The thermal effect of anti-plane crack in a functionally graded piezoelectric strip under electric shock

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Abstract The thermal effect of mode III crack in a functionally graded strip under the electric shock is investigated. This fracture analysis can be expressed through the superposition of two problem solutions. The first solution is the dynamic behaviors of a functionally graded piezoelectric material with central crack subjected to the electric shock. The second solution means the temperature field by calculating the power of point heat source around the crack tip. Based on the Laplace transform and Fourier transform technique, this mixed boundary value problems is reduced to a Cauchy singular integral equation, which is solved numerically by the Cauchy-Chebyshev quadrature technique. Numerical results are presented to show the effects of geometrical of crack and graded quantities of material on the stress intensity factors.

Keywords Anti-plane crack, Functionally graded piezoelectric materials, Electric shock, Thermal effect

1. Introduction

Piezoelectric materials (PMs) have been widely used as a smart material in electromechanical devices due to the demand of transform from mechanical to electrical loadings, and vice versa. To improve the reliability and durability problems arising largely from high residual and thermal stress, poor interfacial bonding strength, the functionally graded piezoelectric materials (FGPMs) as a new class of advanced composites have been developed.

Recently, some researchers start to investigate the fracture behavior in FGPMs. Wang and Node [1] firstly studied the thermo-piezoelectric fracture problem of a functionally graded piezoelectric layer bonded to a metal. They obtained the thermal flow, stress and electric displacement intensity factors and predict the direction of crack extension by using the energy density theory. Wang considered the mode III crack problem in FGPM, where the material properties are assumed in a class of functional form such that an analytic solution is possible. Recently, Li and Lee [2] investigated the fracture behavior of a weak discontinuous interface between two piezoelectric strips under electro-mechanical loads by using the methods of Fourier integral transform and Cauchy singular integral equation. Ding and Li [3] studied the problem of periodic interface cracks in a functionally graded coating-substrate structure. Recently, based on the methods of variable separation and singular integral equation, Ref. [4] investigated the arc-shaped interfacial cracking problem in a hollow cylinder that consists of an inner orthotropic dielectric layer and an outer functionally graded piezoelectric layer.

Although a variety of challenging issues related to certain crack problems in the functionally graded piezoelectric materials have been addressed, one of the remaining problems that need to fully understand is that FGPMs belongs to the dielectric material. Research of Bilyk et al [5] revealed that applying high current on the conductor, the temperature of conductor is lager than the temperature just under force load. To the authors' knowledge, few papers considered the solution for the problem of the heating effect of the crack tip in FGPM under electric shock. Then, this paper discusses the heating effect of the crack tip on piezoelectric medium under the high electric shock load.

2 Formulation of the problem

2.1 Theoretical model

The thermal effect of mode III crack in a functionally graded strip under the electric shock is investigated. This fracture analysis can be expressed through the superposition of two problem solutions. The first solution refers to the dynamic behaviors of a functionally graded piezoelectric material with central crack subjected to the electric shock. The second solution means the temperature field by calculating the power of point heat source around the crack tip. Illustrated in Fig.1 is the fracture model of a functionally graded piezoelectric strip which is assumed to contain a center crack. The crack of length and the thickness of strip are defined as $2c$ and $2h$. In addition, the rectangular coordinate system is established as fig 1. Since the poling directions of piezoelectric materials are orientated along z-axis, the antiplane mechanical field and inplane electric field are coupled. Here, the fundamental solution of crack under a pair of the equivalent electric shock $-D_0H(t)$ and shear traction $-\tau_0H(t)$ acting on the crack surface is considered. $H(t)$ is Heaviside function, and τ_0, D_0 are the range of the impact load of force field and electric field.

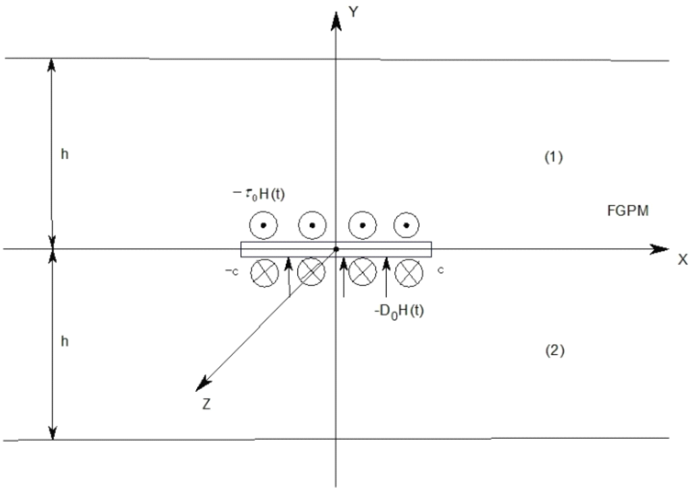

Figure 1. Functionally graded piezoelectric strip with central crack subjected to the electric shock

In fracture analysis of functionally graded piezoelectric strip, for the convenience of employing some standard methods such as Fourier transforms and integral equations, material properties are always assumed to be continuous functions of spatial coordinates, among which the most widely used one is exponential function [4]. Therefore, like many previous literatures, the properties of the functionally graded piezoelectric strip are assumed in power forms along y axis as follows:

$$c_{44} = c_0 e^{\beta y}, \quad e_{15} = e_0 e^{\beta y}, \quad \kappa_{11} = \kappa_0 e^{\beta y}, \quad \rho = \rho_0 e^{\beta y} \tag{1}$$

where $c_0, e_0, \kappa_0, \rho_0$ are the coefficient of the functionally graded strip at $y=0$, such as shear modulus, piezoelectric coefficient, dielectric permittivity and density permeability, respectively. β is the non-homogeneity parameters controlling the material coefficient in the graded layer.

2.2 Governing equations

Firstly, without considering the related effect of temperature field, a theoretical model is developed for the dynamic fracture analysis of a functionally graded piezoelectric material of a finite dimension with central crack subjected to the electric shock. Under axial shear deformation, the constitutive relations can be expressed in terms of polar coordinate system in the form

$$\sigma_{xz} = c_{44}(y)\frac{\partial \omega}{\partial x} + e_{15}(y)\frac{\partial \phi}{\partial x}, \sigma_{yz} = c_{44}(y)\frac{\partial \omega}{\partial y} + e_{15}(y)\frac{\partial \phi}{\partial y}$$
$$D_x = e_{15}(y)\frac{\partial \omega}{\partial x} - \kappa_{11}(y)\frac{\partial \phi}{\partial x}, D_y = e_{15}(y)\frac{\partial \omega}{\partial y} - \kappa_{11}(y)\frac{\partial \phi}{\partial y} \tag{2}$$

where ϕ and w denote the electric potential and anti-plane mechanical displacement, D and σ are the electric displacement and anti-plane stresses, respectively.
The geometric equation can be written as

$$2\varepsilon_{xz} = \frac{\partial \omega}{\partial x}, \quad 2\varepsilon_{yz} = \frac{\partial \omega}{\partial y}, \quad E_x = -\frac{\partial \phi}{\partial x}, \quad E_y = -\frac{\partial \phi}{\partial y}, \quad E_z = -\frac{\partial \phi}{\partial z} \tag{3}$$

Under the static condition, when body forces and body charges are omitted, the stress, electric displacement should satisfy the following equations

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} = \rho \frac{\partial^2 \omega}{\partial t^2}, \quad \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0 \tag{4}$$

Introduce an auxiliary function by

$$\psi = \phi - \frac{e_0}{\kappa_0}\omega \tag{5}$$

Substituting Eqs. (1-3) into Eq. (4) yields the decoupled governing equations in the piezoelectric strip

$$\nabla^2 \omega + \beta \frac{\partial \omega}{\partial y} = s_0 \frac{\partial^2 \omega}{\partial t^2}, \nabla^2 \psi + \beta \frac{\partial \psi}{\partial y} = 0 \tag{6}$$

where

$$s_0 = \rho_0 / (c_0 + \frac{e_0^2}{\kappa_0}) \tag{7}$$

2.3 Boundary conditions

The upper region $y \in [0, h]$ and lower region $y \in [-h, 0]$ is symmetric with respect to the x axis. Therefore, in the following we confine our attention to the upper region $y \in [0, h]$. Then, based on the symmetry, the corresponding boundary conditions are imposed

$$\omega(x,0,t) = 0, \quad \phi(x,0,t) = 0 \ (|x| > c) \tag{8}$$

Here, the fundamental solution of crack under a pair of the equivalent electric shock $-D_0 H(t)$ and shear traction $-\tau_0 H(t)$ acting on the crack surface is considered. The corresponding mechanical boundary conditions on the crack surface are imposed

$$\sigma_{yz}(x,0,t) = -\tau_0 H(t), \quad D_y(x,0,t) = -D_0 H(t) \ (-c < x < c) \tag{9}$$

Assume that the upper surface $y = h$ is free of loading. Therefore, the electric displacement and anti-plane stresses on the upper surface is taken as zero,

$$\sigma_{yz}(x,h,t) = 0, \quad D_y(x,h,t) = 0 \ (-\infty < x < +\infty) \tag{10}$$

3. Solution

3.1 Basic solution expression

Using the Laplace transform and Fourier transform technique, ones can transform Eqs. (6) into a system of decoupled differential equations

$$(-s^2)F(s,y,p)+\frac{\partial^2}{\partial y^2}F(s,y,p)+\beta\frac{\partial}{\partial y}F(s,y,p)=s_0p^2F(s,y,p) \tag{11}$$

$$(-s^2)G(s,y,p)+\frac{\partial^2}{\partial y^2}G(s,y,p)+\beta\frac{\partial}{\partial y}G(s,y,p)=0 \tag{12}$$

where

$$\psi^*(x,y,p)=\int_0^{+\infty}\psi(x,y,t)e^{-pt}dt,\psi(x,y,t)=\int_{Br}\psi^*(x,y,p)e^{pt}dp \tag{13}$$

$$\omega^*(x,y,p)=\int_0^{+\infty}\omega(x,y,t)e^{-pt}dt,\omega(x,y,t)=\frac{1}{2\pi i}\int_{Br}\omega^*(x,y,p)e^{pt}dp \tag{14}$$

$$\omega^*(x,y,p)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}F(s,y,p)e^{-isx}ds,F(s,y,p)=\int_{-\infty}^{+\infty}\omega^*(x,y,p)e^{isx}dx \tag{15}$$

$$\psi^*(x,y,p)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}G(s,y,p)e^{-isx}ds,G(s,y,p)=\int_{-\infty}^{+\infty}\psi^*(x,y,p)e^{isx}dx \tag{16}$$

where $\omega^*(x,y,p),\psi^*(x,y,p)$ are the Laplace transform of $\omega(x,y,t),\psi(x,y,t)$, Br means path integral Bromwich formulation. And $F(s,y,p),G(s,y,p)$ are the Fourier transform of $\omega^*(x,y,p),\psi^*(x,y,p)$.

Afterwards, solving Eqs. (11-12), one can finally express the mechanical displacement, electric potential of the strip in the Laplace domains below

$$\omega^*(x,y,p)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}[A_1(s,p)e^{t_1y}+A_2(s,p)e^{t_2y}]e^{-isx}dx \tag{17}$$

$$\psi^*(x,y,p)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}[A_3(s,p)e^{t_3y}+A_4(s,p)e^{t_4y}]e^{-isx}dx \tag{18}$$

$$\begin{aligned}\phi^*(x,y,p)=&\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{e_0}{\kappa_0}[A_1(s,p)e^{t_1y}+A_2(s,p)e^{t_2y}]e^{-isx}dx\\&+\frac{1}{2\pi}\int_{-\infty}^{+\infty}[A_3(s,p)e^{t_3y}+A_4(s,p)e^{t_4y}]e^{-isx}dx\end{aligned} \tag{19}$$

There are four unknown functions $A_i(s,p),i=1,2,3,4$ which can be solved by considering the boundary conditions and continuity conditions which are formulated by Eqs. (8-10).

$$t_1=-\frac{\beta}{2}+\sqrt{(\frac{\beta}{2})^2+s^2+s_0p^2}\,,t_2=-\frac{\beta}{2}-\sqrt{(\frac{\beta}{2})^2+s^2+s_0p^2}\,,t_3=-\frac{\beta}{2}+\sqrt{(\frac{\beta}{2})^2+s^2}\,,t_4=-\frac{\beta}{2}+\sqrt{(\frac{\beta}{2})^2+s^2} \tag{20}$$

3.2 Transformation to singular integral equation

In order to derive the singular integral equation, two dislocation density functions are introduced as

$$f^*(x,p)=\begin{cases}\dfrac{\partial\omega^*(x,0,p)}{\partial x} & |x|\le c\\ 0 & |x|>c\end{cases} \tag{21}$$

$$g^*(x,p)=\begin{cases}\dfrac{\partial\phi^*(x,0,p)}{\partial x} & |x|\le c\\ 0 & |x|>c\end{cases} \tag{22}$$

Substituting Eqs. (17-19) into Eq. (8), and considering Eqs. (21-22), one can obtain the following relation

$$A_1(s,p)+A_2(s,p)=\frac{1}{-is}\int_{-c}^{c}f^*(t,p)e^{ist}dt \tag{23}$$

$$A_3(s,p)+A_4(s,p)=(-\frac{e_0}{\kappa_0})\cdot(\frac{1}{-is}\int_{-c}^{c}f^*(t,p)e^{ist}dt)+\frac{1}{-is}\int_{-c}^{c}g^*(t,p)e^{ist}dt \tag{24}$$

Substituting Eqs. (2) (17-19) into Eq. (10), it yields

$$(c_0+\frac{e_0^2}{\kappa_0})e^{\beta h}[t_1A_1(s,p)e^{t_1h}+t_2A_2(s,p)e^{t_2h}]+e_0e^{\beta h}[t_3A_3(s,p)e^{t_3h}+t_4A_4(s,p)e^{t_4h}]=0 \tag{25}$$

$$2e_0e^{\beta h}[t_1A_1(s,p)e^{t_1h}+t_2A_2(s,p)e^{t_2h}]+\kappa_0e^{\beta h}[t_3A_3(s,p)e^{t_3h}+t_4A_4(s,p)e^{t_4h}]=0 \tag{26}$$

One can finally obtain the following relations form Eqs. (23-26)

$$A_1=\frac{-t_2e^{t_2h}}{t_1e^{t_1h}-t_2e^{t_2h}}f,\quad A_2=\frac{t_1e^{t_1h}}{t_1e^{t_1h}-t_2e^{t_2h}}f \tag{27}$$

$$A_3=\frac{-t_4e^{t_4h}}{t_3e^{t_3h}-t_4e^{t_4h}}(-\frac{e_0}{\kappa_0}f+g),\quad A_4=\frac{t_3e^{t_3h}}{t_3e^{t_3h}-t_4e^{t_4h}}(-\frac{e_0}{\kappa_0}f+g) \tag{28}$$

where

$$f=\frac{1}{-is}\int_{-c}^{c}f^*(t,p)e^{ist}dt,\quad g=\frac{1}{-is}\int_{-c}^{c}g^*(t,p)e^{ist}dt \tag{29}$$

Then, substituting Eqs. (2) (27-29) into Eqs. (2), the basic solution expression of the electric displacement and anti-plane stresses is obtained. And then, substituting the electric displacement and stress component into Eq. (8), one arrives at an integral equation in the form [6]

$$\frac{1}{\pi}\int_{-c}^{c}\frac{f^*(t,p)}{t-x}dt+\frac{e_0}{\pi}\int_{-c}^{c}\frac{g^*(t,p)}{t-x}dt+\frac{1}{\pi}\int_{-c}^{c}K_{11}(x,t,p)f^*(t,p)dt$$
$$-\frac{e_0}{\pi}\int_{-c}^{c}K_{12}(x,t,p)g^*(t,p)dt=-\frac{\tau_0}{p} \tag{30}$$

$$-\frac{e_0}{\pi}\int_{-c}^{c}\frac{f^*(t,p)}{t-x}dt+\frac{\kappa_0}{\pi}\int_{-c}^{c}\frac{g^*(t,p)}{t-x}dt-\frac{e_0}{\pi}\int_{-c}^{c}K_{21}(x,t,p)f^*(t,p)dt$$
$$+\frac{\kappa_0}{\pi}\int_{-c}^{c}K_{22}(x,t,p)g^*(t,p)dt=-\frac{D_0}{p} \tag{31}$$

where

$$K_{11}=\int_0^{+\infty}[U_1(s,p)-1]\sin s(t-x)ds,\quad K_{12}=\int_0^{+\infty}[U_2(s,p)+1]\sin s(t-x)ds,$$
$$K_{21}=K_{22}=\int_0^{+\infty}[U_2(s,p)-1]\sin s(t-x)ds \tag{32}$$

$$U_1=\frac{1}{s}[\frac{e_0^2}{\kappa_0}\cdot\frac{t_3t_4e^{t_3h}-t_3t_4e^{t_4h}}{t_3e^{t_3h}-t_4e^{t_4h}}-(c_0+\frac{e_0^2}{\kappa_0})\cdot\frac{t_1t_2e^{t_1h}-t_1t_2e^{t_2h}}{t_1e^{t_1h}-t_2e^{t_2h}}],U_2=\frac{1}{s}\cdot\frac{t_3t_4e^{t_3h}-t_3t_4e^{t_4h}}{t_3e^{t_3h}-t_4e^{t_4h}} \tag{33}$$

Because of the symmetry of fracture analysis, the dislocation density function $F(u,p),(u,p)$ must be an odd function of u. Then, it is automatically satisfied

$$f^*(0,p)=0,g^*(0,p)=0 \tag{34}$$

3.3 Numerical solution

Introducing $t=uc,x=rc$, $f^*(t,p)=F(u,p),g^*(t,p)=G(u,p)$, $K_{ij}(x,t,p)=K_{ij}^*(u,r,p)(i,j=1,2)$, Eqs. (30-31) and (34) can be transformed into the standard form of the first kind Cauchy singular integral equations as

$$\frac{1}{\pi}\int_{-1}^{1}\frac{F(u,p)}{u-r}du+\frac{e_0}{\pi}\int_{-1}^{1}\frac{G(u,p)}{u-r}du+\frac{c}{\pi}\int_{-1}^{1}K_{11}{}^{*}(r,u,p)\cdot F(u,p)du$$

$$-\frac{e_0 c}{\pi}\int_{-1}^{1}K_{12}{}^{*}(r,u,p)G(u,p)du=-\frac{\tau_0}{p} \tag{35}$$

$$-\frac{e_0}{\pi}\int_{-1}^{1}\frac{F(u,p)}{u-r}du+\frac{\kappa_0}{\pi}\int_{-1}^{1}\frac{G(u,p)}{u-r}du-\frac{e_0 c}{\pi}\int_{-1}^{1}K_{21}{}^{*}(r,u,p)\cdot F(u,p)du$$

$$+\frac{\kappa_0 c}{\pi}\int_{-1}^{1}K_{22}{}^{*}(r,u,p)G(u,p)du=-\frac{D_0}{p} \tag{36}$$

$$F(0,p)=0\,,\ G(0,p)=0 \tag{37}$$

According to the theory of singular integral equation, the solution of $F(u,p)$ 和 $G(u,p)$ may be expressed as [6]

$$f^{*}(t,p)=F(u,p)=\frac{R(u,p)}{\sqrt{1-u^2}}\,,\quad R(u,p)=\sum_{n=0}^{\infty}C_nT_n(u,p) \tag{38}$$

$$g^{*}(t,p)=G(u,p)=\frac{S(u,p)}{\sqrt{1-u^2}}\,,\quad S(u,p)=\sum_{n=0}^{\infty}D_nT_n(u,p) \tag{39}$$

Based on the Cauchy-Chebyshev collocation method, Eqs. (35-37) are reduced into a system of algebraic equations [6]

$$\sum_{l=1}^{N}[\frac{1}{u_l-r_m}+cK_{11}{}^{*}(r_m,u_l,p)]\frac{R(u_l,p)}{N}+\sum_{l=1}^{N}[\frac{e_0}{u_l-r_m}+e_0cK_{12}{}^{*}(r_m,u_l,p)]\frac{S(u_l,p)}{N}=-\frac{\tau_0}{p} \tag{40}$$

$$\sum_{l=1}^{N}[-\frac{e_0}{u_l-r_m}+e_0cK_{21}{}^{*}(r_m,u_l,p)]\frac{R(u_l,p)}{N}+\sum_{l=1}^{N}[\frac{\kappa_0}{u_l-r_m}+\kappa_0cK_{22}{}^{*}(r_m,u_l,p)]\frac{S(u_l,p)}{N}=-\frac{D_0}{p} \tag{41}$$

$$\sum_{l=1}^{N}\frac{R(u_l,p)}{N}=0\,,\sum_{l=1}^{N}\frac{S(u_l,p)}{N}=0 \tag{42}$$

where

$$u_l=\cos(\frac{2l-1}{2N}\pi)\ (l=1,2,\cdots,N)\,,r_m=\cos(\frac{m}{N}\pi)\ (m=1,2,\cdots,N-1) \tag{43}$$

Solving Eqs. (40-41) and taking (42) into account, the numerical values of the function $R(u_l,p)$ and $S(u_l,p)$ can be obtained. Furthermore, the values of the function $F(u,p)$ and $G(u,p)$ can be obtained numerically. Thus, after obtaining the solutions $f\,,g$ from the Eqs. (19), we may obtain the four unknown functions $A_i(s,p), i=1,2,3,4$ from Eqs. (27-28), furthermore, the mechanical displacement, electric potential of the strip in the Laplace domains will be obtained from Eqs. (17-19).

3.4 Temperature field of the crack tip

Research of Bilyk et al [5] revealed that applying high current on the conductor, the temperature of conductor is lager than the temperature just under force load. Then, this paper discusses the heating effect of the crack tip on piezoelectric medium under the high electric shock load. Secondly, within supposing that it is a heat insulation process in a short time, and thermal field and electromechanical filed is decoupled, the thermal effect is calculated. according to ref. [5] which shows that electromagnetic field diffusion time scale is far less than the heat conduction time scale under action current load, the this process can be approximated as adiabatic process, so this assumption is established.

In accordance with the above assumptions, under the adiabatic conditions, the first approximation of heat conduction equation of the piezoelectric materials is

$$p = c_0 \frac{\partial T}{\partial t} \tag{44}$$

where c_0 denotes specific heat capacity.

For there is no external heat source, according to the ref. [7] of the electric shock which can retard effectively crack propagation, the heat source power can be introduced to the function of equivalent external point heat source

$$p = E \cdot J \tag{45}$$

where J denotes the electric current density vector field of the dielectric materials.

Current density vector of the dielectric material can be expressed as

$$J = \frac{\partial D}{\partial t} \tag{46}$$

Functionally graded piezoelectric material belongs to the dielectric material. Similarly, introducing power of heat source, temperature field of piezoelectric medium is obtained from the time integration of expression which can be obtained by substituting (45-46) into expressions of (44),

$$T(x, y, t) = \frac{1}{c_0} \int_0^t E(x, y, \tau) \frac{\partial D(x, y, \tau)}{\partial \tau} d\tau \tag{48}$$

For the geometric Eq. (3), constitutive Eq. (2) and fundamental solution (17-19), we get

$$\begin{aligned} E_x^*(x, y, p) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} is \cdot \frac{e_0}{\kappa_0} [A_1(s, p)e^{t_1 y} + A_2(s, p)e^{t_2 y}] e^{-isx} ds \\ &+ \frac{1}{2\pi} \int_{-\infty}^{+\infty} is \cdot [A_3(s, p)e^{t_3 y} + A_4(s, p)e^{t_4 y}] e^{-isx} ds \end{aligned} \tag{49}$$

$$\begin{aligned} E_y^*(x, y, p) &= -\frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{e_0}{\kappa_0} [t_1 A_1(s, p)e^{t_1 y} + t_2 A_2(s, p)e^{t_2 y}] e^{-isx} ds \\ &- \frac{1}{2\pi} \int_{-\infty}^{+\infty} [t_3 A_3(s, p)e^{t_3 y} + t_4 A_4(s, p)e^{t_4 y}] e^{-isx} ds \end{aligned} \tag{50}$$

$$\begin{aligned} D_x^*(x, y, p) &= (e_{15} - \kappa_{11} \cdot \frac{e_0}{\kappa_0}) \frac{1}{2\pi} \int_{-\infty}^{+\infty} (-is) \cdot [A_1(s, p)e^{t_1 y} + A_2(s, p)e^{t_2 y}] e^{-isx} ds \\ &- \kappa_{11} \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} (-is) \cdot [A_3(s, p)e^{t_3 y} + A_4(s, p)e^{t_4 y}] e^{-isx} ds \end{aligned} \tag{51}$$

$$\begin{aligned} D_y^*(x, y, p) &= (e_{15} - \kappa_{11} \cdot \frac{e_0}{\kappa_0}) \frac{1}{2\pi} \int_{-\infty}^{+\infty} [t_1 A_1(s, p)e^{t_1 y} + t_2 A_2(s, p)e^{t_2 y}] e^{-isx} ds \\ &- \kappa_{11} \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} [t_3 A_3(s, p)e^{t_3 y} + t_4 A_4(s, p)e^{t_4 y}] e^{-isx} ds \end{aligned} \tag{52}$$

Furthermore, the solution of electric field strength and electric displacement intensity in the Laplace transform domain is

$$E_x^*(x, y, p) = c[\sum_{l=1}^{N} K_{ex1}(u_l, x, y, p) \frac{R(u_l, p)}{N} + \sum_{l=1}^{N} K_{ex2}(u_l, x, y, p) \frac{S(u_l, p)}{N}] \tag{53}$$

$$E_y^*(x, y, p) = c[\sum_{l=1}^{N} K_{ey1}(u_l, x, y, p) \frac{R(u_l, p)}{N} + \sum_{l=1}^{N} K_{ey2}(u_l, x, y, p) \frac{S(u_l, p)}{N}] \tag{54}$$

$$D_x^*(x, y, p) = c[\sum_{l=1}^{N} K_{dx1}(u_l, x, y, p) \frac{R(u_l, p)}{N} + \sum_{l=1}^{N} K_{dx2}(u_l, x, y, p) \frac{S(u_l, p)}{N}] \tag{55}$$

$$D_y^*(x,y,p)=c[\sum_{l=1}^{N}K_{dy1}(u_l,x,y,p)\frac{R(u_l,p)}{N}+\sum_{l=1}^{N}K_{dy2}(u_l,x,y,p)\frac{S(u_l,p)}{N}] \quad (56)$$

where

$$K_{ex1}(u,x,y,p)=\int_0^{+\infty}\frac{e_0}{\kappa_0}[\frac{t_2e^{t_2h+t_1y}-t_1e^{t_1h+t_2y}}{t_1e^{t_1h}-t_2e^{t_2h}}+\frac{t_3e^{t_3h+t_4y}-t_4e^{t_4h+t_3y}}{t_3e^{t_3h}-t_4e^{t_4h}}]\cos[s(cu-x)]ds \quad (57)$$

$$K_{ex2}(u,x,y,p)=\int_0^{+\infty}\frac{t_4e^{t_4h+t_3y}-t_3e^{t_3h+t_4y}}{t_3e^{t_3h}-t_4e^{t_4h}}\cos[s(cu-x)]ds \quad (58)$$

$$K_{ey1}(u,x,y,p)=\int_0^{+\infty}\frac{e_0}{\kappa_0}[\frac{t_1t_2e^{t_1h+t_2y}-t_1t_2e^{t_2h+t_1y}}{t_1e^{t_1h}-t_2e^{t_2h}}+\frac{t_3t_4e^{t_4h+t_3y}-t_3t_4e^{t_3h+t_4y}}{t_3e^{t_3h}-t_4e^{t_4h}}]\sin[s(cu-x)]ds \quad (59)$$

$$K_{ey2}(u,x,y,p)=\int_0^{+\infty}\frac{t_3t_4e^{t_3h+t_4y}-t_3t_4e^{t_4h+t_3y}}{t_3e^{t_3h}-t_4e^{t_4h}}\sin[s(cu-x)]ds \quad (60)$$

$$K_{dx1}(u,x,y,p)=\int_0^{+\infty}(e_{15}-\kappa_{11}\frac{e_0}{\kappa_0})\frac{t_1e^{t_1h+t_2y}-t_2e^{t_2h+t_1y}}{t_1e^{t_1h}-t_2e^{t_2h}}+\kappa_{11}[\frac{t_3e^{t_3h+t_4y}-t_4e^{t_4h+t_3y}}{t_3e^{t_3h}-t_4e^{t_4h}}]\cos[s(cu-x)]ds \quad (61)$$

$$K_{dx2}(u,x,y,p)=\int_0^{+\infty}\kappa_{11}\frac{t_4e^{t_4h+t_3y}-t_3e^{t_3h+t_4y}}{t_3e^{t_3h}-t_4e^{t_4h}}\cos[s(cu-x)]ds \quad (62)$$

$$K_{dx1}(u,x,y,p)=\int_0^{+\infty}(e_{15}-\kappa_{11}\frac{e_0}{\kappa_0})\frac{t_1t_2e^{t_2h+t_1y}-t_1t_2e^{t_1h+t_2y}}{t_1e^{t_1h}-t_2e^{t_2h}}-\kappa_{11}[\frac{t_3t_4e^{t_3h+t_4y}-t_3t_4e^{t_4h+t_3y}}{t_3e^{t_3h}-t_4e^{t_4h}}]\sin[s(cu-x)]ds \quad (63)$$

$$K_{dx2}(u,x,y,p)=\int_0^{+\infty}-\kappa_{11}\frac{t_3t_4e^{t_4h+t_3y}-t_3t_4e^{t_3h+t_4y}}{t_3e^{t_3h}-t_4e^{t_4h}}\sin[s(cu-x)]ds \quad (64)$$

The solution of electric field strength and electric displacement intensity in the time domain are calculated by Laplace numerical inversion method. Furthermore using Eq. (48), the temperature field of the crack tip is obtained.

4. Numerical Examples and discussions

In the numerical computation, the functionally graded piezoelectric strip layer is assumed to be a non-homogeneous $BaTiO_3$ composite and the material constants of $y=0$ are

$$c_{44}=44Gpa,\quad e_{15}=11.4C/m^2,\quad \kappa_{11}=128.3\times10^{10}\,C/Vm,\quad \rho=5700Kg/m^3 \quad (65)$$

Here, the fundamental solution of crack under a pair of the equivalent electric shock $-D_0H(t)$ acting on the crack surface is considered strongly, so it assume that $\tau_0=0$.

Fig. 2 show the the temperature field around the crack tip versus the value of time. It is indicated that the effect of time on the temperature field is simple, i.e., when $c/h=1/2,1/3,1/4$ are specified, the temperature field increases drastically as time increases from 0 to 3. However, when time is larger than 3, the temperature field gradually decrease with time, finally, stabilized. It is also observed that the temperature field increases with the c/h increasing.

Fig. 3 depict the variation of the the temperature field around the crack tip versus the non-homogeneity parameter β. When time is small (t<2), the temperature field decreases as β increases; but when time is larger than 2, the temperature field increases drastically as β increases. Meanwhile, it is obviously indicated that when time is larger (t>2), the effect is larger than that when time is small (t<2). The above results show that the crack tip will cause high temperature change under high electric shock load. In this case, the crack tip temperature effect cannot be ignored

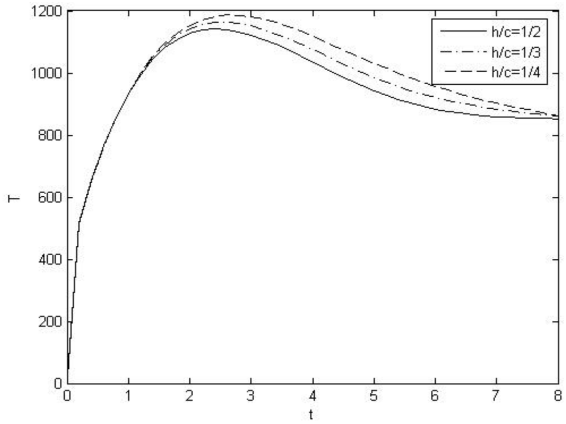

Figure 2. The temperature field around the crack tip versus time（$\beta = 0.5$ $\tau_0 = 0$， $D_0 = 5\times10^4$）

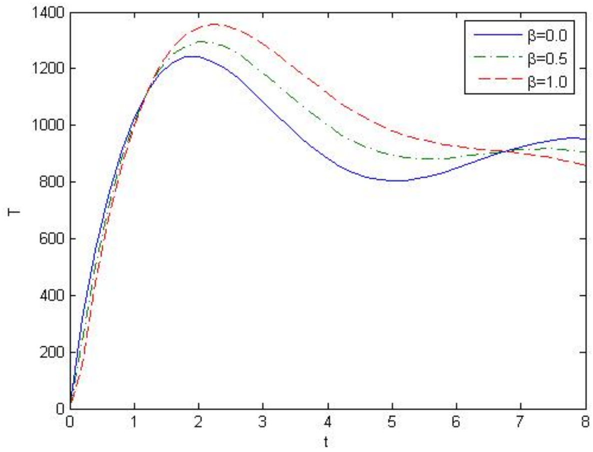

Figure 3. The temperature field around the crack tip versus parameter β （$h = 0.5c$，$\tau_0 = 0$，$D_0 = 5\times10^4$）

5. Conclusions

Research of Bilyk et al [5] revealed that applying high current on the conductor, the temperature of conductor is lager than the temperature just under force load. Then, this paper discusses the heating effect of the crack tip on piezoelectric medium under the high electric shock load. The thermal effect of a mode III crack in a functionally graded strip under the electric shock is investigated. This fracture analysis can be expressed through the superposition of two problems. The first problem refers to the dynamic behaviors of a functionally graded piezoelectric strip with central crack subjected to the electric shock. The second problem means the temperature field by calculating the power of point heat source around the crack tip.

Firstly, without considering the related effect of temperature field, a theoretical model is developed for the dynamic fracture analysis of a functionally graded piezoelectric strip of a finite dimension with central crack subjected to the electric shock. The Laplace transformation and the Fourier transforms are applied to make the transient problem tractable, and singular integral equations is derived with the dislocation density functions of crack as the unknown functions. In particular, the closed-form expressions for the electric field intensity and electric displacements intensity in terms of fundamental functions are derived, which provide a scientific basis for the interpretation of the thermal effect. Secondly, within supposing that it is a heat insulation process in a short time, the thermal effect is calculated. For this problem, thermal field and electromechanical filed is decoupled. So the heat conduction equation contains no electromechanical quantity. Based on that the temperature of conductors increases when high voltage is applied and that the electric field intensity and electric displacements intensity is obtained in the first solution. The power of point heat source of functionally graded piezoelectric strip is deduced. Therefore, from the time integration of power equations of point heat source, the temperature field around the crack tip is calculated.

It is worth noting that the problem of a cracked functionally graded piezoelectric strip under electric shock is a mix mode crack problem. The first problem is an electromechanical coupling problem. And the second problem is the temperature field around the crack tip, which is the basis problem of mode I or II heat crack analysis.

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