On the tolerance to short cracks departing from notch roots

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Abstract

It is well known that it is impossible to guarantee that structural components are really free of cracks smaller than the detection threshold of the non-destructive method. Nevertheless, most components are still designed against fatigue crack initiation using procedures that do not recognize such cracks. Consequently, their "infinite life" predictions may become unreliable when cracks are introduced by any means and not quickly detected or properly removed. Therefore, structural components that must last for very long fatigue lives should be designed to be tolerant to undetectable short cracks. Indeed, continuous work under fatigue loads cannot be guaranteed if any crack can propagate during their service lives. Since most structural components designed for long lives work in spite of not recognizing such cracks, and they certainly are somehow tolerant to short cracks. However, the question "how much tolerant" cannot be answered by traditional fatigue design procedures alone, but such a problem can be avoided by adding proper short crack concepts to their "infinite" life design criteria. This work proposes such a damage-tolerance requirement to quantify the behavior of short cracks. This methodology can also be used to quantify the difference between the fatigue K_f and the static stress concentration factor K_t.

Keywords: Fatigue; Short crack; stress gradient; non-propagating crack; tolerance

1. Introduction

During the manufacture or service of structural components, small defects ranging in length from tens microns to several millimeters are unavoidable. Industrial experiments have shown that the rupture of components is often produced by these "short cracks", which develop in most cases from the instinct faults of these structures and propagate when they are subjected to cyclic loading. As the initiation fatigue life is largely influenced by the behavior of short cracks, not taking into account of the propagation of short cracks can lead to potentially dangerous overestimation of fatigue life [1, 2]. It is therefore essential to thoroughly understand their growth rules and effectively predict the structure life by considering the initiation and propagation of short cracks.

As we know, growing short cracks from sharp notches can stop completely even when the remote applied stress amplitude remains constant. This phenomenon was discovered as early as 1949 by Frost [3] as shown in Fig.2. In the succeeding research of short cracks, inspired by Frost's report about the non-propagating cracks, people normally tend to focus on the behavior of short cracks departing from the sharp notch root which corresponds to the evident notch sensitivity coefficient and significant stress gradient. It has a non-propagating cracks zone where the cracks initiated from a sharp notch ($K_t > 3$); and the driving force ΔK, which should increase along with the short cracks growth, is not high enough to propagate the cracks under the constant remote applied stress and K_t. These analyses contribute to the quantification of fatigue crack initiation life and non-propagating crack length. Hence, it is certainly reasonable to expect that such phenomenon can be used to quantitatively explain why the fatigue stress concentration factor K_f differs from the conventional stress concentration factor K_t of sharp notches. The stop of short cracks has been attributed to the decrease of ΔK as short cracks lengthen, which relates to significant stress gradient at the edge of notch.

Recently, Castro et al. have indicated in [4] that the high stress gradient at the narrow notch root was drastically reduced due to the short crack growth, which may lead to the non-propagating crack phenomena. In the present paper, based on their analytical methodology, a modified method has been provided to quantify the short crack behavior. And the variation of the stress gradient and the resulting ΔK along with the short cracks growth has been presented.

2. Stress gradient at the sharp notch root along with the short cracks growth

It is well known that the stress concentration is usually caused by geometrical discontinuity or heterogeneity of microstructure which involves an appearance of the maximum stress compared to the calculated median values based on the smooth section. In mechanical design, one attaches rather importance to the high stress fields which arise in the majority of the industrial components (shoulder, holes, fillets etc.). The stress concentrations can be regarded as the origin of the fatigue cracks or the unstable ruptures.

In this paper, the authors are devoted to the investigation of the stress concentrations due to the geometry imposed by the industrial design. The notch effect is modeled by an increase of the local stress in a restricted volume, compared to the distribution of nominal stress. However, the concept of stress gradient can be generalized with any cross-section of a specimen or with any volume element of a mechanical component. Therefore, for a plate containing cracks at the bottom of an elliptic notch as illustrated in Fig. 1, the stress σ_y, which acts in the remaining ligament of an infinite plate with an elliptic hole, is given by Eq. (1) at points ($x \geq b$, 0) (see [5]).

$$\frac{\sigma_y}{\sigma_n} - 1 = \frac{\left(b^2 - 2bc\right)\left(x - \sqrt{x^2 - b^2 + c^2}\right)\left(x^2 - b^2 + c^2\right) + bc^2\left(b - c\right)x}{\left(b - c\right)^2\left(x^2 - b^2 + c^2\right)\sqrt{x^2 - b^2 + c^2}} \tag{1}$$

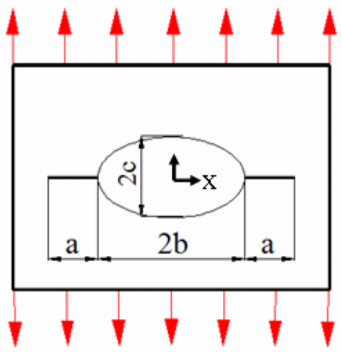


Fig. 1. Centre elliptic notch with a fatigue cracks formed at the notch roots

The σ_y/σ_n can be defined as the stress concentration factor in the presence of short crack K_{tf}. Thus, the gradient of σ_y at the bottom of notch (at the edge of the elliptic hole) is given by the following equation:

$$d\sigma_y / dx\big|_{x=b} = -\left(2K_t + 1\right)\sigma_n / \rho = -\left(3 + 4b/c\right)b\sigma_n / c^2 \tag{2}$$

The stress gradient increases with the increase of K_t and/or the reduction of ρ, where $K_t = 1 + 2\sqrt{b/\rho}$ according to [6]. In this paper, five different configurations with the same dimension of elliptic notch b = 27.5 mm (see Table 1) have been analyzed, and five radii values of which have been tested on a purely comparative basis (ρ = 0.5, 1, 2.5, 3 and 27.5mm), which correspond to the critical value of the elliptic notch.

ρ (mm)	b (mm)	c (mm)
0,5	27,5	0,5
1	27,5	1
2,5	27,5	2,5
3	27,5	3
27,5	27,5	27,5

Table 1. Notches dimensions

The high value $K_{tf} = \sigma_y/\sigma_n$ in the presence of a short crack is always localized close to the edges of the hole according to the curves of Fig. 2. In fact, in the case of an elliptic notch ($b > c$), the ratio of the stress $\sigma_y(x)$ along the remaining ligament and the nominal stress σ_n, i.e. K_{tf}, is roughly equal to 3 for the distance x = 1.1b. This ratio falls to a value of 2 independently of the radius ρ when x reaches approximately 1.2b. The Eq. (3) enables us to quantify the influence of the stress gradient on the variation of the K_{tf} according to x/b.

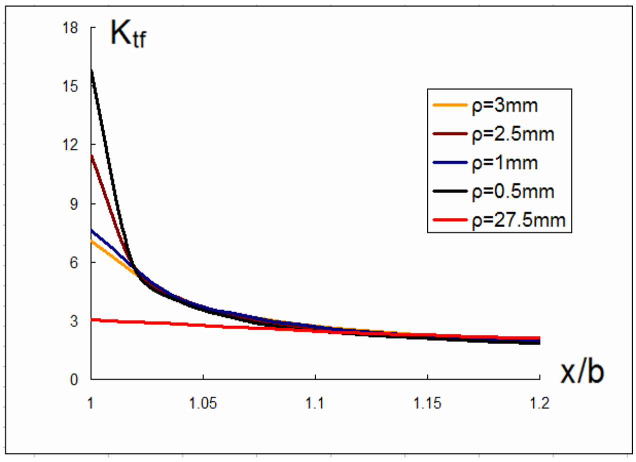


Fig. 2. Evolution of the stress concentration in the presence of a short crack

Fig. 3 illustrates explicitly the relationship between the stress concentration K_t (without crack) and the stress concentration ($K_{tf} = \sigma_y/\sigma_n$) (in the presence of a crack) with respect to the distance away from the edge of the notch. The reduction of K_{tf}/K_t is practically linear and not very sensitive to the value of K_t. The ratio decreases steeply from 1 for a point located at the end of the notch to 0.9 for a point which is very close to the crack length $a = b/20$. This rather high stress gradient can cause an initiation of the fatigue short crack, even in the case of a nominal stress lower than the yield stress.

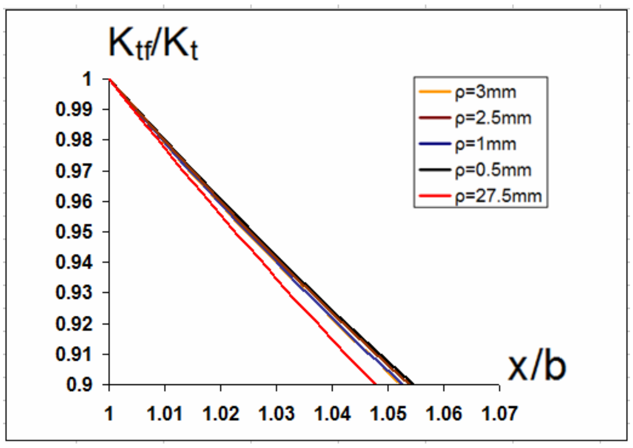


Fig. 3. Evolution of the ratio K_{tf}/K_t with respect to the relative length of the short crack

After the above discussion of the stress gradient with the propagation of short cracks at the sharp notch root, the variation of ΔK which associates with the fatigue stress concentrations K_f can be better understood. The K_f is defined as the ratio of the fatigue limit of a smooth specimen, S_0, to that of a notched specimen, S_0', under the same experimental conditions. Obviously, it is related to the size and geometry of the specimen and dimension of the notch. However, to determine experimentally the K_f with the grand variation of the notch shape (from the blunt notch to the critical condition "crack") is too laborious. Thus the analytical method based on some experimental results to predict the K_f is rather necessary. Indeed, if the crack is regarded as the critical condition of "notch", the nominal stress which can initiate the existing crack will be equal to the value of S_0'. That implies that the K_f is not only the key point for the crack initiation life analysis but also for the short crack growth estimations.

In 1979, El Haddad et al. associated the fatigue limit with the crack threshold through the Eq. (3) – Eq.(4) [7].

$$\Delta K = \eta \cdot \Delta\sigma \sqrt{\pi(a+a_0)} \geq \Delta K_0(a) = \Delta K_0 \cdot \sqrt{a/(a+a_0)} \quad (3)$$

$$a_0 = (1/\pi)\cdot\left(\Delta K_0/(\eta\cdot\Delta S_0)\right)^2 \quad (4)$$

Following Bazant's reasoning in [8], a more general threshold expression as Eq. (5) was presented by introducing an adjustable parameter γ to fit experimental data as described by Eq.(5)

$$\Delta K_0(a) = \Delta K_0 \cdot \left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \quad (5)$$

For the case where the crack is much smaller or much larger than the notch dimensions, the stress intensity range of a semi-elliptical notch has been mentioned in [9] with the expression in Eq.(6) and Eq.(7).

$$\Delta K = \eta \cdot K_t \cdot \Delta\sigma \cdot \sqrt{\pi(a+a_0)} \qquad \text{for} \quad a \ll b \quad (6)$$

$$\Delta K = \eta \cdot \Delta\sigma \cdot \sqrt{\pi(a+b)} \qquad \text{for} \quad a \gg b \quad (7)$$

For the case of very small crack with $a \ll a_0$, $\Delta\sigma$ is the notch root stress range; for the long crack with $a \gg b$, $\Delta\sigma$ tends to the nominal stress $\Delta\sigma_n$. In most cases, the K_t is used to indicate the stress concentration factor of the notch root without the crack growth. Hence, the function $\varphi(a)$ in Eq. (8)

can be used to reflect the effects of the notch root stress concentration instead of K_t. It is remarkable that the $\varphi(a)$ tends to the notch root stress concentration factor as the crack length a tends to zero.

$$\Delta K = \alpha \cdot \varphi(a) \cdot \Delta\sigma \cdot \sqrt{\pi(a + a_0)} \tag{8}$$

According to these theories, Meggiolaro et al. have proposed an effective method which could analytically estimate the K_f (see [9]). They associated ΔK and $\Delta K_{th}(a)$ so that the short crack will propagate when:

$$\Delta K = \eta \cdot \varphi(a/\rho) \cdot \Delta\sigma\sqrt{\pi a} > \Delta K_{th}(a) = \Delta K_0 \cdot \left[1 + (a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{9}$$

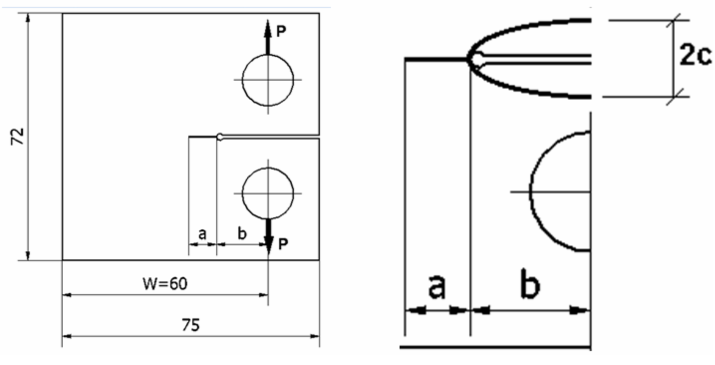
Accordingly, a crack propagation criterion was stated using two dimensionless functions φ and g with the following expression:

$$\varphi\left(\frac{a}{\rho}\right) > \frac{\left(\Delta K_0 / \Delta S_0\sqrt{\rho}\right) \cdot (\Delta S_0/\Delta\sigma)}{\left[\left(\eta\sqrt{\pi a/\rho}\right)^{\gamma} + \left(\Delta K_0/\Delta S_0\sqrt{\rho}\right)^{\gamma}\right]^{1/\gamma}} \equiv g\left(\frac{a}{\rho}, \frac{\Delta S_0}{\Delta\sigma}, \frac{\Delta K_0}{\Delta S_0\sqrt{\rho}}, \gamma\right) \tag{10}$$

The second term $\Delta S_0/\Delta\sigma$ in the function g will be equal to the K_f when the $\Delta\sigma$ corresponds to the minimum stress range that can cause crack initiation and propagation from the notch border by fatigue, without arrest.
In general, K_f and a_{max} can always be found by solving the system

$$\begin{cases} \varphi/g = 1 \\ \partial(\varphi/g)/\partial x = 0 \end{cases} \Rightarrow \begin{cases} \varphi\left(\dfrac{a_{max}}{\rho}\right) = g\left(\dfrac{a_{max}}{\rho}, K_f, \kappa, \gamma\right) \\ \partial\varphi\left(\dfrac{a_{max}}{\rho}\right) \Big/ \partial x = \partial g\left(\dfrac{a_{max}}{\rho}, K_f, \kappa, \gamma\right) \Big/ \partial x \end{cases} \tag{11}$$

To verify the accuracy of the models to evaluate the tolerance to short cracks, a methodology is proposed to design notched fatigue test specimens specially conceived to induce non-propagating short cracks. The geometric configuration is that of a modified compact tension specimen C(T), having a machined notch with a circular hole with radio ρ at the tip, as is shown in 错误！未找到引用源。a. The line of size a represents the crack length. The external dimensions and the notch length $b = 15(mm)$ is chosen according to ASTM E647 recommendations to assure that the loading condition at the crack tip is not influenced by the loading-pin holes.



(a) (b)

Fig. 4 a) Modified C(T) specimens, dimensions in mm.

b) Approximation to a semi-elliptical notch.

The specimen notch is approximated by a semi-elliptical notch with semi-axes b and c (see **Fig.** b). The value of c can be expressed as a function of the notch root tip ρ, $\rho = c^2 / b$.

From an elastic stress analysis, considering the C(T) as a cantilever beam, the nominal stress range applied at the notch can be calculated as

$$\Delta\sigma_n = \frac{\Delta P}{t \cdot (W-b)} \cdot \left(1 + 3 \cdot \frac{W+b}{W-b}\right) \quad (1)$$

Therefore, after the value of $\Delta\sigma_n$ is determined by solving the system of equations in Eq. (11), the load ΔP to be applied to the specimen will be found by applying Eq. (12), in the next section are presented the numerical results for that combination.

3. Numerical results

Assuming the fatigue limit of smooth specimen as $S_L' = 0.5 \cdot S_u$, for a load ratio $R = -1$ (fully reversed loading), by Goodman it can be interpolated for $R = 0$ (pulsating tension), resulting in $\Delta\sigma_0 = 2 \cdot S_u / 3$.

Following the Frost's statement, it is the difference between K_t and K_f that define the generation of non-propagating cracks. The figure also shows the stress concentration factor K_t and how its value tends to K_f as the notch root ρ increases. Therefore, for this material and specimen configuration, notch with root radii $\rho <\sim 1.5$ will be able to generate non-propagating cracks (see **Fig. 5**).

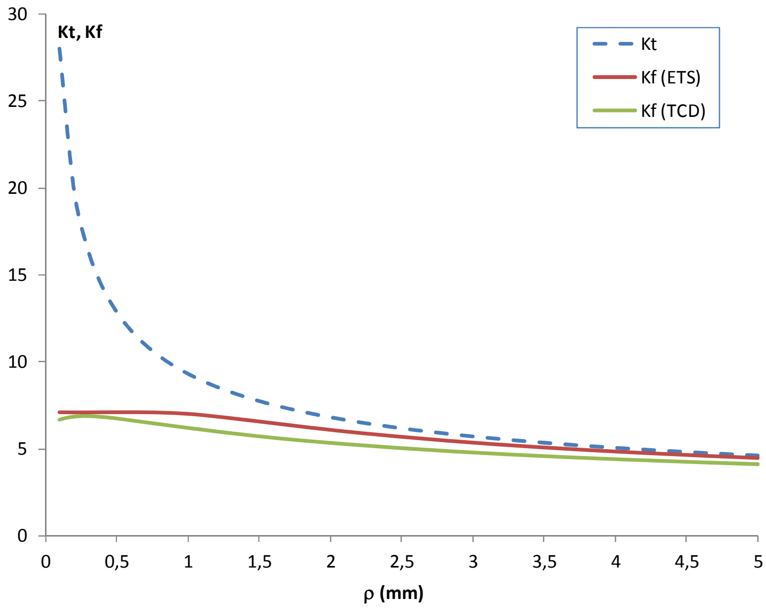


Fig. 5 Comparison of predictions of the notch fatigue factor K_f with the stress concentration K_t as a function of the notch root ρ.

In addition to K_f, the model also allows calculating the largest non-propagating crack $a_{\max}$ that can arise from fatigue alone. **Fig.** shows the value of $a_{\max}$ as a function of the notch root ρ.

Ideally, it would be better to deal with high values of ρ because they are easier to machine at the notch tip. In the other hand, the smaller the notch root radius ρ, the greater the maximum non-propagating crack $a_{\max}$ is, and, consequently, the more reliably the method can be applied to predict non-propagating cracks that can be robustly measured. According to the numerical results shown in **Fig.** , for $\rho <\sim 1.5$ it can be expected that the maximum non-propagating crack size should be $0.309mm < a_{\max} < 0.83mm$. Those values can be easily measured by an optical microscope.

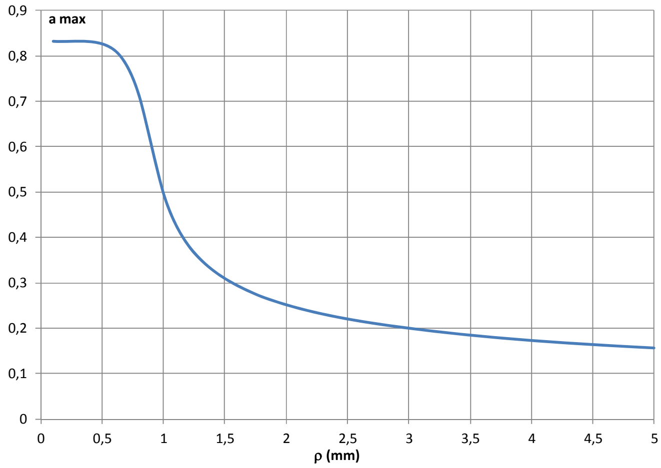


Fig. 6 Comparison of predictions of the maximum non-propagating crack a_{max} as a function of the notch root ρ.

4. Conclusions

A method was used to predict the notch factor K_f and, therefore, the fatigue limit for a notched specimen can be designed to induce non-propagating short cracks at the notch root. The configuration of the notch was approximated as a semi-elliptical notch and its fatigue limit was determined as a function of the radius of the notch root ρ. The largest non-propagating crack length can be calculated.

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