

# Simulating the bluntness of TBM Disc Cutters in Rocks using Displacement Discontinuity Method

Hadi Haeri<sup>1,\*</sup>, Kourosh Shahriar<sup>2</sup>, Mohammad Fatehi Marji<sup>3</sup>, Parviz Moarefvand<sup>4</sup>

<sup>1</sup>PhD candidate of rock mechanics, Mining Engineering Department, Science and Research Branch, Islamic Azad University, Poonak, Hesarak, Tehran, Iran

<sup>2</sup>Prof., Department of Mining and Metallurgical Engineering, Amirkabir University of Technology, Tehran, Iran

<sup>3</sup>Associate Prof., Head of Mine exploitation Engineering Department, Faculty of Mining and Metallurgy, Institution of Engineering, Yazd University, Yazd, Iran

<sup>4</sup>Assistant Prof., Department of Mining and Metallurgical Engineering, Amirkabir University of Technology, Tehran, Iran

\* Corresponding author: hadihaeri@ymail.com

---

## Abstract

Underground accessibility includes several steps which the cutting step is the one of most basic and most important of them. Indentation of TBM disc cutters into the rock and producing of the chips in different scales are well known as an indentation process. This phenomena passé from producing the micro cracks to the coalescing into macro cracks. The present research focuses on the linear elastic fracture mechanics of rock and maximum shear stress criterion to investigate the micro crack propagation and its direction under disc cutters. A higher order indirect boundary element method (using quadratic displacement discontinuity elements) is used to estimate the stress intensity factors in rocks under single disc cutter. Also to eliminate errors caused by stress and displacement singularity near the crack tip, three special crack tip elements are used. As the TBM disc cutters will be eroded after a period of working, the effects of eroded and not eroded discs are numerically modeled, analyzed and compared with each other. To create the eroded disc model, we considered 4 small elements to generate the curvature of the cutter tip, which reducing the computing efficiency and increasing the required specific energy for chips formation.

**Keywords:** eroded disc, micro crack propagation, specific energy, SDDM, TBM

---

## 1. Introduction

One of the most complex engineering problems is cutting and indentation of mechanical tools into the rock which has been considered by human primitively. The investigation on rock fracture mechanics with mechanical tools in mining and civil operations is significant duo to its widespread applications. These investigations can be used to predict and estimate rock fracture mechanism for optimization and risk-reducing of planning, temporal and economic schematization [1]. The key parameters in TBM head design are head diameter, number of cutters, thrust force, rolling force, RPM, penetration depth, and cutter spacing [2, 3 and 4]. Many experimental studies and numerical simulations have been reported for the breakage mechanism of rock under disc cutters of tunnel boring machines (TBMs) [5-16]). Recently, Cho et al [4] investigated the optimum spacing of TBM

disc cutters using numerical simulations. Their results are in good agreement with those obtained from experimental studies (LCM). Propagation and coalescence mechanism of micro cracks due to indentation of TBM disc cutters into the rock and production of chips in indent process is highly complex and important. The mechanism of micro crack propagation and coalescence has not been widely studied and its real reasons (micro crack propagation and coalescence) have not been fully found. Because of the complexity of the micro crack propagation and coalescence problem under TBM disc cutters, nowadays, numerical or analytical-numerical approaches are mostly used for simulating the rock breakage mechanism [17]. A numerical model, the higher order semi-infinite displacement discontinuity (HOSDD<sub>2D</sub>) code, a two-dimensional code based on the linear elastic fracture mechanics (LEFM) which uses quadratic displacement discontinuity formulation with three special crack tip elements at each micro crack initial and end is used to simulate the rock breakage mechanism. Based on this numerical method, stresses near the crack tip and distribution of displacements can be clearly defined to determine the accurate strain energy release rate and stress intensity factors. There are three important fracture initiation criteria, which are applicable in action: the maximum tangential stress ( $\sigma_\theta$ -criterion), the minimum energy density criterion (S-criterion) and the maximum energy release rate (G-criterion) or any modified form of those mentioned issues, (e.g. F-criterion which is A modified energy release rate criterion) has mostly been used to study the breakage mechanism of rock [18-24,14]. Although this criteria act prosperously for predicting the crack initiation under TBM disc cutters, the maximum tangential stress criterion is used here to predict the direction of micro crack initiations resulted from artifact cracks of TBM disc cutters. In the present research, the rock breakage mechanism under eroded disc cutters of tunnel boring machines (TBMs) is modeled and studied by the proposed method. A comparison of results between eroded and non-eroded disc models is presented.

## 2. Higher Order Displacement Discontinuity In a Half-plane

The semi-infinite displacement discontinuity method (SDDM) is an indirect boundary element method that is solved problems on the basis of fracture mechanics principles given the boundary conditions and calculates stresses and displacements at discontinuities in all boundary elements. cruch and starfield [25] used the analytical solution of a constant element displacement discontinuity, over the line segment  $|x| \leq a, y = 0$  in the semi-infinite area  $y \leq 0$  as shown in Fig. 1. For complete computaion of displacements and stresses in semi-infinite body, due to the real displacement discontinuity, its portrait and its resulting from the supplementary solution are denoted by  $u_i^R$  and  $\sigma_{ij}^R, u_i^P$  and  $\sigma_{ij}^P, u_i^S$  and  $\sigma_{ij}^S$ , respectively.

The complete solution for the semi-infinite plane  $y \leq 0$  can be expressed as:

$$u_i = u_i^R + u_i^P + u_i^S \quad \text{and} \quad \sigma_{ij} = \sigma_{ij}^R + \sigma_{ij}^P + \sigma_{ij}^S \quad (1)$$

$\bar{x}, \bar{y}$  and  $x, y$  are local coordinates and global coordinates, respectively, that can be transformed by the following two formulas:

$$\begin{aligned} \bar{x} &= (x - c_x) \cos \varphi + (y - c_y) \sin \varphi \\ \bar{y} &= -(x - c_x) \sin \varphi + (y - c_y) \cos \varphi \end{aligned} \quad (2)$$

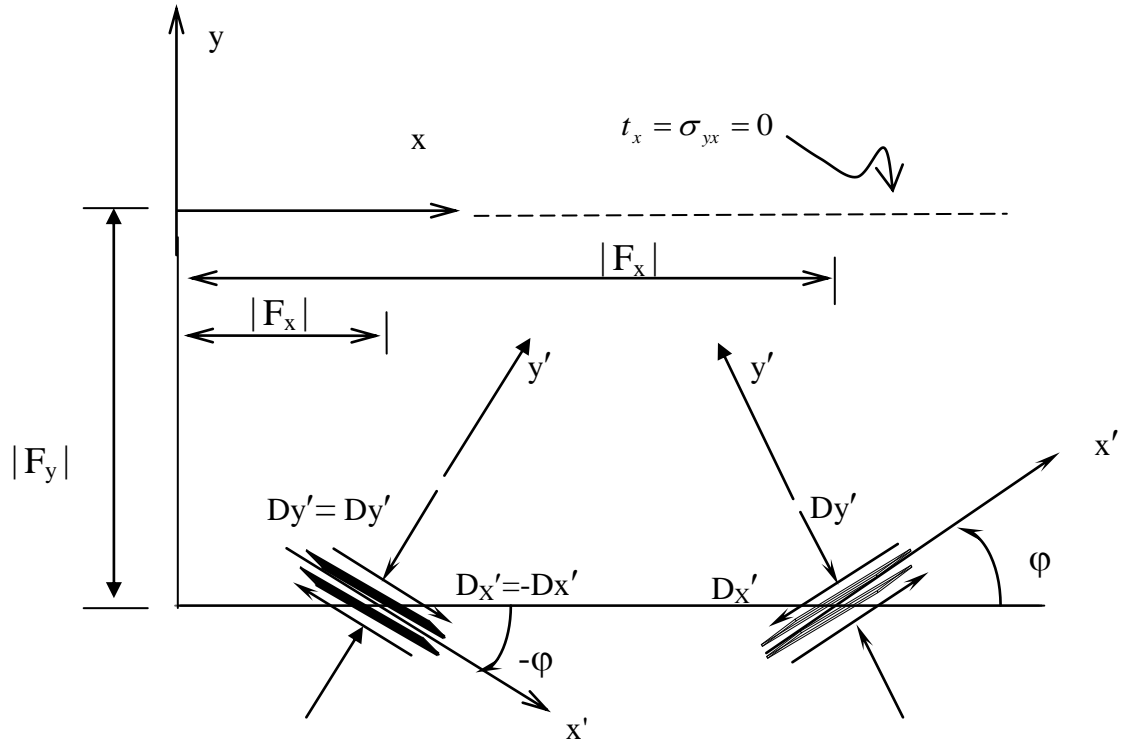


Figure 1. Real and portrait displacement discontinuities in half-plane  $y \leq 0$ , with a traction-free surface (after Crouch and Starfield [25]).

In the present study, in order to obtain high precision, quadratic collocation displacement discontinuity modified for half plane crack problems with a traction free surface are used.  $D_k(\xi)$  is equation that can be used to calculate two fundamental variables of each element (the opening displacement  $D_y$  and sliding displacement  $D_x$ ).

$$D_j(\xi) = \sum_{i=1}^n \Pi_i(\xi) D_i^k, k = x, y \quad (3)$$

Where  $D_1^k$  (i.e.  $D_1^x$  and  $D_1^y$ ),  $D_2^k$  (i.e.  $D_2^x$  and  $D_2^y$ ),  $D_3^k$  (i.e.  $D_3^x$  and  $D_3^y$ ) are the quadratic displacement discontinuities, and using the equal length of each sub element  $a_1 = a_2 = a_3$

$$\begin{aligned} \Pi_1(\xi) &= -(3a_1^3 - a_1^2\xi - 3a_1\xi^2 + \xi^3)/(48a_1^3), \\ \Pi_2(\xi) &= (9a_1^3 - 9a_1^2\xi - a_1\xi^2 + \xi^3)/(16a_1^3), \\ \Pi_3(\xi) &= (9a_1^3 + 9a_1^2\xi - a_1\xi^2 - \xi^3)/(16a_1^3) \end{aligned} \quad (4)$$

are their quadratic element shape functions.

In Fig. 2, A quadratic displacement discontinuity (DD) element is divided into three equal sub-elements (each sub-element contains a central node for which the nodal displacement discontinuities are evaluated numerically) [26].

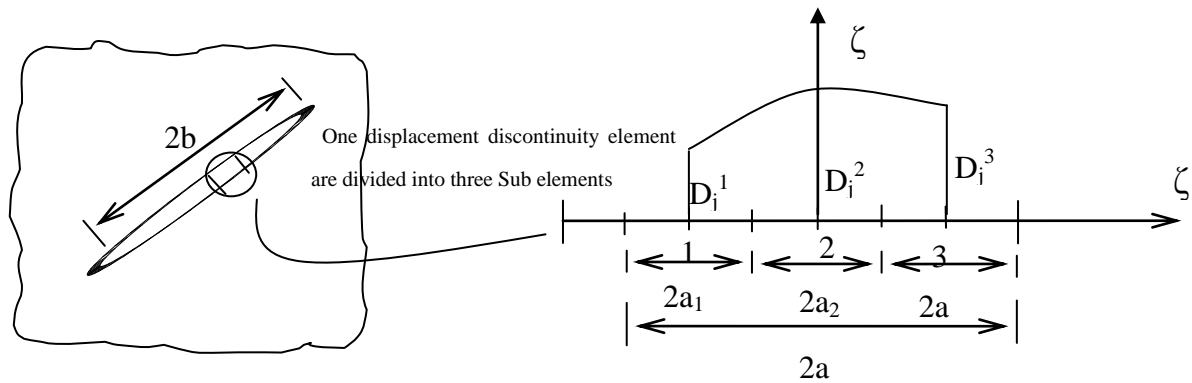


Figure 2. Higher order displacement discontinuity (HODD) elements by quadratic elements

To increase the accuracy in computing of the displacement discontinuities and stresses near original crack tips, the special crack tip elements are used. In previous works, usually, one or two special elements for crack tip were used, but in the present research, three special elements are used. Following equation can be used three elements, particularly for crack tip:

$$D_i(\xi) = [\Pi_{T_1}(\xi)]D_i^1(b) + [\Pi_{T_2}(\xi)]D_i^2(b) + [\Pi_{T_3}(\xi)]D_i^3(b) \quad (5)$$

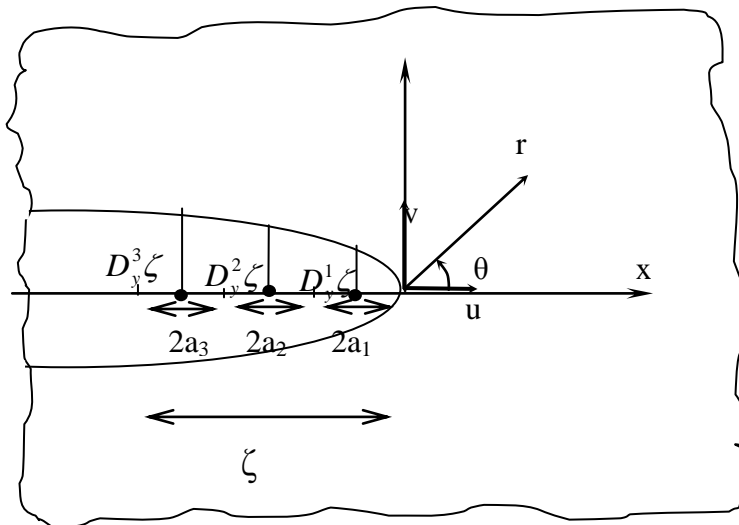


Figure 3. Quadratic element for the higher order displacement discontinuity variation

### 3. Verification of the semi-infinite displacement discontinuity method (using quadratic elements)

Verification of this method is made through the solution of a simple example problem i.e. a center inclined micro crack in a semi-infinite body which is shown in Fig. 4. The tensile stress is acting parallel to the x axis at infinity. Considering a 30 degrees center inclined micro crack under uniform normal tension  $\sigma = 10 \text{ MPa}$ , a half micro crack length  $b = 1 \text{ meter}$ , fracture toughness  $K_{IC} = 2 \text{ MPa}$

$\sqrt{m}$ , modulus of elasticity  $E=10\text{ GPa}$ , Poisson's ratio  $\nu=0.2$  are assumed. The analytical solution of mode I and mode II stress intensity factors,  $K_I$  and  $K_{II}$ , for the infinite body problem are given as [27]:

$$K_I = \sigma(\pi b)^{\frac{1}{2}} \sin^2 \beta \text{ and } K_{II} = \sigma(\pi b)^{\frac{1}{2}} \sin \beta \cos \beta \quad (6)$$

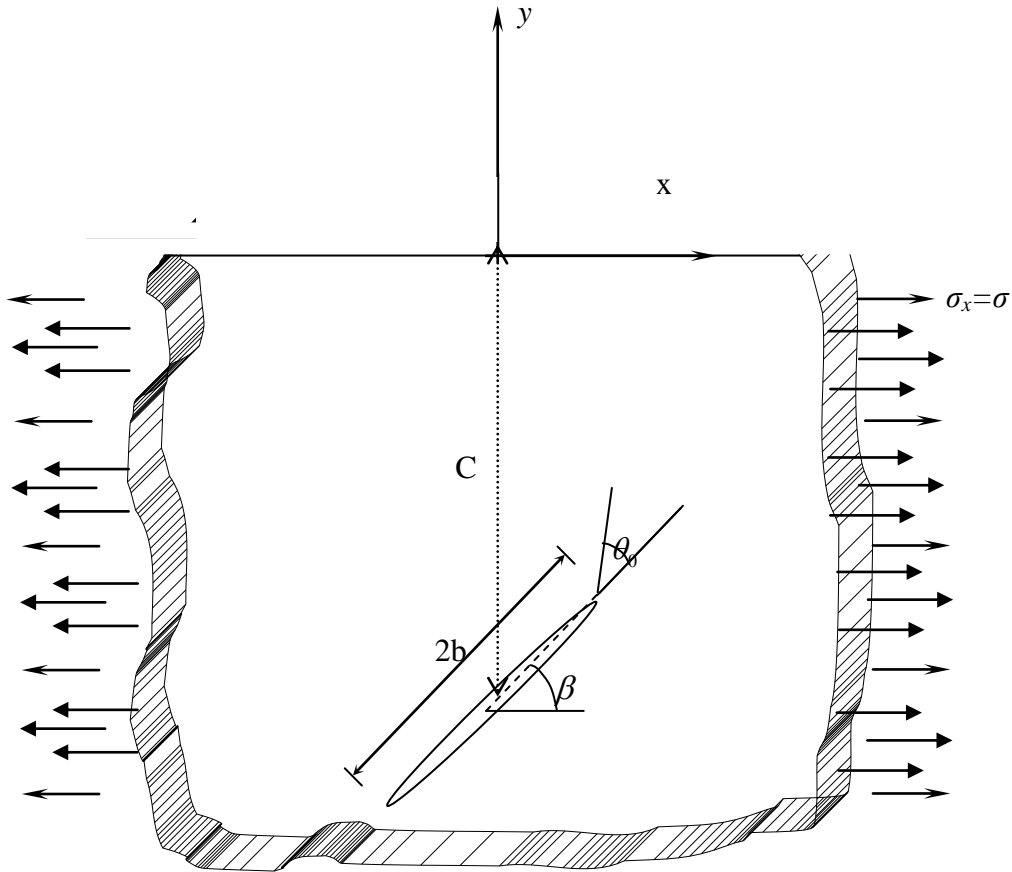


Figure 4. A center inclined micro crack in a semi- infinite body.

The semi-infinite center inclined micro crack problem is selected to verify the proposed code (see Fig. 4).

Fig.5 is based on the normalized stress intensity factors which compare the different results obtained for the upper cracks (the cracks near to the free surface of the half plane), and the lower cracks, with the available analytical results of the center inclined micro crack in an half plane.

The numerical results show that as the depth of the micro crack ( $C/R$  ratio) increases the mixed mode stress intensity factors  $K_I$  and  $K_{II}$ , and the crack initiation angle  $\theta_0$  tend to their corresponding analytical values of the center inclined micro crack in an infinite plane.

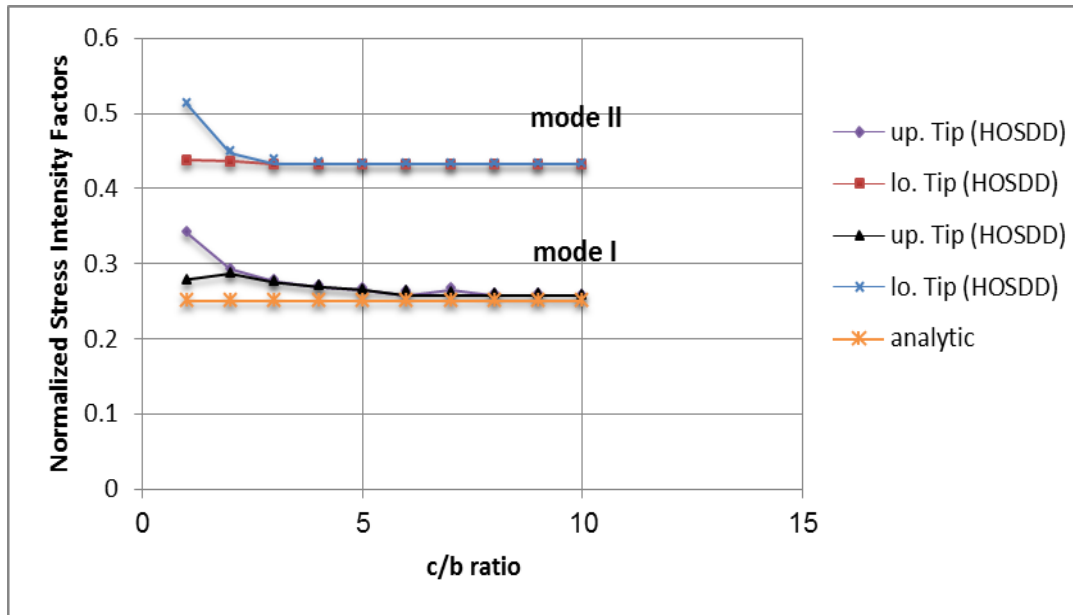


Figure 5. The normalized stress intensity factors  $K_I / (p\sqrt{\pi\rho R})$  and  $K_{II} / (p\sqrt{\pi\rho R})$ , for different  $C/R$  ratios, for a center inclined micro crack, in a semi-infinite plane

#### 4. Modeling of eroded disc cutter

Since, after a while, TBM disc cutters erosion caused by contact with the rock, therefore, the effect of erosion is modeled numerically. For this purpose, four special elements are used to simulate the disc cutter tips. This model estimates the required curvature near the disc cutter tips, reducing the computing efficiency and increases the required specific energy for chips formation under disc cutters during a rock indentation process. In the Figure 6, the micro crack propagation path for artifact cracks of AC1, AC2, AC3 and AC4 which represents the coalescing of propagated micro cracks from artifact cracks, and also this path for  $S/P_d=10$ , disc edge angle  $\psi=60$ ,  $P_d=6mm$  and extended length of  $\delta=4mm$  (8 iteration) are estimated. Micro cracks propagate with respect to direction of the artifact cracks, then after several (8) iteration propagated micro cracks from artifact cracks coalesce to each other at the region between cutters. Considering a typical TBM with a thrust of 250 kN per cutter, with disc cutters of 15 mm width and 432 mm diameter. The typical rock is the Aspo diorite with the Mode I and Mode II fracture toughnesses  $K_{IC}=3.83 MPa m^{1/2}$ , and  $K_{IIC}=5.09 MPa m^{1/2}$  respectively. The other parameters of this rock as quoted by Backers (2004) are:  $\sigma_t = 15 MPa$ ,  $\sigma_c = 220 MPa$ , and  $E= 68 GPa$  and  $\nu = 0.24$  [20].

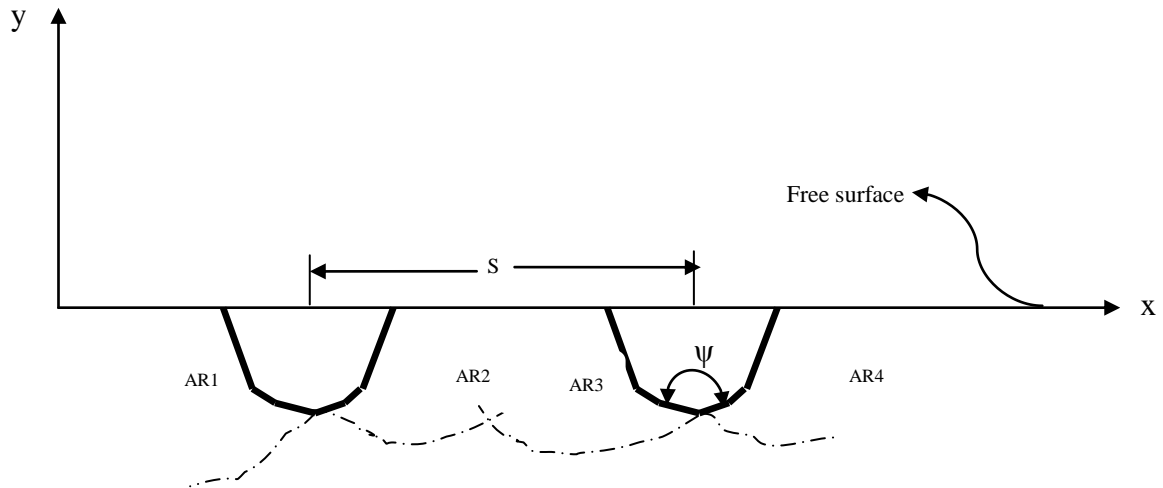


Figure 6. The micro crack propagation path for  $S/Pd=10$  edge angle  $\psi = 60$ ,  $Pd=6$ , mm and extended length of  $\delta=4mm$  (8 iteration).

In order to investigate the effect of erosion on the TBM, the results of modeling as illustrate in Figs. 7- 9. According to the Fig. 7, because the cutters are blunted under erosion, as a result, more specific energy for eroded cutters than non-eroded cutters is required. The figure shows that in both cases, the optimum ratio  $S/Pd$  is in range of 7.5–15, which is in well agreement with the experimental results. Figs. 8 shows comparison between eroded and non-eroded disc cutters for the Mode I and Mode II stress intensity factors,  $K_I$ ,  $K_{II}$  ( $MPa m^{1/2}$ ) versus different  $S/Pd$  ratios. Figs. 9 shows comparison between eroded and non-eroded disc cutters for the Mode I and Mode II stress intensity factors,  $K_I$ ,  $K_{II}$  ( $MPa m^{1/2}$ ) versus different  $Pd$  ratios.

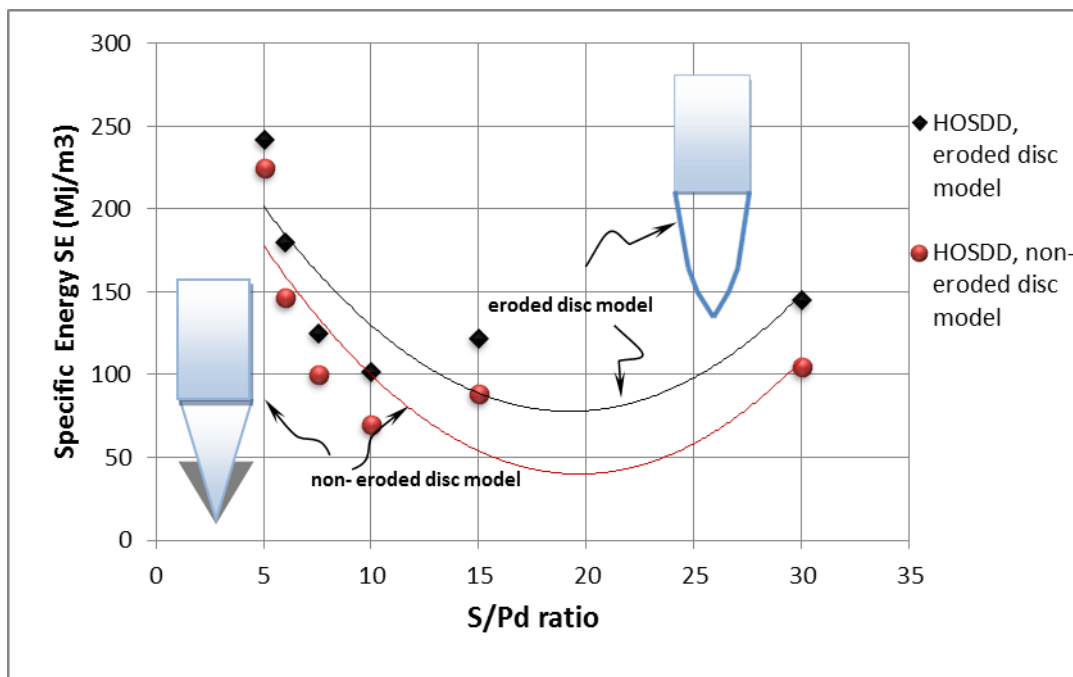


Figure 7. Comparison between eroded and non-eroded disc cutters for specific energy  $SE$  ( $Mj/m^3$ ) versus different  $S/Pd$  ratios

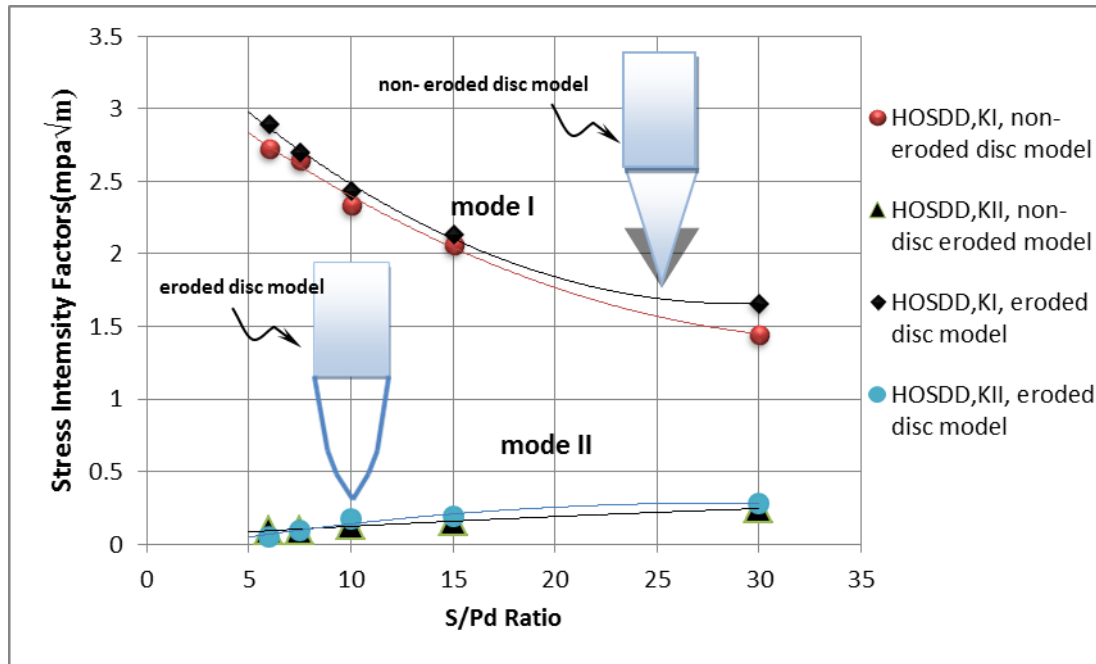


Figure 8. Comparison between eroded and non-eroded disc cutters for the Mode I and Mode II stress intensity factors,  $K_I$ ,  $K_{II}$  ( $MPa m^{1/2}$ ) versus different  $S/Pd$  ratios

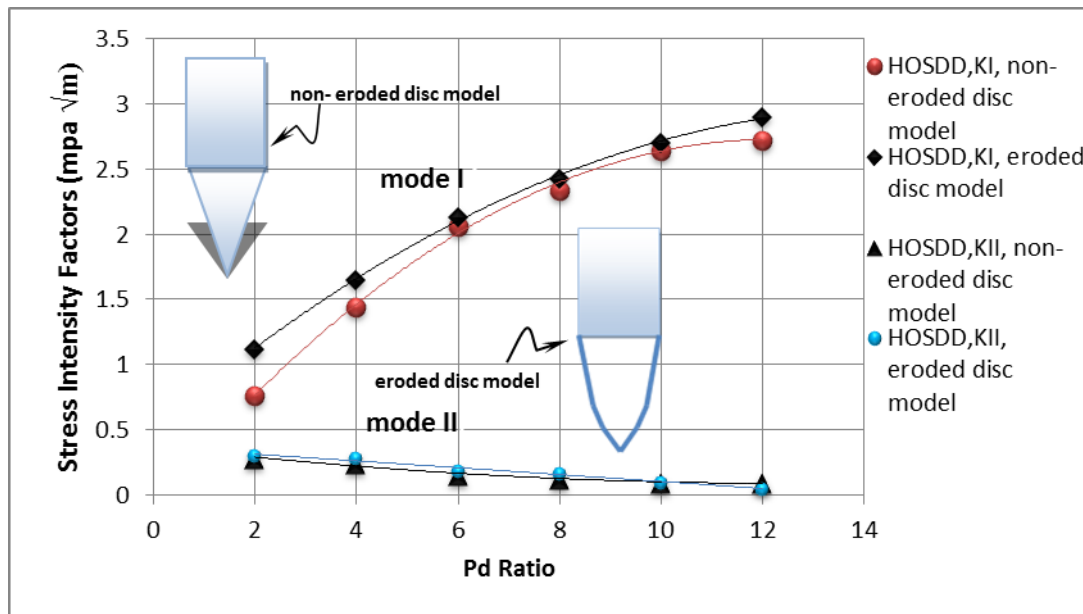


Figure 9. Comparison between eroded disc cutter and non-eroded disc cutters for the Mode I and Mode II stress intensity factors,  $K_I$ ,  $K_{II}$  ( $MPa m^{1/2}$ ) for various penetration depths  $Pd$

## 5. Conclusions

A two-dimensional code based on a semi-infinite displacement discontinuity method (Known as HOSDD<sub>2D</sub>) is used to simulate rock-cutting behavior under TBM disc cutter during rock indentation. The produced micro cracks under the disc cutters are coalesced during their propagation into the



underneath rock mass. The disc edges are assumed to behave as artifact cracks. The propagation and coalescence mechanism of the micro cracks emanating from each artifact crack tips are investigated by discretizing it into three ordinary and three special crack tip elements at the tip. The maximum tangential tensile stress criterion ( $\sigma_{\theta}$ -criterion) is implemented into HOSDD<sub>2D</sub> code. In the present research, the effects of erosion on the Mode I and Mode II stress intensity factors and specific energy (SE) for TBM disc cutters are investigated by simulating the penetrating artifact cracks (disc cutters) into a rock mass. The main purpose of the present modeling was to compare the results obtained for the eroded disc cutters with those of non-eroded disc cutters. As a result, for the eroded disc cutters, higher forces (more specific energy) are required to propagate the micro cracks emanating from the artifact cracks compared to those for non-eroded disc cutters

## 6. References

- [1] X. C. Tan, S. Q. Kou, P. A. Lindqvist, Simulation of Rock Fragmentation by Indenters Using DDM and Fracture Mechanics, Rock Mechanics, Tools and Techniques; Aubertin M., Hassani F., and Mitri H. (Eds.); Balkema, Rotterdam, 1996
- [2] J. Rostami, , L. Ozdemir, A new model for performance prediction of hard rock TBMs. In: Proceedings, Rapid Excavation and Tunneling Conference (RETC), 1993, pp. 793–809.
- [3] O. Acaroglu, L. Ozdemir, B. Asbury,. A fuzzy logic model to predict specific energy requirement for TBM performance prediction. Tunn. Undergr. Space Technol, 23 (2008) 600–608.
- [4] J.W. Cho, S. Jeon, S.H. Yu, S.H. Chang, Optimum spacing of TBM disc cutters: A numerical simulation using the three-dimensional dynamic fracturing method, Tunnelling and Underground Space Technology 25 (2010) 230–244.
- [5] N.G.W. Cook, M. Hood, F. Tsai, Observations of crack growth in hard rock loaded by an indenter. Int. J. Rock Mech. Min. Sci. Geomech. Abstr, 21 (1984) 97– 107.
- [6] Y. Uga, K. Sakoi, S. Sugiyama, Y. Kondo, K. Nishimura, H. Ono,. Development of new tunnel boring machine with slurry transport system – penetration efficiency of disc cutters, Kawasaki Heavy Industry Report 91, 1–8 (in Japanese), 1986.
- [7] B. Nilsen, L. Ozdemir, Hard rock tunnel boring prediction and field performance. In: Proceedings, Rapid Excavation and Tunneling Conference (RETC), 1993, pp. 833–852.
- [8] N. Bilgin, H. Tuncdemir, C. Balci, H. Copur, S. Eskikaya, A model to predict the performance of tunneling machines under stressed conditions, In: Proceedings, AITES-ITA 2000 World Tunnel Congress, 2000, pp. 47–53.
- [9] H.Y. Liu, S.Q. Kou, P.A. Lindqvist, C.A. Tang, Numerical simulation of the rock fragmentation process induced by indenters. Int. J. Rock Mech. Min. Sci. 39 (2002) 491–505.
- [10] S.H. Baek, H.K. Moon, A numerical study on the rock fragmentation by TBM cutter penetration. Tunn. Undergr. Space (J. Korean Soc. Rock Mech.) 13 (2003) 444–454 (in Korean).
- [11] N. Bilgin, C. Feridunoglu, D. Tumac, M. Cinar, Y. Palakci, O. Gunduz, L. Ozyol, The performance of a full face tunnel boring machine (TBM) in Tarabya (Istanbul). In: Proceedings, 31st ITA-AITES World Tunnel Congress, 2005, pp. 821–826.
- [12] S. Eskikaya, N. Bilgin, C. Balci, H. Tuncdemir,. From research to practice – development of rapid excavation technologies. In: Proceedings, 31st ITA-AITES World Tunnel Congress, 2005, pp. 435–441.

- [13] N. Bilgin, M.A. Demircin, H. Copur, C. Balci, H. Tuncdemir, N. Akcin, Dominant rock properties affecting the performance of conical picks and the comparison of some experimental and theoretical results. *Int. J. Rock Mech. Min. Sci.* 43 (2006) 139–156.
- [14] S.H. Chang, S.W. Choi, G.J. Bae, S. Jeon., Performance prediction of TBM disc cutting on granitic rock by the linear cutting test. *Tunn. Undergr. Space Technol*, 21 (2006), 271.
- [15] K.I. Park, S.H Chang, S.W Choi, S. Jeon, Prediction of the optimum cutting condition of TBM disc cutter in Korean granite by the linear cutting test. In: *Proceedings, Korean Society for Rock Mechanics Conference, 2006*, pp. 217–236 (in Korean).
- [16] R.Gertsch, L. Gertsch, J. Rostami, Disc cutting tests in Colorado red granite: implications for TBM performance prediction. *Int. J. Rock Mech. Min. Sci.* 44 (2007) 238–246.
- [17] H. Haeri., Numerical Modeling of the Interaction between Micro and Macro Cracks in The Rock Fracture Mechanism Using Displacement Discontinuity Method. PhD Thesis, department of mining engineering, Science and Research branch, Islamic Azad University, Tehran, Iran, during work, 2011.
- [18] A. R. Ingraffea, Numerical Modeling of Fracture Propagation, *Rock Fracture Mechanics*; Rossmann H. P. (Editor); Springer Verlagwien; New York, 1983, pp. 151-208.
- [19] S. Melin., When does a crack grow under mode II condition? *International Journal of Fracture*, 30(1986) 103-114.
- [20] D. Broek., *The Practical Use of Fracture Mechanics*, 4<sup>th</sup> Edition, Kluwer Academic Publishers, Netherland.70. Whittaker BN, Singh RN, Sun G. *Rock fracture mechanics: Principles, design and applications*. Amsterdam: Elsevier, 1989.
- [21] B. N. Whittaker, R. N Singh, G. Sun, *Rock Fracture Mechanics, Principles, Design and Applications*, Elsevier, Netherlands, 1992 .
- [22] B.Shen, O. Stephansson ‘Modification of the G-criterion for Crack Propagation Subjected to Compression’, *Engng. Fract. Mech.*, 47(1994)177-189.
- [23] J.F. Shao, J.W. Rudnicki., A micro crack-based continuous damage model for brittle geomaterials. *Mechanics of Materials*, 32(2000)607-619
- [24] D.J.C Bremaecker, MC. Ferris, D. Ralph, Compressional fractures considered as contact problems and mixed complementarity problems, *Eng. Fract. Mech.*, 66(2000) 287-303.
- [25] S. L Crouch, A. M. Starfield, *Boundary Element Methods in Solid Mechanics*; Allen and Unwin, London, 1983.
- [26] M.F. Marji, H. Hosseini-Nasab, A. H Kohsary, On the uses of special crack tip elements in numerical fracture mechanics, *Int. Journal of solids and structures* 43(2006) 1669-1692.
- [27] C. Scavia, *Fracture Mechanics Approach to Stability Analysis of Crack Slopes*, *Engng. Fract. Mech.*, 35(1990)889-910