the pile foundation and how to develop the relevant prevention measures become a challenging topics for the structural health evaluators.

A variety of pile detection technology at home and abroad have been developed to solve the pile quality problems in the construction. Chen studied the application of core drilling method for testing the damage of piles[5], but the core drilling method can not accurately judge the fracture and the development of piles foundation in service. Xiao studied the application of ultrasonic transmission method for the pile foundation defect detection[6]. Liu et al. [7]applied the low strain method to detect the defects of piles. Wu et al.[8] proposed a approach of foundation piles test which combines application of core drilling method , ultrasonic transmission method and low strain integrity testing. Actually the results demonstrated the reliability of the technical advantages and the measures. The above studies are directed in the pile foundation detection of bridges or buildings under construction. Pile foundation construction of bridges or buildings, when the need to determine its integrity, due to the limitations of the superstructure, the usual pile testing methods are not fully applicable.

Johnson et al. [9] and Luo et al.[10]gradually proposed the low strain dual velocity method, super shock, equality seismic detection methods to detect damage of the pile foundation in service. In 1995, Japanese scientists explore the application hole camera technology roughly detect the foundation piles cracking and damage after southern Hyogo earthquake, but they did not achieve good results for various reasons[11]. Yuan et al. [12] determined the integrity of the bridge piles in service, applying combined horizontal impact load displacement signal collected under static loading. Qi proposed the application of full-bore wall imaging technology to detect damage of the pile foundation in service, and achieved very good detection accuracy[13]. But the piles will produce some damage when the cores of piles are drilling. In order to weaken the flat-slab superstructure on the foundation pile test signal, Zhang et al. [14] used wavelet analysis method to eliminate the interference signal, and the applied wavelet analysis technology to detect the high-pile integrity.

In summary, scientists at home and abroad have done a series of studies on the pile injury, and they have achieved a certain amount of research. However, the in-service structure damage identification of pile foundation is only in the exploratory stage, a lot of work needs to be further in-depth research if the methods would be used in engineering practice. Especially for the small initial crack damage detection of pile foundation, it need more theoretically further analysis and further explore in practice. This article established a recognition method of small injury on the pile positioning and degree. The method take full use the advantage of the natural frequency on the integrity of the damage to determine, the sensitivity of the element modal strain energy method on small injuries, the efficient local analysis capabilities of time domain wavelet analysis. Finally, the engineering application examples are used to verify the accuracy and efficiency of the damage identification method for the piles in service.

2. Basic theory

2.1. Structural vibration eigenvalue equation with fracture damage

The general performance of the structure fracture damage is that the local stiffness of the structure reduced, and it has nothing to do with the quality of the structure. Therefore, according to the perturbation theory, the structural vibration injury eigenvalue equation is as follows[15]:

$$[(K+\Delta K)-(\lambda_i+\Delta\lambda_i)M)](\phi_i+\Delta\phi_i)=0 \quad (1)$$

Where,
$$\Delta K=\sum_{j=1}^{n}\Delta K_j=-\sum_{j=1}^{n}\alpha_j K_j \quad （0\le\alpha_j\le1） \quad (2)$$

$$\Delta\lambda_i=\Delta\omega_i^2=\phi_i^T\Delta K\phi_i \quad (3)$$

M,K represent the mass matrix and stiffness matrix of the structure respectively; ϕ_i represents the i^{th} order modal vector of the structure; λ_i represents the eigenvalue of i^{th} modal of the vibration system, $\lambda_i=\omega_i^2=(2\pi f)^2$; ΔK represents Changes of structural stiffness; α_j represents j^{th} element damage factor of the structure; K_j represents structural stiffness matrix of the j^{th} element before damage; n represents the total number of elements of the structure.

Expand the Eq.(1) and finish structural vibration eigenvalue equation, the Eq.(1) of the structure can be expressed as:

$$\phi_i^T\Delta K\phi_i+\delta_i\phi_i^T K\phi_i+\phi_i^T\Delta K\Delta\phi_i+\delta_i\phi_i^T K\Delta\phi_i=0 \quad (4)$$

Where, $\delta_i=-\Delta\lambda_i/\lambda_i$ is structure Eigenvalue change rate, it is used to determine sensitivity of the i^{th} modal structural damage detection for the structure; therefore, it is called modal damage sensitivity factor.

2.2. Structural damage identification equation

According to the principle of energy balance, cracks release of elastic strain energy into plastic strain energy and surface energy when the pile cracks, thus contributing to the crack propagation, eventually lead to structural strain can be reduced. Based on this, the Eq. (4) can be expressed in the form of the strain energy. For piles structures, define the i^{th} modal j^{th} element strain energy as:

$$MSE_{ij} = \phi_i^T K_j \phi_i \qquad （5）$$

$$MSE_{ij}^d = (\phi_i^d)^T K_j^d \phi_i^d = (1-\alpha_j)(\phi_i^d)^T K_j \phi_i^d \qquad （6）$$

Where, the superscript "d" represents the damaged structure.
According to the above formulas we can get:

$$\phi_i^T K \phi_i = \sum_{j=1}^{n} \phi_i^T K_j \phi_i = \sum_{j=1}^{n} MSE_{ij} = MSE_i \qquad （7）$$

$$\phi_i^T \Delta K \phi_i = \sum_{j=1}^{n} \phi_i^T \Delta K_j \phi_i = -\sum_{j=1}^{n} \alpha_j \phi_i^T K_j \phi_i = -\sum_{j=1}^{n} \alpha_j MSE_{ij} \qquad （8）$$

Ignore second-order or more high-end items, we can get:

$$\begin{aligned} \phi_i^T K \Delta\phi_i &= \sum_{j=1}^{n} \phi_i^T K_j \Delta\phi_i = \sum_{j=1}^{n} [(\phi_i^d)^T K_j \phi_i^d - \phi_i^T K_j \phi_i + (\phi_i^d)^T \Delta K_j \phi_i^d] \\ &= \sum_{j=1}^{n} [(1-\alpha_j)(\phi_i^d)^T K_j \phi_i^d - \phi_i^T K_j \phi_i] = \sum_{j=1}^{n} (MSE_{ij}^d - MSE_{ij}) \end{aligned} \qquad （9）$$

$$\phi_i^T \Delta K \Delta\phi_i = \sum_{j=1}^{n} \phi_i^T \Delta K_j \Delta\phi_i = -\sum_{j=1}^{n} \alpha_j \phi_i^T K_j \Delta\phi_i = -\sum_{j=1}^{n} \alpha_j (MSE_{ij}^d - MSE_j) \qquad （10）$$

Eq.(7), Eq. (8), Eq. (9) and Eq. (10) substituted into Eq. (4), Eq. (4) can be expressed as:

$$\delta_i \cdot MSE_i^d = \sum_{j=1}^{n} (\alpha_j \cdot MSE_{ij}^d) \qquad （11）$$

Where, $MSE_i^d = \sum_{j=1}^{n} MSE_{ij}^d$, the total strain energy of the i^{th} modal of Injury structure.

The formula (11) is structural damage identification equation, which is composed by modal damage sensitivity factor, modal strain energy of injury structure and the element damage factor.The strain energy is more sensitive to the small damage of the structure[16], Measuring modal the damage sensitivity factor is relatively easy, and it has a high measuring accuracy[17]. So the Eq.(11) can be used to identify small damage of the pile structure with higher accuracy.

δ_i is the indicator used to determine the modal sensitivity of structural damage, the value of δ_i is the larger, the modal damage identification of structural damage is the more sensitive, or vice versa. Therefore, according to the modal damage sensitivity factor δ_i, we can select the mode used for structural damage identification. Practical application is setting a threshold based on the environment and damage of the structure, it is the efficient mode when the threshold value greater than the set threshold value.

2.3. Element modal strain energy calculation

For the pile structure, in the case of element is sufficiently small, element modal strain energy can be represented by the formula as follow:

$$MSE_{ij} = \phi_i^T K_j \phi_i = \frac{1}{2}\int_{b_j}^{b_{j+1}} (EI)_j \left(\frac{\partial^2 \phi_i}{\partial z^2}\right)^2 dz \tag{15}$$

$$MSE_{ij}^d = (1-\alpha_j)(\phi_i^d)^T K_j \phi_i^d = \frac{1}{2}\int_{b_j}^{b_{j+1}} (EI)_j^d \left(\frac{\partial^2 \phi_i^d}{\partial z^2}\right)^2 dz \tag{16}$$

$(EI)_j$ and $(EI)_j^d$ represent the structural flexural rigidity of j^{th} element before and after damage respectively. b_j and b_{j+1} represent z coordinates of node j and node $j+1$ respectively.

The bending stiffness $(EI)_j^d$ of structure with damage is unknown for experimental modal analysis, it can be substituted by bending stiffness before injury ($(EI)_j$)in the case of small injuries. Consider the selected elements is relatively small, the bending stiffness $(EI)_j$ of the j^{th} element can be approximated as a constant, it can be mentioned the integral sign. Eq. (15) and (16) can be rewritten as:

$$MSE_{ij} = \frac{1}{2}(EI)_j \int_{b_j}^{b_{j+1}} \left(\frac{\partial^2 \phi_i}{\partial z^2}\right)^2 dz \tag{17}$$

$$MSE_{ij}^d = \frac{1}{2}(EI)_j \int_{b_j}^{b_{j+1}} \left(\frac{\partial^2 \phi_i^d}{\partial z^2}\right)^2 dz \tag{18}$$

$\left(\frac{\partial^2 \phi_i}{\partial z^2}\right)^2$ in Eq. (17) can be instead by the mean value of $(\phi_{ij}'')^2$ and $(\phi_{i(j+1)}'')^2$, so Eq.(17) can be expressed as:

$$MSE_{ij} = \frac{1}{4}(EI)_j[(\phi_{ij}'')^2 + (\phi_{i(j+1)}'')^2](b_j + b_{j+1}) \tag{19}$$

Similarly, the equation (18) can be rewritten as:

$$MSE_{ij}^d = \frac{1}{4}(EI)_j\{[(\phi_{ij}^d)'']^2 + [(\phi_{i(j+1)}^d)'']^2\}(b_j + b_{j+1}) \tag{20}$$

Where, $(\phi_{ij}'')^2$、$(\phi_{i(j+1)}'')^2$、$[(\phi_{ij}^d)'']^2$ and $[(\phi_{i(j+1)}^d)'']^2$ can be got by measuring displacement mode shapes.

2.4.Damage location determination

The original signal of injury can be got by the difference of element modal strain energy of the before and after damage:

$$f_j(z) = MSE_{ij}^d - MSE_{ij} \tag{21}$$

Fitting signal $f_j(z)$ using cubic spline interpolation, the signal are transformed by wavelet function, we get the transform coefficients which can be used to detect the damage location.

$$DI_{ij} = C(a,b)_{i,j} \tag{22}$$

$C(a,b)_{i,j}$ represents the j^{th} element transform coefficient, DI_{ij} represent element injury positioning indicators. So index DI_{ij} can be used to determine the damage element, if the index DI_{ij} is a large value, we can judge the j^{th} element occur injury.

In order to reduce the impact of random noise from the test mode shapes , multi-order efficient modality are used to diagnose structural damage location:

$$DI_j = \frac{1}{N}\sum_{m} DI_{ij} \tag*{（23）}$$

Where, N is the efficient modal number of selection. A given threshold value of DI_j to determine whether the structure has occurred injury can be applied in practical applications.

2.5.Degree of damage calculation

After the injury element is determined, the formula (11) can be used to calculated the degree of element damage. The element damage factor α_j can be used to represent the degree of damage. If there is only one location injury, assuming k element injury, then $\alpha_j = 0(j \neq k)$, only $\alpha_k \neq 0$. The Eq.(11) can be expressed:

$$\alpha_k = \frac{\delta_i \cdot MSE_i^d}{MSE_{ik}^d} \tag*{（24）}$$

If there are multiple locations injury, assuming m element（k_1、 k_2、 $\cdots k_m$） injury $(m \leq n)$, then need m efficient mode（q_1、q_2、$\cdots q_m$）to be calculated. m equations can solve the corresponding damage factor α. The equations are as follows:

$$\overline{\delta} \cdot MSED = \overline{\alpha} \cdot MSEDD \tag*{(25)}$$

Where,

$$\overline{\delta} = \begin{bmatrix} \delta_{q_1} & \delta_{q_2} & \delta_{q_3} & \ldots & \delta_{q_m} \end{bmatrix}$$

$$MSED = \begin{bmatrix} MSE_{q_1}^d \\ MSE_{q_2}^d \\ MSE_{q_3}^d \\ \ldots \\ MSE_{q_m}^d \end{bmatrix}$$

$$\overline{\alpha} = \begin{bmatrix} \alpha_{k_1} & \alpha_{k_2} & \alpha_{k_3} & \ldots & \alpha_{k_m} \end{bmatrix}$$

$$MSEDD = \begin{bmatrix} MSE_{q_1k_1} & MSE_{q_2k_1} & MSE_{q_3k_1} & \ldots & MSE_{q_mk_1} \\ MSE_{q_1k_2} & MSE_{q_2k_2} & MSE_{q_3k_2} & \ldots & MSE_{q_mk_2} \\ MSE_{q_1k_3} & MSE_{q_2k_3} & MSE_{q_3k_3} & \ldots & MSE_{q_mk_3} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ MSE_{q_1k_m} & MSE_{q_2k_m} & MSE_{q_3k_m} & \ldots & MSE_{q_mk_m} \end{bmatrix}$$

If $p < m$, extended modal can be applied to the calculation.

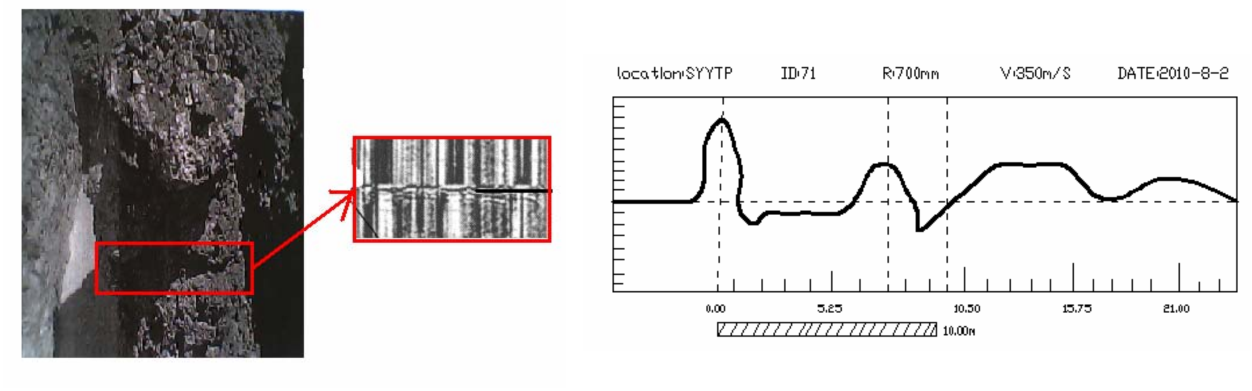
3. Instance of verification

3.1. Project overview

Artificial hollowed piles are used to pile foundation of Ouhai Sports Center Stadium. Effective pile length is 4.00m ~ 30m, pile diameter is 600mm ~ 1000mm, the pile concrete is C25. The project total pile is 129, according to the verify results of low strain detection and excavation, 9.6% of the damage occurred at 7.4m from the top of the pile of NO.71 piles. No. 71 piles pile length is 21m, diameter is 1m. Real damage of No.71 pile is shown in Fig.1.

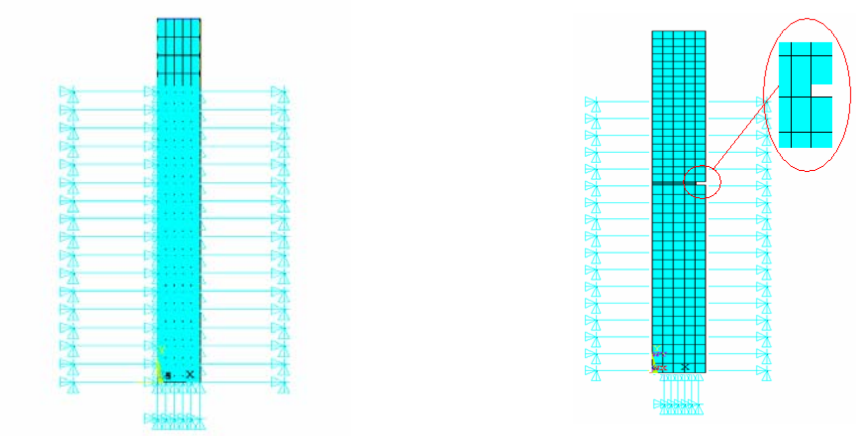
3.2.Finite Element Modeling

Applying of finite element software ANSYS, according to the design drawings, we establish finite element model of the role of interaction between pile and soil for NO.71 pile (Fig.2(a)). According to actual damage on the No. 71 pile, pile model with cracks was established at 7.4 meters from the top of the pile. Crack depth is 9.6% of the diameter of pile foundation (Fig.2(b)). The element type of pile foundation model is SOLID45. Pile-soil interaction applies the nonlinearity the spring element COMBIN139 to simulate. In order to establish the finite element computation model which are in line with the environment of the actual engineering, model updating technology based on the measured data was used to improve the finite element model.



（a）Site excavation detection （b）Detection of low strain

Figure 1 The actual damage location of No. 71 piles



（a）Before damage （b）After damage

Figure 2 Finite element model of pile-soil interaction

3.3.Efficient mode selection

The rate of change of the natural frequencies before and after the injury is shown in Fig. 3, efficient frequency threshold is set as $\delta_i \geq 0.4\%$. As Fig. 3 shows, the damage determination efficient modes are mode 1 and mode 4.

3.4. Damage location identification

Information function is established using the changes of element strain energy before and after injury, wavelet transform is used to determine the parameters (DI) for the location of damage. DI_j of efficient modal shown in Fig. 4. By setting the threshold （ $DI > 2$ ）, we can clearly determine that the element 55 to 59 crack damage occurs. This is is very consistent with the actual damage location. Results of location damage identification shown as table 1. Identification error is 3%, it proved that the method of damage location identification has a higher accuracy.

3.5. Identification of injury severity

Element damage factor is calculated applying Eq.25, the results of calculation shown as table 1. As shown in Table 1, actual injury and calculation of injury is very close. Identification error is 6.2%, it proved that the method of damage severity identification has a higher accuracy.

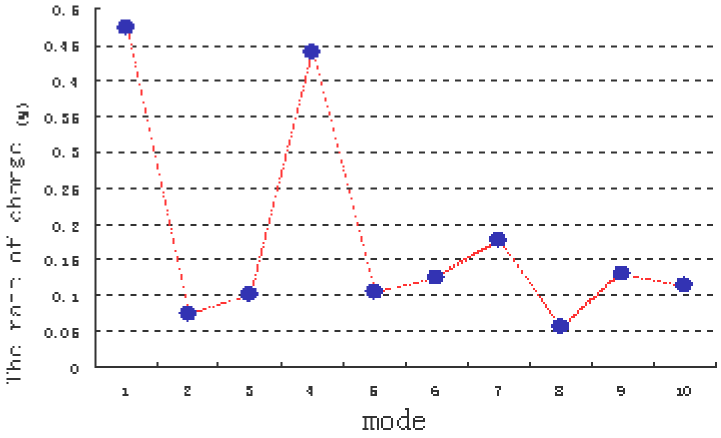


Figure 3 The rate of change of the natural frequency

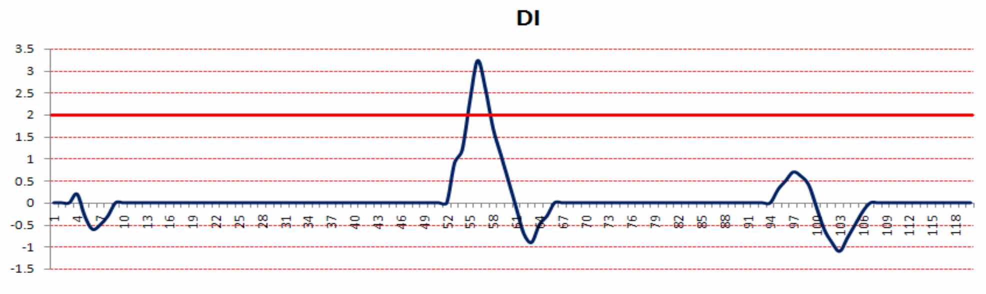


Figure 4　DI value for each element applying of efficient modal

Table 1 Results of damage identification

	Actual injury	Low strain method/error	New method/error
damage location	7.4m	6.71m/9.3%	7.08m/4.3%
damage severity	9.6%	Small damage	10.2%/6.2%

4 Conclusion

This paper selected efficient modal for damage identification applying the change rate of eigenvalues firstly, then established the small injury accurate recognition method based on the application of sensitivity of element modal strain energy to small structural injury and efficient local analysis capabilities of wavelet analysis. The efficiency and accuracy of the proposed method is verified by small cracks damage identification of the pile of Ouhai Sports Center. The concrete conclusions are as follows:

(1) Fracture identification method proposed in this paper for the pile foundation can effectively identify the fracture damage location of the hidden structures in service, this method has a high recognition accuracy.

(2) Fracture identification method proposed in this paper for the pile foundation can effectively identify the fracture damage severity of the hidden structures in service; it is able to quantify the degree of damage; it provides an important reference for the pile foundation fracture injury assessment.

(3) The method presented in this paper provides a new approach to concealed structure damage identification.

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