High Cycle Fatigue Simulation using Multi-temporal Scale Method coupled with Continuum Damage Mechanics

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Abstract A multiple temporal scale computational approach for assessing the fatigue life of engineering materials and components is presented. This full-scale simulation approach is developed in light of the challenges in employing the traditional computational method based on Finite Element Method (FEM)) and semi-discrete schemes for fatigue design and analysis. Simulating loading conditions with cycles on the order of hundreds of thousands and beyond is generally an impractical task for FEM even with the high-performance computing platform. Two critical aspects are addressed, i.e., the multiple time scales associated with the fatigue loading condition and the fatigue initiation/growth representations. Detailed implementation of integrating a multiscale space-time representation with a robust material model for the fatigue failure is outlined and demonstrated in the context of a common choice of industrial metal.

Keywords Space-time FEM, High Cycle Fatigue, Multi-temporal Scale Method, Enrichment

1. Introduction

In fatigue-based design of mechanical components and structures, safe-life and damage-tolerance approaches have been widely employed in practice. In safe-life approach, stress or strain are related to the number of cycles to failure under fatigue loading. Empirical curves developed from such a kind of relation are utilized to predict the life or acceptable level of fatigue load. Damage-tolerance approach based itself on the fracture mechanics concepts established by Griffith [1] and Irwin [2]. Using these concepts, Paris [3] proposed the relationship between the rate of crack growth and the range of applied stress intensity factor, which is used to predict the fatigue crack growth under cyclic loading history. While both methodologies have been widely used by the industry, they have significant limitations due to the empirical nature of the method. In addition, there can be significant scatterings in the fatigue test, which makes the curve-fitting unreliable. Further, the extrapolation of constant amplitude (CA) test data to predict life under variable amplitude (VA) and random loading conditions is a challenging task [4].

Motivated by these limitations, a multiple temporal scale computational approach is developed to assess the fatigue life of structural components. Examples include columns, beams, plates, and shells that are extensively used in assembly of mechanical components. This full-scale simulation approach is established in light of the challenges in employing the traditional computational method based on Finite Element Method (FEM) and semi-discrete schemes for fatigue design and analysis. Semi-discrete schemes such as the popular central difference or Newmark-β methods are known to suffer from either the time-step constraints or lack of convergence due to the oscillatory nature of the fatigue loading condition. As such, simulating loading conditions with cycles on the order of hundreds of thousands and beyond is generally an impractical task for FEM even with the high-performance computing platform. On the other hand, there is a great demand for such a computational capability as factors such as stress history and triaxiality, nonlinear coupling among the loads, complex geometry are known to critically influence the fatigue failure and generally not fully accounted for in the empirical design approaches that are in practice today.

More specifically, an enriched space-time finite element method (XTFEM) based on the time discontinuous Galerkin formulation is developed to handle the multiple temporal scales in fatigue problems. XTFEM is coupled with the two-scale continuum damage mechanics model for evaluating fatigue damage accumulation, with a damage model governing the fatigue crack-initiation and propagation as the simulation progresses. High Cycle Fatigue (HCF) simulations are performed using the developed methodology on a notched specimen of AISI 304L steel to predict total fatigue life under cyclic conditions.

2. Technical Approach

2.1 A Computational Framework of Enriched Space-time FEM

As the name suggests, space-time method introduces discretization both in the spatial and time domains. The main advantage of the space-time method over the semi-discrete scheme based methods is reflected in the ability to introduce approximations in the temporal domain. Space-time methods are also known for their capability of reducing artificially oscillations and handling loads with sharp gradients. These features of space-time method make it an attractive platform for fast and accurate simulation of the fatigue loading. In the semi-discrete scheme, approximations are established in space **x** for each time instance *t*. In the space-time formulation, the approximations are built simultaneously with both space **x** and time *t*. If finite element method is used, we have the following approximation in the space/time description for a general three-dimensional case

$$\mathbf{u}(\mathbf{x},t)=\sum_{I} N_{\mathbf{I}}(\mathbf{x},t)\mathbf{d}_{\mathbf{I}} \tag{1}$$

in which $N_I(\mathbf{x},t)$ is the finite element shape functions at nodes indexed by *I* and $\mathbf{d}_I$ is the corresponding nodal displacement vector. Note that although N_I and $\mathbf{d}_I$ look similar to the ones in the semi-discrete scheme, they are defined on a space/time grid as opposed to space only.

To describe our approach what is also known as the time discontinuous Galerkin method (TDG) [5], the 2D space/time grid plotted in Figure 1 will be used as a reference. We consider a space/time domain *Q*, which is the Cartesian product of space domain Ω and time duration $(0,\mathrm{T})$, i.e., $Q=\Omega\times(0,\mathrm{T})$.

As can be seen from Figure 1, the time domain is subdivided into time slabs with the *n*-th time slab given as $Q_n=\Omega\times(t_{n-1},t_n)$. In the current method, we will restrict the presence of discontinuities to the mechanical field only. We define a jump operator $[\ ]$, given as

$$[w(t)]=w(t^+)-w(t^-) \text{ and } [w(\mathbf{x})]=w(\mathbf{x}^+)-w(\mathbf{x}^-) \tag{2}$$

with $w(t^\pm)=w(t\pm\varepsilon)$, $w(\mathbf{x}^\pm)=w(\mathbf{x}\pm\varepsilon\mathbf{n})$, ε represents infinitesimal perturbation, $\mathbf{n}$ is the normal to the surface of discontinuity.

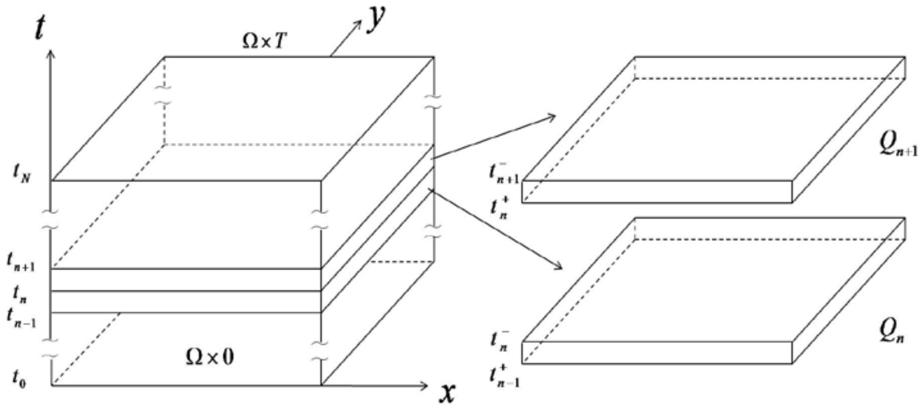


Figure 1: Illustration of TDG discretization of space-time finite elements.

By applying the variational principle to the governing equations of momentum within each time slab, we develop the so-called weak form that serves as the basis for computational implementation. For the the *n*-th time slab, we have the following bilinear form:

$$B_{DG}\left(\mathbf{w}^h,\mathbf{u}^h\right)_n = L_{DG}\left(\mathbf{w}^h\right)_n \tag{3}$$

with

$$B_{DG}\left(\mathbf{w}^h,\mathbf{u}^h\right)_n = \int_Q \dot{\mathbf{w}}^h\cdot\rho\ddot{\mathbf{u}}^h dQ_n + \int_Q \nabla\dot{\mathbf{w}}^h\cdot\boldsymbol{\sigma}\left(\nabla\mathbf{u}^h\right)dQ_n + \int_\Omega \dot{\mathbf{w}}^h\left(t_{n-1}^+\right)\cdot\rho\dot{\mathbf{u}}^h\left(t_{n-1}^+\right)d\Omega + \int_\Omega \nabla\mathbf{w}^h\left(t_{n-1}^+\right)\cdot\boldsymbol{\sigma}\left(\nabla\mathbf{u}^h\left(t_{n-1}^+\right)\right)d\Omega$$

$$L_{DG}\left(\mathbf{w}^h\right)_n = \int_Q \dot{\mathbf{w}}^h\cdot\rho\,\mathrm{b}dQ_n + \int_\gamma \dot{\mathbf{w}}^h\cdot\mathrm{t}d\left(\gamma_t\right)_n + \int_\Omega \dot{\mathbf{w}}^h\left(t_{n-1}^+\right)\cdot\rho\dot{\mathbf{u}}^h\left(t_{n-1}^-\right)d\Omega + \int_\Omega \nabla\mathbf{w}^h\left(t_{n-1}^+\right)\cdot\boldsymbol{\sigma}\left(\nabla\mathbf{u}^h\left(t_{n-1}^-\right)\right)d\Omega + \int_\gamma \dot{\mathbf{w}}^h\cdot\left[\mathrm{n}\cdot\boldsymbol{\sigma}\right]d\left(\gamma_i\right)_n$$

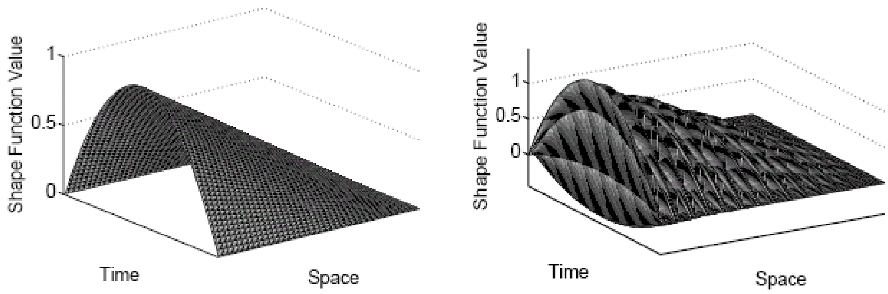
(4-5)

in which $\mathbf{u}^\mathbf{h}$ and $\mathbf{w}^\mathbf{h}$ are respectively the trial and test function for displacement. In Eqs.(4) and (5), the combination of the first two terms represents the momentum balance and traction boundary conditions. The 3rd and 4th terms in Eqs.(4) and (5) combined gives the continuity conditions between the time slabs. The last term in Eq.(5) is due to internal discontinuity.

In addressing the multiple temporal scales, we propose that the space-time approximation from Eq.(1) can be further improved by introducing enrichment [6-8]. In the enrichment formulation, the displacement field is approximated as

$$\mathbf{u}^h\left(\mathbf{x},t\right) = \sum_{I\in\tilde{N}} N_I\left(\mathbf{x},t\right)\mathbf{d}_I + \sum_{J\in\tilde{N}} N_J\left(\mathbf{x},t\right)\left(\phi\left(\mathbf{x},t\right)-\phi\left(\mathbf{x}_J,t_J\right)\right)\mathbf{a}_J \tag{6}$$

in which $N_I\left(\mathbf{x},t\right)$ is the regular space/time finite element shape functions, $\phi\left(\mathbf{x},t\right)$ is the called an enrichment function and is selected based on the physics of the problem. Correspondingly, $\mathbf{d}_I$ is the nodal displacement, and $\mathbf{a}_J$ is the enriched degree of freedom. The multiscale decomposition in Eq.(6) can be further written in direct notation, i.e., $\mathbf{u}^h(\mathbf{x},t) = \mathbf{N}\mathbf{u} = \bar{\mathbf{N}}\mathbf{d} + \tilde{\mathbf{N}}\mathbf{a}$ where $\bar{\mathbf{N}}$ is the FEM shape function matrix and $\tilde{\mathbf{N}}$ is the enrichment shape function matrix. $\mathbf{N} = \left[\bar{\mathbf{N}}, \tilde{\mathbf{N}}\right]$ and $\mathbf{u} = [\mathbf{d}, \mathbf{a}]^T$ with $\mathbf{d}$ the nodal displacement vector and $\mathbf{a}$ the vector for the enriched degree of freedom. Note that the way that the enrichment function $\phi(\mathbf{x},t)$ being built in Eq.(6) is an equivalent statement of the "partition-of-unity" concept. It ensures the consistency of the approximation. An example of regular and enrichment space-time shape functions is shown in Figure 2. In this case, we have used a fine scale harmonic function for $\phi(\mathbf{x},t)$ with a temporal scale that is smaller than the temporal element size of the regular space-time FEM.



(a) Regular FEM shape function (b) Enrichment shape function

Figure 2: Examples of regular and enrichment shape functions.

We have employed *harmonic* enrichment function considering the fine scale harmonic components of the loading and the solution (displacement) in the case of HCF. i.e., $\phi(\mathbf{x},t) = \sum_k \sin(\omega_k t)$ in which ω_k represents the *k*-th characteristic frequency. Complete spatial domain will be enriched with the time enrichment function as the whole structure is subjected to fatigue loads. With the addition of the enrichment, the final discretized equation is in the form of

$$\begin{bmatrix} \mathbf{K}_{\mathbf{rr}} & \mathbf{K}_{\mathbf{re}} \\ \mathbf{K}_{\mathbf{er}} & \mathbf{K}_{\mathbf{ee}} \end{bmatrix} \begin{Bmatrix} \mathbf{d} \\ \mathbf{a} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_{\mathbf{r}} \\ \mathbf{F}_{\mathbf{e}} \end{Bmatrix} \tag{7}$$

with subscripts "r" and "e" representing the contribution from the regular and enrichment components, respectively.

2.2 Modeling of Material Failure under Cyclic Loading

Modeling damage under fatigue loading is challenging due to the strong coupling to the loading path. Many techniques have been developed, including fracture mechanics, porous plasticity and continuum damage mechanics (CDM). In HCF, we introduce the two-scale CDM approach proposed by Lemaitre et al. [9] and Desmorat et al. [10]. This approach assumes that material behavior at mesoscale is elastic and damage can be treated as quasi-brittle in HCF. This quasi-brittle damage is modeled by introducing damage variables that represent the microcracks and microvoid due to HCF at microscale. These microdefects cause the plasticity at microscale but the mesoscale behavior remains elastic. In light of the the basic features of HCF, a two-scale damage model is developed. A schematic of the model is shown in Figure 3. All the quantities with subscript μ are defined at the microscale.

$$\boxed{\begin{array}{c}\text{Meso-scale}\\ \text{(elasticity)}\\ \boldsymbol{\sigma},\boldsymbol{\varepsilon}\end{array}} \longrightarrow \boxed{\begin{array}{c}\text{Localization law}\\ \boldsymbol{\varepsilon}^{\mu} = f\left(\boldsymbol{\varepsilon}, a, b, D\right)\end{array}} \longrightarrow \boxed{\begin{array}{c}\text{Plasticity and damage}\\ \boldsymbol{\varepsilon}^{\mu}, \boldsymbol{\varepsilon}^{\mu p}, \dot{D}, \tilde{\boldsymbol{\sigma}}^{\mu}, \mathbf{X}^{\mu}\end{array}}$$

Figure 3: A schematic for the two-scale damage model.

The development of the model can be described by three steps. First of all, stress-strain $(\boldsymbol{\sigma}-\boldsymbol{\varepsilon})$ data at mesoscale can be obtained using the elastic material properties such as Young's modulus and Poisson's ratio (E,ν). Secondly, the scale transition from the meso to micro will be described by a localization law based on the modified Eshelby-Kroner law. Localization law relates the microscale strain $\boldsymbol{\varepsilon}^{\mu}$ to the mesoscale strain $\boldsymbol{\varepsilon}$ through material parameters *a*, *b* and damage *D* given in the second box in Figure 3. Here *a*, *b* are functions of ν based on Eshelby's inclusion theory. Finally we will formulate a plasticity-damage model at the microscale as shown in the third box of Figure 3. The key component of this model is the damage evolution law, which assumes that the damage rate is proportional to the effective plastic strain rate $\dot{p}^{\mu}$, given as

$$\dot{D} = \left(\frac{Y^{\mu}}{S}\right)^{s} \dot{p}^{\mu} \quad \text{if } \dot{p}^{\mu} \geq \dot{p}^{\mu}_{D} \text{ or } w_s \geq w_d \tag{8}$$

Here s, S are the material parameters to be calibrated from available experimental data on fatigue, Y^{μ} is the damage energy release rate and w_s is the stored energy in the material, w_d is the damage threshold energy, $\dot{p}^{\mu}_{D}$ is the threshold effective plastic strain. Threshold values act as a barrier for damage initiation. Once these conditions are met, damage accumulation begins. Threshold values depend on the material and the type of loading condition. The major advantage of employing Eq.(8) is from the fact that fatigue failure is regarded as a function of increment in strain or stress as opposed to function of the load cycles.

A constitutive solver has been developed for the two-scale damage model outlined here and integrated with the space-time FEM framework. The specific algorithm will be similar to that of a standard rate-dependent plasticity-damage model. A mixed explicit-implicit scheme has been implemented.

3. Numerical Example

The developed methodology is applied to the simulation of fatigue response of a single notched rectangular steel plate subjected to cyclic load. The geometry and boundary condition are shown in Figure 4. A fully reversed fatigue load at $\omega = 20Hz$ is applied to the top face of the plate in a form of uniformly distributed pressure of 70 MPa while the bottom surface is fixed, which replicates the loading conditions during the actual fatigue test where the bottom surface is clamped in grips and top surface is subjected to push-pull loading. The plate thickness is taken as $2mm$. The problem is modeled as plane stress problem. Motivation behind this problem is to simulate the complete fatigue loading history using the presented formulation, calculate damage at gauss points during the simulation, and predict the fatigue crack initiation and propagation. Mode I crack growth is expected in this loading situation.

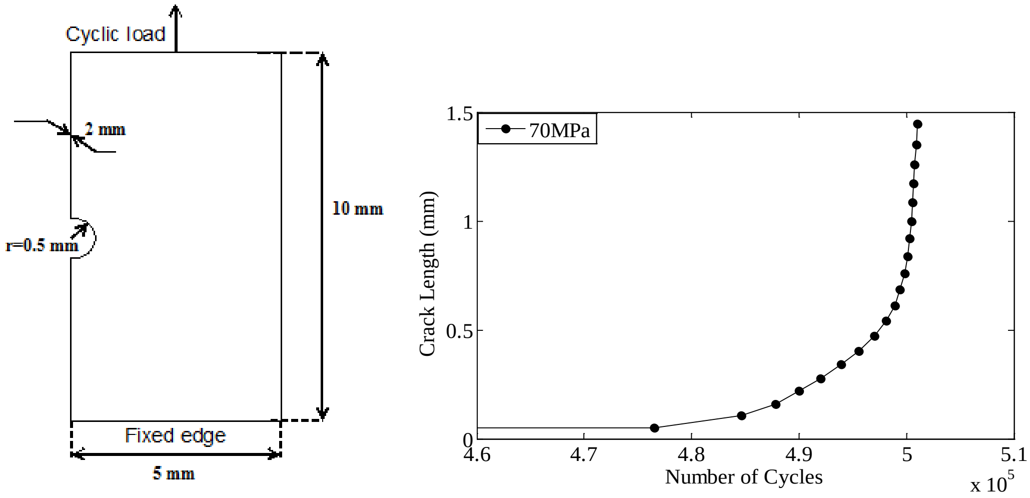


Figure 4: (a) Configuration of the fatigue simulation of a single notched plate. (b) Crack growth vs. number of cycles for the case of completely reversed load of 70 MPa

The space-time discretization employs a multiplicative form of the shape function so that the spatial discretization is independent of the temporal discretization. Advantage of this treatment is that the existing meshing scheme widely used in the case conventional FEM can be directly adopted. In the present case, the plate is discretized by quadrilateral mesh with non-uniform mesh density. Fine mesh is prescribed close to the notch in order to capture crack initiation and growth. In the temporal dimension, quadratic shape function is employed for each space-time slab. The regular space-time shape functions are enriched with a harmonic function with frequency of $20\,Hz$. With the two-scale damage model introduced here, crack initiation and propagation are governed by the evolution of the damage. Numerically, once the damage-based the crack criteria is met, the corresponding space element is eliminated. The material parameters chosen correspond to 304L steel and are based on the earlier work by Lemaitre and Desmorat [9-12]. Detailed implementations of the model is documented in an upcoming paper [13] and will not be presented here due to page limitation.

Crack growth as a function of the load cycles predicted by the simulation is shown in Figure 4. It is worth noting that up to half-million load cycles are successfully simulated with the proposed numerical approach. In addition, the nature of the crack growth data is similar to that obtained from the experiments. This suggests that, with the calibrated material model accurate fatigue simulation for the HCF loading can be performed using the methodology outlined in this paper.

4. Conclusions

In summary, the proposed multiscale space-time formulation with harmonic enrichment function successfully simulates the practical fatigue loading histories. To capture the fatigue crack initiation and propagation, a two-scale damage model algorithm is implemented. Damage model algorithm is integrated with the multiscale space-time FEM formulation. Standard techniques of *element deletion* are used to predict the fatigue crack initiation and propagation. HCF simulations are performed on the notched plate without any *ad hoc* procedure to predict the total fatigue life. Based on the simulations, crack length verses number of cycles curve is generated. The trend of the results is similar to what is observed in the experiments. Mode I crack growth is successfully using XTFEM. Future plan is to verify the predictions by comparing with either literature data or analytical model (e.g., Paris law crack growth predictions). In addition, different fatigue-based material model can be integrated with the framework presented.

Acknowledgements

The authors gratefully acknowledge the financial support of the start-up fund from the University of Texas at Dallas and the ASEE Air Force Summer Faculty Fellowship. This work was also supported in part by an allocation of computing time from the Ohio Supercomputer Center.

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