A Prediction Model for Low Cycle Fatigue Crack Initiation under Axial Loading

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Abstract: A prediction model for low cycle fatigue crack initiation was proposed, based on damage mechanics theory, which indicated the relation of equivalent value *d* of low cycle fatigue crack initiation to stress attenuation percentage $\triangle S/S$. Then, parameter *d* will be calculated if $\triangle S/S$ is given, and the corresponding cyclic life is initiation life of fatigue crack. To validate the model, a group of 10CrNiMo structural steel specimens with a round-section were tested through low cycle fatigue test controlled by constant strain under axial loading, and the results manifested that the model is dependable and applicable; and the error of *d* to really measured value is not more than 5.0%; when the total strain amplitude is 0.6%, *d* is close to 2.0mm and the initiation life is approximately 900 cycles, which is nearly 80% of the entire life.

Keywords: Low cycle fatigue, Crack initiation，Prediction model, Initiation life

1. Introduction

In engineering, a crack which can be observed through a microscope with a low magnification (10×～20×), usually was defined as crack initiation and its length is 0.13mm to 0.25mm [1,2]. In design, crack initiation was deemed as a range from 0.25mm to 2.5mm [3]. When a crack increases to a certain length such as 0.1mm, 0.5mm, or 1.0mm, relevant fatigue cycle is called initiation life of fatigue crack. In general, there are three methods to measure crack initiationi, replication technique, potential drop method and observation through a microscope with a long focal length. Basically, the three methods have a high precision, and can meet engineering requirements. But all of them are inconvenient in real application [4]. In the fields of crack initiation and life prediction, many scholars proposed several models based on different parameters, for examples, Manson-Coffin Model [5] beginning with macro-mechanics, Mesomechanics Model [6] to describe microcosmic dispersivity using macroscopic parameters, Probability Model [7] calculating crack initiation life taking microcrack bulk density as index and so on. Many problems appeared when above models applied in reality [8]. In this paper, a new prediction model for low cycle fatigue crack initiation was proposed, based on damage mechanics theory, by which equivalent value of crack initiation could be calculated in case of an assuming condition, and crack initiation life could be measured combining with some simple tests, which would provide some reference for engineering design.

2. A prediction model for low cycle fatigue crack initiation based on damage mechanics

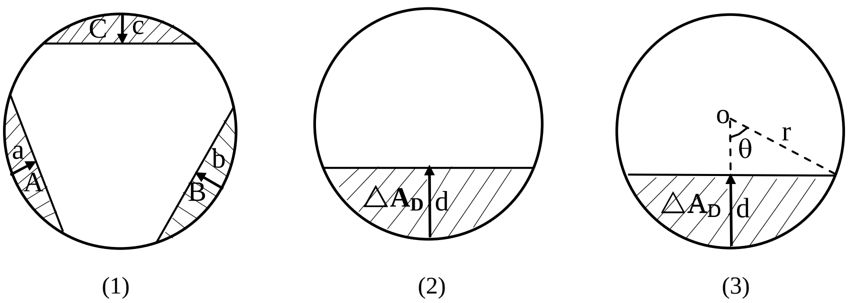
Damage mechanics is a subject studying evolution law of damage field of the deformable solid, including damage and its effect on mechanical properties of materials under action of outside factors such as load, changing temperature and corrosion and so on. Based on damage mechanics theory, a relation was deduced in literature [8] as follows:

$$\frac{\Delta A}{A} = \frac{\Delta S}{S} \tag{1}$$

where △A and △S were after damaged area, stress amplitude , accordingly.

Constant strain is admitted as control parameter generally, so low cycle fatigue is also called strain fatigue. Under cyclic action of constant strain, grains in material may come into microcracks because of repeated slips, which will damage the material. Simultaneously, effective area of the material will decrease gradually and capacity enduring the load may drop and nominal stress acted on the material will attenuate step by step. Supposing a crack or several cracks initiated, and damage effect produced by several cracks initiated in the same section or different sections is equivalent to one crack, then attenuation of the stress and the length of the crack should meet a certain mathematical relation. When cracks don't initiate, stress will keep constant; and cracks initiated and propagated to fracture the material, stress would drop to zero. That's to say, ratio of reduction of the specimen area perpendicular to the direction of the crack propagation to original specimen area should be equal to ratio of attenuation of the current stress to steady stress, that is Eq. (1). Reduction of the specimen area and length of the crack propagation should satisfy some mathematic relation, which would provide a basis of damage mechanics theory for setting up the expression of a equivalent value of crack initiation to stress attenuation. Attenuation of stress can be measured by a test, and the equivalent value of crack initiation be also obtained through the built expression, and corresponding cycle is called crack initiation life.

Literatures [9,10] indicated that several cracks initiated in the course of low cycle fatigue. It could be seen that several cracks sometimes initiated in the same section perpendicular to loading orientation, sometimes not from many tests. No matter one crack or several cracks initiated in the same section or not, damage effect should be deemed as that of one crack in one section. Figure 1 reveals above analysis. Parameters a, b and c are crack lengths located in one section or different sections, and parameters A, B and C are corresponding areas swept by above three cracks, see figure 1(1) for details. Damage effect produced by above three cracks or more equals to that by one crack *d*, with a corresponding swept area $\triangle A_{D,}$ see figure1(2). Figure1(3) shows the geometry relations of equivalent crack length *d*, its swept area $\triangle A_D$ and specimen radius *r*, and also is calculation model to predict crack initiation.



(1) (2) (3)

(1) Several cracks and corresponding swept areas (2) Equivalent to one crack and an area (3) Sketch of calculation model

Fig.1 Calculation model predicting crack initiation based on damage mechanics theory

From figure 1(3), some equations may be obtained:

$$\theta = \arccos\frac{r-d}{r} \tag{2}$$

$$\Delta A_D = r^2\theta - r(r-d)\sin\theta \tag{3}$$

$$d = r - \frac{r\theta - \pi\Delta A_D\theta}{\sin\theta} \tag{4}$$

From above analysis, ratio of reduction of the specimen area to original area $\Delta A_D/A_D$ should equal to ratio of attenuation of nominal stress to steady stress ΔS/S, namely:

$$\frac{\Delta A_D}{A_D} = \frac{\Delta S}{S} \tag{5}$$

Based on above equations, when specimen radius r is determined (e.g. 5mm), the relation of equivalent length d of crack initiation and ΔS/S will be built. Through test data processed (see figure2), the following equation would be set up:

$$\frac{\Delta S}{S} = -0.1186d^3 + 1.7765d^2 + 4.1698d - 0.5006 \quad (R=1) \tag{6}$$

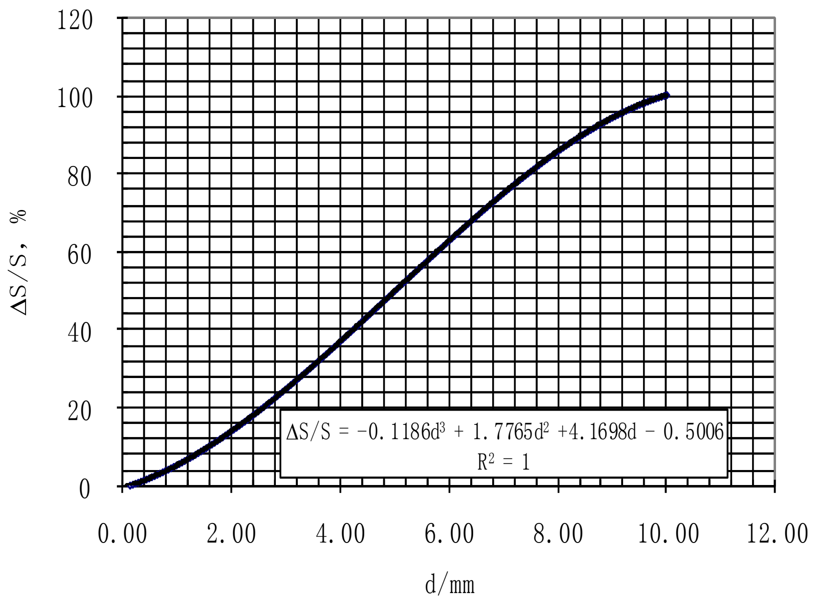


Fig.2 Relation curve and corresponding equation of △S/S-*d*

3. Correction for prediction model for low cycle fatigue crack initiation

Considering deviation of damage effect of one crack equivalent to several cracks in one section or several sections, damage coefficient λ (0～1) was introduced. Simultaneously, many factors including some measurement errors of attenuation of stress, areas swept by cracks and section distortion when specimen fractured and so on, which would directly influence calculation precision of equivalent crack length. Supposing calculating error is d_0 (a constant relevant to material), then expression (6) can be further amended:

$$\frac{\Delta S}{\lambda S} = -0.1186(\mathrm{d}+\mathrm{d}_0)^3 + 1.7765(\mathrm{d}+\mathrm{d}_0)^2 + 4.1698(\mathrm{d}+\mathrm{d}_0) - 0.5006 \quad (R=1) \tag{7}$$

From equation (7), we know, firstly, through a simple test, for each specimen, solve damage coefficient λ and material constant d_0, then, whatever equivalent length *d* of crack initiation is defined,

△S/S will be calculated .Vice versa, we can get *d* from the known ΔS/S and corresponding cycle is crack initiation life.

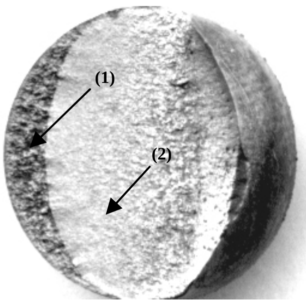
It could be seen from many tests, when ΔS/S reached 50% (corresponding equivalent crack length was 5mm), specimen fractured immediately. So, for formulas (6) and (7), the following boundary conditions will be satisfied: $0 \leqslant \triangle S/S \leqslant 0.5$, $0 \leqslant d \leqslant r$.

4. Verification of prediction model for low cycle fatigue crack initiation

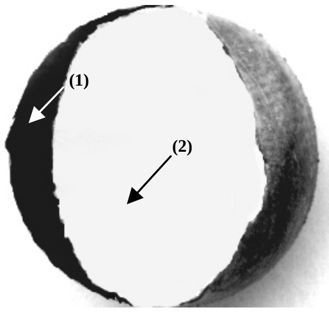
In order to ensure correctness of the built prediction model, related test were carried out, with the following test conditions: four pieces used with series of number 1 to 4, constant strain amplitude 0.6%, test frequency 0.2Hz; when percentage of stress attenuation increased to 5.33%, 9.35%, 14.0%, 19.17% respectively (corresponding equivalent crack length was 1.00mm, 1.50mm, 2.00mm and 2.50mm accordingly), test ended. Afterwards, the tested specimens were heated about 30 minutes at a temperature from 300℃ to 400℃ and then fatigued again, and finally all specimens were fractured into two parts. Adopting PHOTOSHOP tool software, daubing fatigue crack propagation district black and re-fatigue and fractured distracts white, see figure 3(a) and (b).

Figure 3 showed that the specimen fracture appearance had different degree distortion, and measured area swept due to crack propagation was not uniform with the real area by reason of measurement error. So, it is necessary to correct the prediction model.

Percentage of black distract to the entire fractured area was measured by OLYCIAm3 Image Analysis System, and equivalent crack length was calculated per equation (7). Relative errors of real crack length to calculated crack length from prediction model, λ value of each specimen, and corrected value d_0 (difference between average of real equivalent crack length and that from prediction model) were listed in table1.



(a) before (magnification 5×)



(b) after (magnification 5×)

(1) District of fatigue crack propagation (2) Districts of re-fatigue and fracture

Fig.3 Fracture appearance of the specimen processed before and after

Tab.1 Comparison of real value and calculated value from prediction model of equivalent length of crack initiation

Spec. No.	Specimen diameter /mm	ΔS/S /%	Damage coefficient λ	Measured $\Delta A_D/A_D$ /%	△A_D /mm^2	Calculated crack length /mm	Constant d_0/mm	Real crack length /mm	Corrected crack length /mm	Relative error /%
1	10.07	5.33	0.33	16.32	12.82	1.0	1.10	2.20	2.10	4.39
2	10.08	9.35	0.47	19.85	15.59	1.5		2.54	2.60	-2.46
3	10.07	14.00	0.53	26.56	20.86	2.0		3.15	3.10	1.61
4	10.10	19.17	0.57	33.545	26.35	2.5		3.74	3.60	3.72

Table 1 shows that parameter d_0 really is a constant. Although there is some difference between average of real equivalent crack length and that from prediction model, relative error is not more than 5.0%, which manifests that the prediction model is reliable and applicable.

5. Predetermination crack initiation life by means of the prediction model

Generally, for low cycle fatigue test using smooth specimen, specimen will fail quickly once crack initiates, due to larger stress and strain acted on the specimen. For most of metallic materials, crack initiation life will dominantly take up 80% or so of the entire life. It is very difficult to directly measure crack initiation life through normal low cycle fatigue tests using smooth specimen. In order to investigate applicability of the prediction model, relation of crack initiation life to entire life was studied below.

Simple-specimen-method was employed in order to avoid data distribution. Amount of cycle, corresponding to percentage of stress attenuation which relates to some equivalent crack length, is crack initiation life of the equivalent crack length. Different literatures defined different crack initiation, and it is difficult to determine one proper equivalent value for steel such as 10CrNiMo structural steel, which will rely on concrete test. In this paper, three of specimens were performed, with series of number 5 to 7, and percentage of stress attenuation was supposed to 0.65%, 2.0%, 5.33%, 9.35%, 14.0%, 19.17%, 20%, 30% and 50% from a low-level to a high-level, recording relevant repeated cycles. See figure 4 for final test results.

Curve of △S/S-N in figure 4 indicates that with cycle increasing, percentage of stress attenuation is improved, and when the cycle is close to 800 to 1000 cycles, specimens fail quickly with a high speed. Based on above analysis, when repeated cycle attains to 800 to 1000 cycles, it is suitable taking corresponding equivalent crack length as crack initiation for 10CrNiMo steel. In the range from 800 to 1000 cycles, △S/S of the three specimens are all 5.33% and corresponding equivalent length of crack initiation is 1.0mm. Considering of correctness of the prediction model, real crack initiation should be as 2.1mm. See table 2 for data analysis of three specimens.

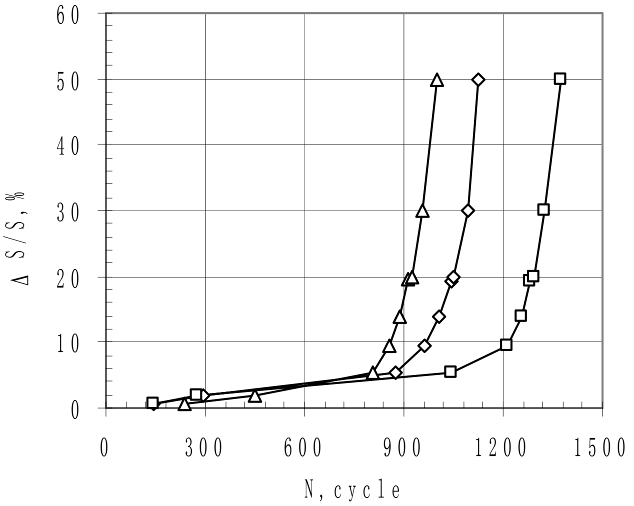


Fig.4 Relations of percentage of stress attenuation to cycle N

Table.2 Data of crack initiation and initiation life

Specimen No.	Percentage of stress attenuation ΔS/S， %	Crack initiation length a_0/mm	Crack initiation Life N_i	Entire life N	N_i/N /%
5	5.33	2.10	808	1003	80.6
6	5.33	2.10	876	1228	77.7
7	5.33	2.10	1044	1372	76.1
average	5.33	2.10	909	1168	78.1

Table 2 indicate that equivalent value of crack initiation of the structural steel 10CrNiMo is close to 2.0mm and initiation life is approximately 900 cycles, which is nearly 80% of the entire life.

6. Conclusions

(1)Any equivalent value of crack initiation, corresponding percentage of stress attenuation, can be calculated and crack initiation life be predetermined, according to prediction model for low cycle fatigue crack initiation proposed by damage mechanics theory, combining with some simple tests.

(2)Relative error between average of real equivalent crack length and that from prediction model is not more than 5.0%, which manifests that the prediction model is reliable and applicable.

(3) When the total strain amplitude is 0.6%, equivalent value of crack initiation of the structural steel 10CrNiMo is close to 2.0mm and initiation life is approximately 900 cycles, which is nearly 80% of the entire life, on the basis of prediction model and related tests, which will be referred to in engineering application and design.

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