# A PARTIALLY PERMEABLE MIXED-MODE CRACK EMBEDDED IN A FUNCTIONALLY GRADED MAGNETO ELECTRO ELASTIC LAYER

M. Rekik[1], S. El-Borgi[1,2*] and Z. Ounaies[3]

[1]*Applied Mechanics and Systems Research Laboratory, Tunisia Polytechnic School, University of Carthage, B.P. 743, La Marsa 2078, Tunisia*

[2]*Texas A&M University at Qatar, Engineering Building, Mechanical Engineering Program, PO. Box 23874 Education City, Doha, Qatar*

[3]*Department of Mechanical Engineering, The Pennsylvania State University, 157B Hammond Building, University Park, PA 16802, USA*

*mongi.rekik@hotmail.com*, *sami.el_borgi@qatar.tamu.edu*, *zxo100@engr.psu.edu*

**Abstract**

This ~~work~~ paper considers the problem of a partially permeable mixed-mode crack embedded in a graded magneto electro elastic layer subjected to magneto electro mechanical loads. The medium is graded in the direction orthogonal to the crack plane and is modeled as a non homogeneous medium with anisotropic constitutive laws. Using Fourier transform, the resulting magneto electro elasticity equations are converted analytically into singular integral equations which are then solved numerically to yield the crack-tip mode I and II stress, electric displacement and magnetic induction intensity factors. The main objective of this work is to study the influence of material non homogeneity, crack position and magneto electric permeabilities on the fields' intensity factors for the purpose of gaining better understanding on the behavior of fractured graded magneto electro elastic layers. Results showed that fields' intensity factors increase with nonhomogeneity, and decrease with magnetic and electric permeabilities, and as the crack become closer to the layer's center.

## Introduction

Smart structures possessing the ability of magneto electro mechanical energy conversion have found increasing application in several engineering fields such as magnetic field probes, electric packaging, acoustic, hydrophones, medical ultrasonic imaging, microwave electronics, optoelectronics, electronic instrumentation, transducers, sensors and actuators. Research has focused on the use of Functionally Graded Magneto Electro Elastic Materials (FGMEEM) in smart structures to improve their performance. But, the manufacturing of FGMEEMs may lead to cracks that can eventually propagate and cause premature failure. Therefore, it is of a great importance to study the fracture behavior of magneto electro elastic composites.

A number of authors considered FGMEEM crack problems. Ma et al (2007) studied the mode III crack problem in a functionally graded magneto electro elastic strip accounting for ideal crack surface magneto electric permeability. Ma et al (2009) considered the problem of a surface crack in a functionally graded magneto electro elastic coating homogeneous elastic substrate subjected to anti-plane mechanical and in plane magneto electrical loading for the ideal crack surface magneto electric permeability. Zhou et al (2004) examined the problem of two parallel symmetric permeable cracks in functionally graded materials under anti-plane shear loading. Feng et al (2007) analyzed the dynamic behavior of magneto electrically impermeable cracks in functionally graded magneto electro elastic plates. Feng et al (2006) studied the dynamic problem of a crack embedded in a graded magneto electro elastic strip assuming ideal crack surface permeability. Jun (2007) examined the scattering of harmonic anti-plane shear stress waves by a crack in functionally graded magneto electro elastic materials assuming purely permeable crack surfaces. Zhou et al (2008) solved the mode I crack problem in a FGMEEM infinite medium assuming air permeability within the crack. Li et al (2008) considered the anti-plane problem of a permeable crack intersecting the interface between two FGMEEM layers. Li et al (2008) analyzed the anti-plane problem of a crack in the interface of tow symmetrically bonded FGMEEM assuming a linear variation of the magneto electromechanical properties. Guo et al (2009) solved the anti-plane problem of a crack in bonded FGMEEM strip sandwiched between two functionally graded strips assuming ideal magneto electrical permeability on the crack faces. Rekik et al (2012) considered the problem of magneto electrically impermeable crack embedded in a graded infinite medium subjected to magneto electro mechanical loading.

The present work consists of studying the plane problem of a partially magneto electrically permeable crack embedded in a graded magneto electro elastic layer. The applied magneto electro mechanical loading will give rise to coupled fields intensity factors; namely, mode I and II stress, electric displacement and magnetic induction intensity factors denoted respectively $k_1, k_2, k_D$ and $k_B$. To the best of the authors' knowledge, this problem was not considered in the open literature to-date.

**Problem description and formulation**

As shown in Figure 1, the problem under consideration consists of a functionally graded magneto electro elastic layer containing an embedded crack of length *2a* along the *x*-axis. The crack surfaces are assumed to be partial magneto electrically permeable using the magnetic and electric permeability parameters $k_m$ and $k_e$ varying in between 0 and 1 representing the cases of completely impermeable and completely permeable crack surfaces, respectively. Consequently, crack faces are subjected to mechanical tangential and normal tractions $\omega_1(x)$ and $\omega_2(x)$, electric displacement $(1-k_e)E(x)$, and magnetic induction $(1-k_m)B(x)$. The graded layer is modeled as a nonhomogeneous elastic medium with magneto electromechanical properties varying in the depth direction (y-coordinate) as follows:

$$(c_{11}, c_{13}, c_{33}, c_{44}) = (c_{110}, c_{130}, c_{330}, c_{440})\,\mathrm{e}^{\beta y}, \quad (e_{15}, e_{31}, e_{33}) = (e_{150}, e_{310}, e_{330})\,\mathrm{e}^{\beta y}, \quad \forall y, \tag{1a,b}$$

$$(f_{15}, f_{31}, f_{33}) = (f_{150}, f_{310}, f_{330})\,\mathrm{e}^{\beta y}, \quad (\varepsilon_{11}, \varepsilon_{33}) = (\varepsilon_{110}, \varepsilon_{330})\,\mathrm{e}^{\beta y}, \quad \forall y, \tag{1c,d}$$

$$(g_{11}, g_{33}) = (g_{110}, g_{330})\,\mathrm{e}^{\beta y}, \quad (\mu_{11}, \mu_{33}) = (\mu_{110}, \mu_{330})\,\mathrm{e}^{\beta y}, \quad \forall y. \tag{1e,f}$$

where $c_{ij0}, e_{ij0}, f_{ij0}, \varepsilon_{ij0}, g_{ij0}, \mu_{ij0}$ are the value of the magneto electromechanical coefficient in the FGMEEM layer along the axis $y = 0$ and $\beta$ is the nonhomogeneity parameter controlling the variation of these coefficient in the graded layer.

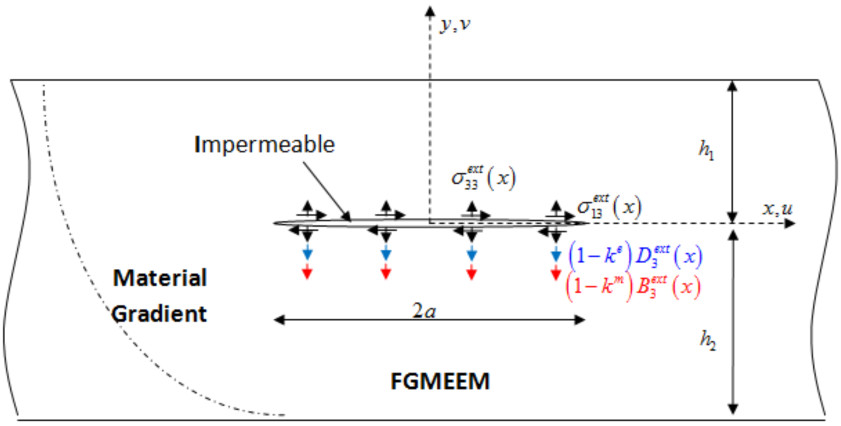


**Figure 1.** Geometry and loading of the crack problem

Neglecting body forces and local electric charge, assuming small deformations and considering linear constitutive laws, the basic equations consisting of equilibrium equations and Gauss's laws for electricity and magnetism can be combined, resulting in the following governing magneto electro elasticity equations:

$$c_{11}\frac{\partial^2 u}{\partial x^2} + c_{33}\left(\frac{\partial^2 u}{\partial y^2} + \beta\frac{\partial u}{\partial y}\right) + c_{12}\frac{\partial^2 v}{\partial x \partial y} + c_{33}\left(\frac{\partial^2 v}{\partial x \partial y} + \beta\frac{\partial v}{\partial x}\right) + e_{21}\frac{\partial^2 \phi}{\partial x \partial y} + e_{13}\left(\frac{\partial^2 \phi}{\partial x \partial y} + \beta\frac{\partial \phi}{\partial x}\right) + f_{21}\frac{\partial^2 \psi}{\partial x \partial y} + f_{13}\left(\frac{\partial^2 \psi}{\partial x \partial y} + \beta\frac{\partial \psi}{\partial x}\right) = 0, \tag{2a}$$

$$c_{33}\frac{\partial^2 u}{\partial x \partial y} + c_{12}\left(\frac{\partial^2 u}{\partial x \partial y} + \beta\frac{\partial u}{\partial x}\right) + c_{33}\frac{\partial^2 v}{\partial x^2} + c_{22}\left(\frac{\partial^2 v}{\partial y^2} + \beta\frac{\partial v}{\partial y}\right) + e_{13}\frac{\partial^2 \phi}{\partial x^2} + e_{22}\left(\frac{\partial^2 \phi}{\partial y^2} + \beta\frac{\partial \phi}{\partial y}\right) + f_{13}\frac{\partial^2 \psi}{\partial x^2} + f_{22}\left(\frac{\partial^2 \psi}{\partial y^2} + \beta\frac{\partial \psi}{\partial y}\right) = 0, \tag{2b}$$

$$e_{13}\frac{\partial^2 u}{\partial x \partial y} + e_{21}\left(\frac{\partial^2 u}{\partial x \partial y} + \beta\frac{\partial u}{\partial x}\right) + e_{13}\frac{\partial^2 v}{\partial x^2} + e_{22}\left(\frac{\partial^2 v}{\partial y^2} + \beta\frac{\partial v}{\partial y}\right) - \varepsilon_{11}\frac{\partial^2 \phi}{\partial x^2} - \varepsilon_{22}\left(\frac{\partial^2 \phi}{\partial y^2} + \beta\frac{\partial \phi}{\partial y}\right) - g_{11}\frac{\partial^2 \psi}{\partial x^2} - g_{22}\left(\frac{\partial^2 \psi}{\partial y^2} + \beta\frac{\partial \psi}{\partial y}\right) = 0, \tag{2c}$$

$$f_{13}\frac{\partial^2 u}{\partial x\partial y}+f_{21}\left(\frac{\partial^2 u}{\partial x\partial y}+\beta\frac{\partial u}{\partial x}\right)+f_{13}\frac{\partial^2 v}{\partial x^2}+f_{22}\left(\frac{\partial^2 v}{\partial y^2}+\beta\frac{\partial v}{\partial y}\right)-g_{11}\frac{\partial^2\phi}{\partial x^2}-g_{22}\left(\frac{\partial^2\phi}{\partial y^2}+\beta\frac{\partial\phi}{\partial y}\right)-\mu_{11}\frac{\partial^2\psi}{\partial x^2}-\mu_{22}\left(\frac{\partial^2\psi}{\partial y^2}+\beta\frac{\partial\psi}{\partial y}\right)=0, \tag{2d}$$

where $u$ and $v$ are, respectively, the x and y components of the mechanical displacement vector, $\phi$ and $\psi$ are, respectively, the electric and magnetic potentials.

The above magneto electro elasticity equations are subjected to the following boundary conditions:

$$\sigma_{13}\left(x,0^+\right)=\sigma_{13}^{ext}\left(x\right),\quad \sigma_{33}\left(x,0^+\right)=\sigma_{33}^{ext}\left(x\right),\quad |x|\le a, \tag{3a,b}$$

$$D_3\left(x,0^+\right)=\left(1-k^e\right)D_3^{ext}\left(x\right),\quad B_3\left(x,0^+\right)=\left(1-k^m\right)B_3^{ext}\left(x\right),\quad |x|\le a, \tag{3c,d}$$

$$\sigma_{13}\left(x,0^+\right)=\sigma_{13}\left(x,0^-\right),\quad \sigma_{33}\left(x,0^+\right)=\sigma_{33}\left(x,0^-\right),\quad \forall x, \tag{4a,b}$$

$$D_3\left(x,0^+\right)=D_3\left(x,0^-\right),\quad B_3\left(x,0^+\right)=B_3\left(x,0^-\right),\quad \forall x, \tag{4c,d}$$

$$u\left(x,0^+\right)=u\left(x,0^-\right),\quad v\left(x,0^+\right)=v\left(x,0^-\right),\quad |x|\ge a, \tag{5a,b}$$

$$\phi\left(x,0^+\right)=\phi\left(x,0^-\right),\quad \psi\left(x,0^+\right)=\psi\left(x,0^-\right),\quad |x|\ge a, \tag{5c,d}$$

$$\sigma_{13}\left(x,h_1\right)=0,\quad \sigma_{33}\left(x,h_1\right)=0,\quad \forall x, \tag{6a,b}$$

$$D_3\left(x,h_1\right)=0,\quad B_3\left(x,h_1\right)=0,\quad \forall x, \tag{6c,d}$$

$$\sigma_{13}\left(x,-h_2\right)=0,\quad \sigma_{33}\left(x,-h_2\right)=0,\quad \forall x, \tag{7a,b}$$

$$D_3\left(x,-h_2\right)=0,\quad B_3\left(x,-h_2\right)=0,\quad \forall x, \tag{7c,d}$$

Eqs. (3a-d) describe the applied magneto electro mechanical loadings on the crack faces. Eqs. (4a-d) represent the continuity of stresses, electric displacement and magnetic fields along the crack plane. Eqs. (5a-d) describe the continuity of the mechanical displacement and the magnetic and electric potentials outside the crack. Eqs. (6a-d) and (7a-d) represent the free layer' surfaces boundary conditions.

**Singular integral equations and their solutions**

The magneto electro elasticity equations (2a-d) are solved using Fourier transform to yield the mechanical displacement and electric and magnetic potentials in the composite medium. The density functions which represent the discontinuity of the mechanical displacement, electric and magnetic fields across the crack are now introduced

$$\varphi_1\left(x\right)=\frac{\partial\left(u^+-u^-\right)}{\partial x},\qquad \varphi_2\left(x\right)=\frac{\partial\left(v^+-v^-\right)}{\partial x}, \tag{8a,b}$$

$$\varphi_3\left(x\right)=\frac{\partial\left(\phi^+-\phi^-\right)}{\partial x},\qquad \varphi_4\left(x\right)=\frac{\partial\left(\psi^+-\psi^-\right)}{\partial x}. \tag{8c,d}$$

Applying the boundary conditions and after a lengthy analysis, we obtain four coupled singular integral equations in which the unknowns are the density functions $\varphi_1,\varphi_2,\varphi_3$ and $\varphi_4$. After extracting the Cauchy and logarithmic singularities from the kernels, the four equations take the following form:

$$\begin{aligned}&\frac{k_{11}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_u\left(t\right)dt+\frac{1}{\pi}\int_{-a}^{a}k_{11}\left(t,x\right)\omega_u\left(t\right)dt-\frac{k_{12}^1}{\pi}\int_{-a}^{a}\beta\ln|t-x|\omega_v\left(t\right)dt+\\&\frac{1}{\pi}\int_{-a}^{a}k_{12}\left(t,x\right)\omega_v\left(t\right)dt-\frac{k_{13}^1}{\pi}\int_{-a}^{a}\beta\ln|t-x|\omega_\phi\left(t\right)dt+\frac{1}{\pi}\int_{-a}^{a}k_{13}\left(t,x\right)\omega_\phi\left(t\right)dt-\\&\frac{k_{14}^1}{\pi}\int_{-a}^{a}\beta\ln|t-x|\omega_\psi\left(t\right)dt+\frac{1}{\pi}\int_{-a}^{a}k_{14}\left(t,x\right)\omega_\psi\left(t\right)dt=\sigma_{13}^{ext}\left(x\right),\qquad |x|\le a,\end{aligned} \tag{9a}$$

$$-\frac{k_{21}^1}{\pi}\int_{-a}^{a}\beta\ln|t-x|\omega_u(t)dt+\frac{1}{\pi}\int_{-a}^{a}k_{21}(t,x)\omega_u(t)dt+\frac{k_{22}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_v(t)dt+$$
$$\frac{1}{\pi}\int_{-a}^{a}k_{22}(t,x)\omega_v(t)dt+\frac{k_{23}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_\phi(t)dt+\frac{1}{\pi}\int_{-a}^{a}k_{23}(t,x)\omega_\phi(t)dt+$$
$$\frac{k_{24}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_\psi(t)dt+\frac{1}{\pi}\int_{-a}^{a}k_{24}(t,x)\omega_\psi(t)dt=\sigma_{33}^{ext}(x), \qquad |x|\le a, \tag{9b}$$

$$-\frac{k_{31}^1}{\pi}\int_{-a}^{a}\beta\ln|t-x|\omega_u(t)dt+\frac{1}{\pi}\int_{-a}^{a}k_{31}(t,x)\omega_u(t)dt+\frac{k_{32}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_v(t)dt+$$
$$\frac{1}{\pi}\int_{-a}^{a}k_{32}(t,x)\omega_v(t)dt+\frac{k_{33}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_\phi(t)dt+\frac{1}{\pi}\int_{-a}^{a}k_{33}(t,x)\omega_\phi(t)dt+$$
$$\frac{k_{34}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_\psi(t)dt+\frac{1}{\pi}\int_{-a}^{a}k_{34}(t,x)\omega_\psi(t)dt=\left(1-k^e\right)D_3^{ext}(x), \qquad |x|\le a, \tag{9c}$$

$$-\frac{k_{41}^1}{\pi}\int_{-a}^{a}\beta\ln|t-x|\omega_u(t)dt+\frac{1}{\pi}\int_{-a}^{a}k_{41}(t,x)\omega_u(t)dt+\frac{k_{42}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_v(t)dt+$$
$$\frac{1}{\pi}\int_{-a}^{a}k_{42}(t,x)\omega_v(t)dt+\frac{k_{43}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_\phi(t)dt+\frac{1}{\pi}\int_{-a}^{a}k_{43}(t,x)\omega_\phi(t)dt+$$
$$\frac{k_{44}^0}{\pi}\int_{-a}^{a}\frac{1}{t-x}\omega_\psi(t)dt+\frac{1}{\pi}\int_{-a}^{a}k_{44}(t,x)\omega_\psi(t)dt=\left(1-k^m\right)B_3^{ext}(x), \qquad |x|\le a, \tag{9d}$$

where the functions $k_{ij}(t,x)$, where $i,j=1...4$, are known continuous and bounded kernels that depend on the nonhomogeneity parameter $\beta$ .

The solution of (9a-d) subject to the single-valuedness conditions may be expressed as $\varphi_i(t)=w(t)\vartheta_i(t),\ (i=1..4)$ . In this solution, $w(t)=1/\sqrt{1-t^2}$ is the weight function which is obtained from the nature of the singularity at the crack tips and which is associated with the Chebyshev polynomial of the first kind $T_n(t)=\cos(n\arccos(t))$. The functions $\vartheta_i(t),\ (i=1..4)$ are continuous and bounded functions in the interval $[-1,1]$ which may be expressed as truncated series of Chebyshev polynomial of the first kind. Using a suitable collocation method, a linear algebraic system of the unknown coefficients of the density functions is obtained. As a result, the stress intensity factors, the electric displacement intensity factor and the magnetic induction intensity factor can be expressed as:

$$k_1(1)=-\sum_{n=1}^{N}\left(k_{22}^0\tilde{b}_n+k_{23}^0\tilde{c}_n+k_{24}^0\tilde{d}_n\right), \qquad k_1(-1)=-\sum_{n=1}^{N}(-1)^n\left(k_{22}^0\tilde{b}_n+k_{23}^0\tilde{c}_n+k_{24}^0\tilde{d}_n\right), \tag{10a,b}$$

$$k_2(1)=-k_{11}^0\sum_{n=1}^{N}\tilde{a}_n, \qquad k_2(-1)=-k_{11}^0\sum_{n=1}^{N}(-1)^n\tilde{a}_n, \tag{10c,d}$$

$$k_D(-1)=-\sum_{n=1}^{N}\left(k_{32}^0\tilde{b}_n+k_{33}^0\tilde{c}_n+k_{34}^0\tilde{d}_n\right), \qquad k_D(-1)=-\sum_{n=1}^{N}(-1)^n\left(k_{32}^0\tilde{b}_n+k_{33}^0\tilde{c}_n+k_{34}^0\tilde{d}_n\right), \tag{10e,f}$$

$$k_B(-1)=-\sum_{n=1}^{N}\left(k_{42}^0\tilde{b}_n+k_{43}^0\tilde{c}_n+k_{44}^0\tilde{d}_n\right), \qquad k_B(-1)=-\sum_{n=1}^{N}(-1)^n\left(k_{42}^0\tilde{b}_n+k_{43}^0\tilde{c}_n+k_{44}^0\tilde{d}_n\right). \tag{10g,h}$$

## Results and discussion

The formulation using singular integral equations to determine the stress intensity factors of cracked functionally graded magneto electro elastic materials was established. For validation, the results for the case of a sufficiently thick layer, were compared to those of infinite medium (Rekik et al 2012) and a good agreement was obtained (Fig. 2).

Then, a number of simulations, for different magneto electro mechanical load cases, were performed by varying the nonhomogeneity parameter, crack position, crack length and permeability parameters. The considered material is a bimorph composed of the Barium Titanium Oxide, BaTiO3 and the Cobalt Iron Oxide, CoFe2O4.

Fig. 3 illustrates the effect of varying the nonhomogeneity parameter $\beta a$ on the fields’ intensity factors for normal tractions, tangential tractions, magnetic and electric loadings in case of a central crack as large as the layer thickness and electromagnetic permeability of 25%. For each loading case, the corresponding field’’s intensity factors showss lower sensibility to material’s nonhomogeneity while remaining fields’ intensity factors increase monotonouslymonotonically.

Fig. 4 illustrates the effect of varying the crack position on the fields’ intensity factors for the same loading cases in the case of a crack as large as the layer thickness and electromagnetic permeability of 25%. In case of normal traction, electric or magnetic loadings, $k_1$, $k_B$ and $k_D$ show parabolic variation with crack position while $k_2$ varies oddly. Similarly, in case of tangential traction loading, $k_2$ varies parbolicallyparabolically while the $k_1$, $k_B$ and $k_D$ vary oddly.

Fig. 5 illustrates the effect of varying magneto electric permeabilities parameter in case of a central crack as large as the layer thickness. Fields’ intensity factors vary linearly with increasing impermeability.

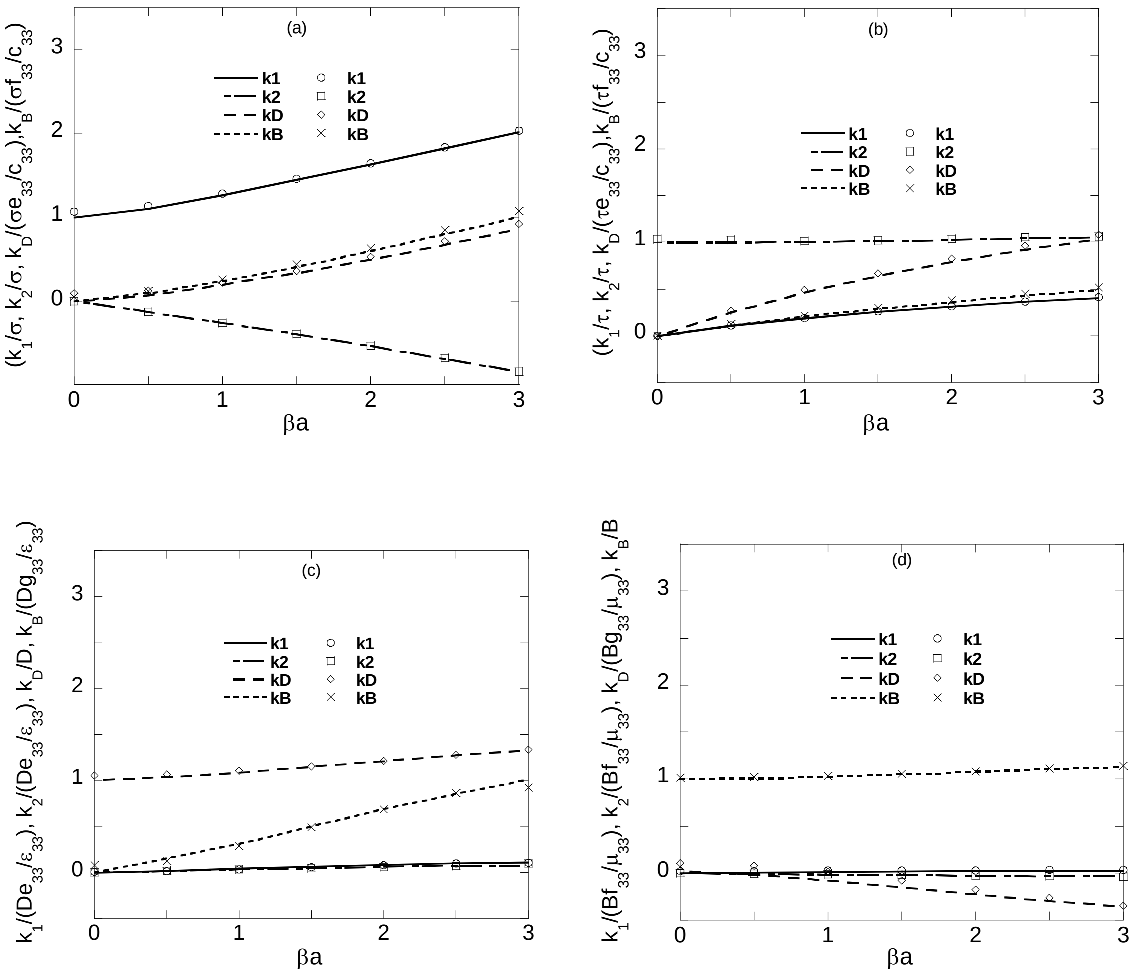


**Figure 2:** Comparison of normalized fields' intensity factors (markers) with those published by Rekik et al (2012) (continuous lines) for the case of a central crack, magneto electrically impermeable, embedded in an FGMEEM layer four times as thick as the crack length, subjected to uniform normal tractions (a), tangential tractions (b), electric displacement (c), and magnetic induction (d)

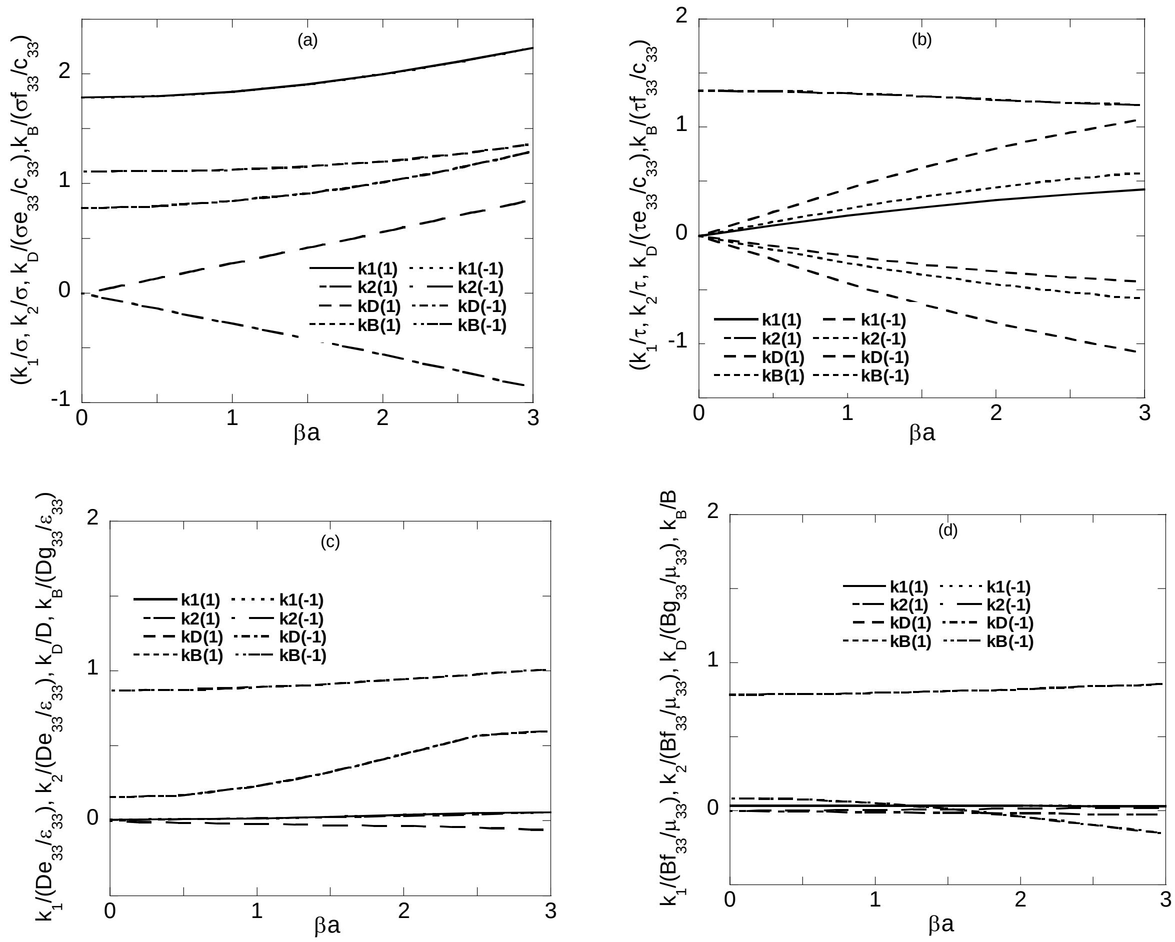


**Figure 3:** Effect of the non homogeneity parameter $\beta a$ on the normalized fields' intensity factors under constant normal (a) and tangential (b) crack surface tractions in addition to constant electric displacement (c) and magnetic induction (d) in the case of central crack with $h_1 = h_2 = a,$ and $k^e = k^m = 0.25$.

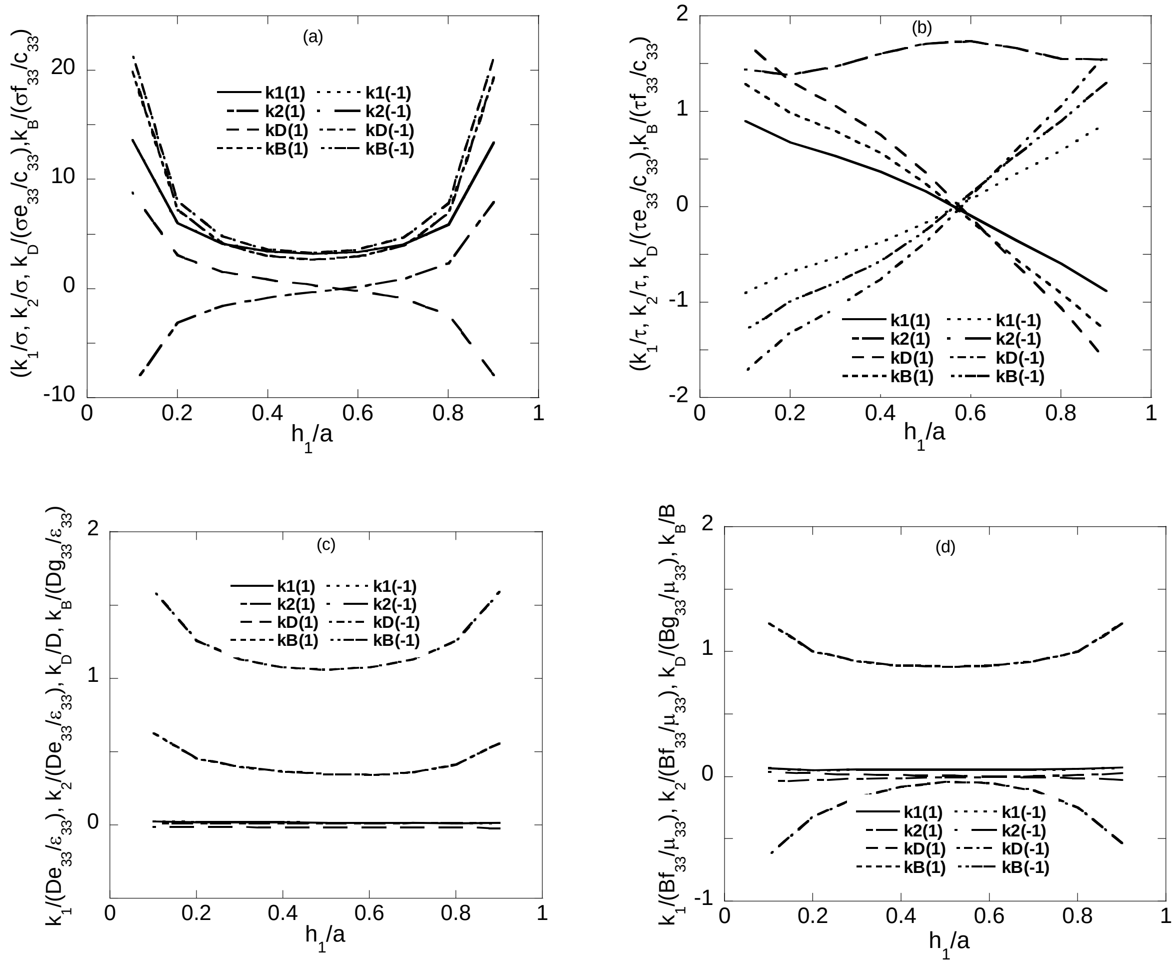


**Figure 4:** Effect of the crack position on the normalized fields' intensity factors under constant normal (a) and tangential (b) crack surface tractions in addition to constant electric displacement (c) and magnetic induction (d) in the case of $\left(h_1 + h_2\right) = a$, $\beta a = 1$ and $k^e = k^m = 0.25$.

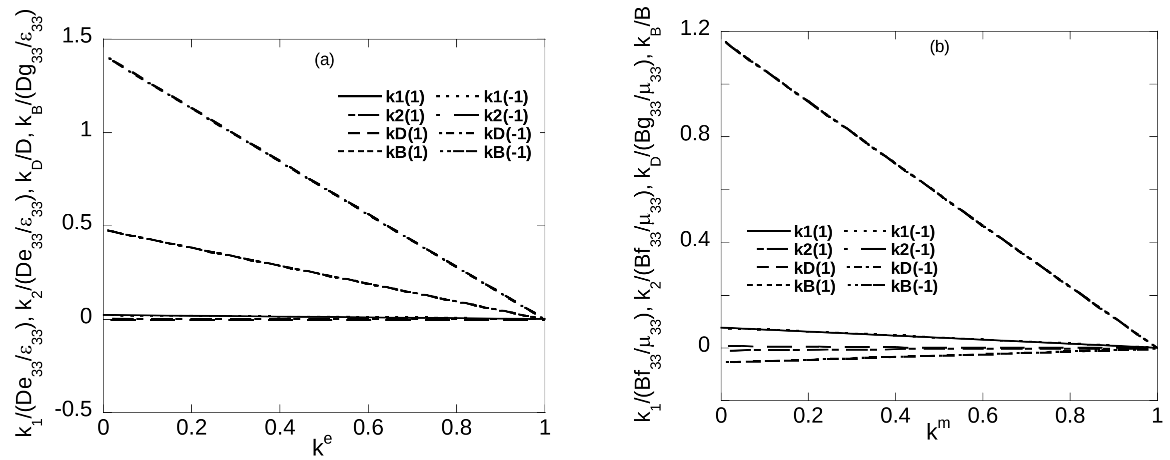


**Figure 5:** Effect of the magneto electric partial permeability parameters on the normalized fields' intensity factors under constant electric displacement (a) and magnetic induction (b) loadings in the case of central crack, $h = a$ and $\beta a = 1$.


## Acknowledgement

The first two authors are grateful for the funding provided to their laboratory by the Tunisian Ministry of Higher Education and Scientific Research. This work was performed within the framework of the International Institute for Multifunctional Materials for Energy Conversion which is funded by the US National Science Foundation.


## References


Mode III crack problem in a functionally graded magneto-electro-elastic strip Li Ma, Jia Li, Radhi Abdelmoula, Lin-Zhi Wu, International Journal of Solids and Structures 44 (2007) 5518–5537

Surface crack problem for functionally graded magneto electro elastic coating homogeneous elastic substrate system under anti-plane mechanical and in-plane electric and magnetic loading, Li Ma, Lin-Zhi Wua, Lin-Ping Feng , Engineering Fracture Mechanics 76 (2009) 269–285

Two parallel symmetry permeable cracks in functionally graded piezoelectric/piezomagnetic materials under anti-plane shear loading, Zhen-Gong Zhou, Biao Wang, International Journal of Solids and Structures 41 (2004) 4407–4422

Dynamic fracture behaviors of cracks in a functionally graded magneto-electro-elastic plate, Feng Wenjie, Su Raykaileung, European Journal of Mechanics A/Solids 26 (2007) 363–379

Dynamic internal crack problem of a functionally graded magneto-electro-elastic strip W.J. Feng, R.K.L. Su, International Journal of Solids and Structures 43 (2006) 5196–5216

Scattering of harmonic anti-plane shear stress waves by a crack in functionally graded piezoelectric/piezomagnetic materials, Liang Jun, Acta Mechanica Solida Sinica, Vol. 20, No. 1, March, 2007

Basic solution of a Mode-I limited-permeable crack in functionally graded piezoelectric/piezomagnetic materials, Zhen-Gong Zhou, Zeng-Tao Chen, International Journal of Solids and Structures 45 (2008) 2265–2296

Anti-plane crack intersecting the interface in a bonded smart structure with graded magneto electro elastic properties, Y.D. Li, K.Y. Lee, Theoretical and Applied Fracture Mechanics 50 (2008) 235–242

Fracture analysis and improved design for a symmetrically bonded smart structure with linearly non-homogeneous magneto electro elastic properties, Yong-Dong Li, Kang Yong Lee, Engineering Fracture Mechanics 75 (2008) 3161–3172

Crack in a bonded functionally graded magneto-electro-elastic strip, Li-Fang Guo, Xing Li, Sheng-Hu Ding, Computational Materials Science 46 (2009) 452–458

An embedded mixed-mode crack in a functionally graded magnetoelectroelastic infinite medium, Rekik M., El-Borgi S., Ounaies Z., 2012 International Journal of Solids and Structures 49 (2012) 835–845