The Model and Application of Fatigue Life Based on Fracture Mechanics and Fuzzy Theory

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Abstract: Based on fracture mechanics and fuzzy theory, a model of fatigue life is developed to predict the fatigue life of structure. The dangerous point and stress amplitude can be gotten by finite element analysis method. According to fracture mechanics theory, considering fuzziness of fatigue life factors, the Paris' formula is combined with fuzzy theory and the model of fatigue life based on fuzzy theory is developed. The fuzziness of fatigue life factors is considered in the model so the results are more close to real situation. The model offers reference for fatigue life analysis.

Keywords : fracture mechanics, fuzzy, fatigue

1. Introduction

In vessels, pipelines, bridges, ships, offshore structures and many other engineering structures, there were a lot of disastrous accidents because of fatigue cracks. A report from American National Standards Institute indicates that cracks cause America a loss of 119 billion dollars every year. It is about 4 percents of American cross national products in 1982. According to a survey result of Committee of the Fatigue and Fracture, a sub-committee of ASCE, about 80-90% destruction of steel structures relates to fatigue cracks. From 1978 to 1981, America did a research against 20 states and Ontario Province in Canada. They collected the data of 142 bridges which had cracks. Among them, they found that the cracks on 115 bridges resulted from fatigue fracture[1]. How to analyze and estimate the fatigue life of member in construction correctly has become an important issue. This paper proposes a analysis modal which is based on fuzzy theory and combined with fracture mechanics.

2. Fuzzy phenomenon in analysis of fatigue life

The fuzziness of objects means the boundary is not clear, we cannot distinguish the difference between the right and wrong in its implication correctly, and we cannot divide its boundaries in our discussion range. The fuzziness is a natural characteristic of objects, it is a transient process between difference of objects.[2,3]

Broadly speaking, strength includes fatigue life, fracture toughness, residual strength and so on. In order to judge a structure is invalid or reliable, we must know the strength of material and structure. But because we get the data of strength from experiment, so it’s inevitable to bring fuzziness.[4,5] It’s feasible to apply the method of S-N curve and fracture mechanics. This paper is based on fuzzy theory, applys the method of fracture mrchanics to predict the fatigue life of structure.

Paris formula[9] is a always used to calculate the fatigue life in fracture mechanics, namely

$$\frac{da}{dN} = C\left(\Delta K\right)^m \quad , \tag{1}$$

Among it, a-crack size; N-number of load cycles; C, m-material constants; ΔK - stress strength

amplitude
Based on theory of fracture mechanics,

$$\Delta K = Y\Delta\sigma\sqrt{a\pi} \quad , \tag{2}$$

Y-coefficient of crack shape (generally it is 1),
$\Delta\sigma$ -alternative stress amplitude
We make (2) type substitute (1) type, we can get

$$N = \frac{1}{C}\int_{a_0}^{a_c}\frac{da}{\left(\Delta K\right)^m} = \frac{1}{C}\int_{a_0}^{a_c}\frac{da}{\left(\Delta\sigma\sqrt{a\pi}\right)^m} \quad , \tag{2-1}$$

Integrating, we can get

$$N = \begin{cases} \dfrac{a_c^{\,1-m/2} - a_0^{\,1-m/2}}{C\pi^{m/2}\left(1-m/2\right)\left(\Delta\sigma\right)^m} & m \neq 2 \\ \dfrac{\ln a_c - \ln a_0}{C\pi\left(\Delta\sigma\right)^2} & m = 2 \end{cases} , \tag{1}$$

Implied by some statistical data, initial crack length a_0 of welded structure is about 0.1~1.0mm. On average, we can think that a_0 is equal to 0.5mm[6]. Critical crack size a_c depends on the fracture toughness and stress level of material. According to the theory of fracture, we can get:

$$a_c = \frac{1}{\pi}\left(\frac{K_c}{\sigma}\right)^2 \quad , \tag{2}$$

Among it, K_c -the fracture toughness of material, we can get it from material data;
σ -structure stress
Because of some uncertain factors, such as fabrication technology, material properties and environment for use, we cannot get a most realistic number of C, so there are different numbers of C in different documents. For this reason that C is a variable quantity with fuzzifrfication, we can select some point among C's feasible region and on its edge. Then we put them into formula (3) and get their fatigue life respectively. Finally, based on the method of fuzzy comprehensive evaluation, we can make a forecast of its fatigue life.

3. Model of fuzzy comprehensive evaluation

3.1 Degree of membership

Assuming $C = (c_1, c_2, \ldots, c_m)$ which is the factor set that affect structure's fatigue life, $V = (v_1, v_2, \ldots, v_n)$ which is the set of evolution level. Among them, $c_i (i = 1, 2, \ldots, m)$ means number i influencing factors, $v_j (j = 1, 2, \ldots, n)$ means evolution level. So the array of degree of membership of set V is:

$$R=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}=\left(r_{ij}\right)_{m\times n} \quad , \tag{5}$$

among it, $i=1,2,\cdots m$, $j=1,2,\cdots n$ 。

In the factor set, for reflecting the different importance of each factor, we give each factor c_i a corresponding weight. The degree of membership of proposal j[7]:

$$u_j=\frac{1}{1+\left(\frac{\sum_{i=1}^{m}\left[w_i\left(r_{ij}-1\right)\right]}{\sum_{i=1}^{m}\left(w_i r_{ij}\right)}\right)^2} \tag{6}$$

We can get the fatigue life through the weighting factor method:

$$\bar{N}=\sum_{j=1}^{n}u_j N_j \tag{3}$$

Among it, N_j is the fatigue life of structure when evolution level is j.

2.2 Weight vector

For decreasing the affection of human factors, the method of getting the weight w_i is based on its relative degree of fuzziness membership. Generally speaking, the higher degree of membership of target, the bigger attention will be pay. In other words, the bigger weight will be given. According to the fuzzy set, we can regard the degree of membership as the weight. So we transpose the array R, and get the array of relative degree of membership which is target to the "importance", namely

$$W=\begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1m} \\ w_{21} & w_{22} & \cdots & w_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & w_{nm} \end{bmatrix}=(w_{ji})_{n\times m}=R^T \tag{7-1}$$

From array W, we can know the vector $\vec{w}_{(i)}=(w_{1i},w_{2i},\cdots,w_{ni})^T$ of relative degree of membership. It is a vector of n proposals that are about factors c_i to the "importance". Because each proposal in set V completes fairly, so n proposals have the same weight to the importance of factor c_i. So weight vector is[7]:

$$w_{(i)}=(1+d_{yi}^2\cdot d_{zi}^{-2})^{-1} \tag{4}$$

Among it: $d_{yi}=\left\{\sum_{j=1}^{n}(1-w_{ji})^p\right\}^{1/p}$, $d_{zi}=\left\{\sum_{j=1}^{n}w_{ji}^p\right\}^{1/p}$

p is the index of distance, when p is one，it is Hamming distance; when p is two, it is Euclidean distance

4. Calculation example

The stiffening girder of steel truss on one over-sea bridge is composed by main truss, main beam and upper and lower bracing. Main truss is connected by integral joint plate, and all the members' sections are closed box. Main truss is composed by top chord members, lower chord members, vertical web members and inclined web members. Main truss is 10m tall, and each standard segment is 10m long. The structure of integral joint on top chord member is shown in figure1. It is made of Q345steel. This joint is under multi-direction stress, it has a complicated stress field. It is connected by large numbers of variable welding lines, so vehicle load could cause fatigue problem easily.

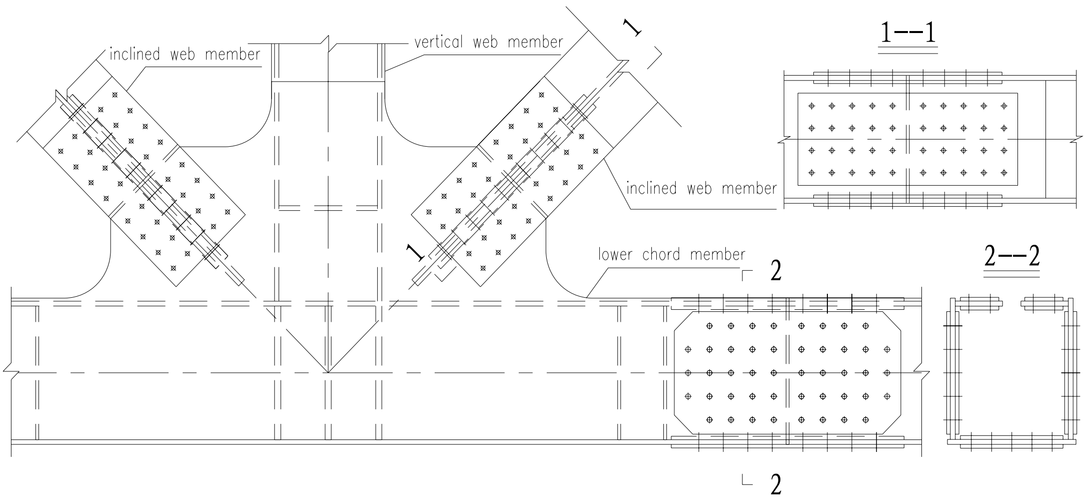

Fig.1 Schematic diagram of integral joint

4.1 Analysis based on crack mechanics

Through finite element analysis, we know that there is obvious stress concentration at the intersection of integral joint plant and inclined web member. As shown in figure2, the biggest stress amplitude $\Delta\sigma = 84.4Mpa$.

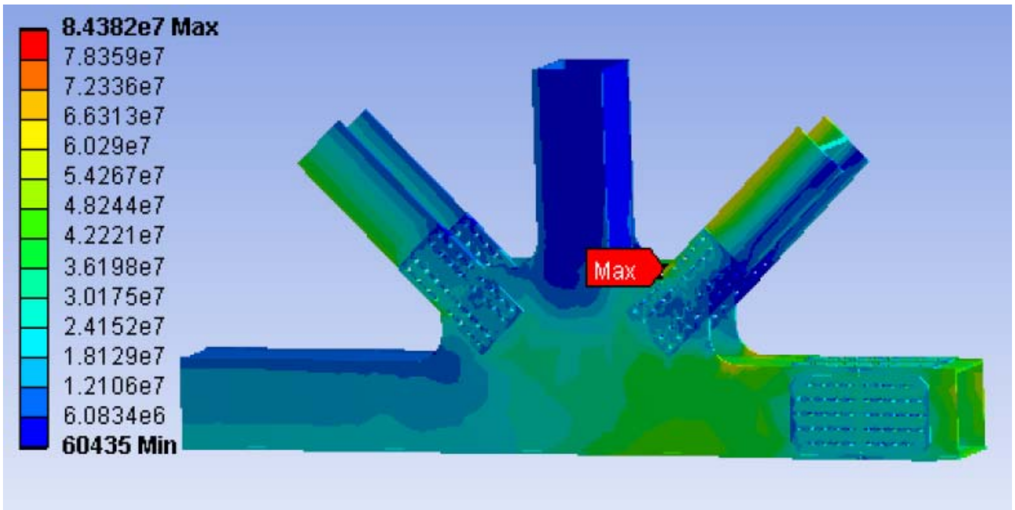

Fig.2 Equivalent stress nephogram of integral joint

The crack toughness of Q345steel is: $K_c = 207.91 Mpa \bullet mm^{\frac{1}{2}}$. According to formula (4), we can get the critical crack size:

$$a_c = \frac{1}{\pi}\left(\frac{K_c}{\sigma}\right)^2 = \frac{1}{\pi}\left(\frac{207.91}{84.4}\right)^2 = 1.93mm$$

Initial crack size: $a_0 = 0.5mm$, material constant $m = 3.0$, material constant C, average $\mu_c = 2.12 \times 10^{-13}$, coefficient of variation $C_v = 0.15$ [8].
According to formula (3), we can get the fatigue life is 2310340 times, 2123015 times, 1963789 times, 1826781 times and 1707643 times under different levels.

4.2 Create the factor set

Considering the specific condition of truss structure, the influencing factors of fatigue life of integral joint plate are as follows:
$C = (c_1, c_2, c_3)$

Among it, c_1 means fabrication technology, c_2 means material properties, c_3 means environment for use.

4.3 Create the evolution set

According to the trends consistency principle, five evolution indexes should be created which are shown in table1.

Tab.1 Grade of each evaluation factors

	v_1	v_2	v_3	v_4	v_5
c_1	excellent	good	average	bad	worse
c_2	excellent	good	average	bad	worse
c_3	excellent	good	average	bad	worse

4.4 Create the array of degree of membership

By the evolution of experts and technical staffs, array of degree of membership should be created as follows:

$$R = \begin{pmatrix} 0.10 & 0.45 & 0.30 & 0.10 & 0.05 \\ 0.05 & 0.50 & 0.35 & 0.05 & 0.05 \\ 0.05 & 0.10 & 0.25 & 0.45 & 0.15 \end{pmatrix}$$

According to (6) and (8) formula, we can get:

$$u = (0.010, 0.511, 0.322, 0.139, 0.018).$$

So the fatigue life is:

$$\overline{N} = \sum_{j=1}^{5} u_j N_j = 2024965 \text{ times.}$$

5. Conclusion

In this paper, we propose an analysis model of fatigue life which is based on fuzzy theory. If we can combine it with actual construction situation, combine the fuzzy theory with crack mechanics, and we consider the fuzziness of factors which affect fatigue life, we can get a more accurate calculation result, and offer reference for fatigue life analysis.

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