# An Equivalent Strain Gradient Theory for Evolvement of Damage in void Material

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**Abstract** In this article, the onset conditions and evolvement path of the crack initiated from void edge in void material are investigated. For this end, the plane stress numerical simulation is implemented for a series of void embedded strip samples in Al-alloy. The stress and strain distribution for the whole plane sample is analyzed by elastic-plastic FEM. Based on the correspondence between equivalent strain distributions, the trend of equivalent strain gradient and the connected style of voids in experiment, a damage evolvement theory for the crack initiation and extension from vole edge is suggested. According to the suggested theory, as the loading increased, the crack is initiated from the edge of the void where the equivalent strain around the edge of the void reached its maximum; next, the initiated crack will propagated along the direction where the absolute gradient value of equivalent strain reached its maximum. It is indicated from the related experiments that the theory can reasonably explain the phenomena of the different linkage style of two holes.

**Keywords** void material; path of the crack extension; equivalent strain grade; voids converge.

## 1. Introduction

In practical structure, the damage or degradation of material is usually the fatal factor influencing the reliability of mechanical engineering structure. However, from microscopic view, the damage in macroscopic is always induced by the accumulation of a series of voids converge, i.e., the voids nucleation, expansion and convergence under loading.

Although the crack problem has been paid great attention by mechanical scientists in fracture mechanics field near a century, a lot of fracture theory and criteria had been proposed to describe the mechanisms of crack initiation and extension. However, the void convergence problem has received fewer attentions relatively till today. Especially in the problem of convergence process of voids, much less for the theory. Only fewer studies related to the void convergence can be retrieved from the open literatures. Most of them are concentrated to the numerically simulation for the void expension and convergence and other factors such as size and direction of grain [1-4].

Thereby, thoroughly understanding the mechanism of the linkage of voids is the precondition for the processing of material and the reliability of structure during its operation. However, the process of the voids linkage under practical stress state is rather complicated considering the relation between relative location of voids and the stress direction. In this article, the problem is simplified to study the void linkage mechanism and control parameter for a multi-hole embedded plate under single loading condition.

# 2. Experiment

## 2.1. Sample

In order to investigate the damage and its evolution of the voids, a group of two-hole stripe sample is selected for the uniaxial tension fatigue test. The sample size is cut to 125mm length, 27mm width and 3mm thickness (Fig.1). The hole pair with the diameter 1mm and the hole center distance 1.5mm are embedded in the center zone around the center point of the plate, and the holes are machined to through the whole thickness, i.e., through hole. The angle of the center line of two holes with the cross section α is selected as 0 and 15 degree. Considering the sample thickness is rather small contrasted to the other sizes, the plain stress condition is applied in this test. The material of specimen is selected as Al-alloy, with the Young's module 56.36GPa, Poisson's ratio 0.33, Yield stress 308.6MPa and Ultimate strength 450.8MPa.

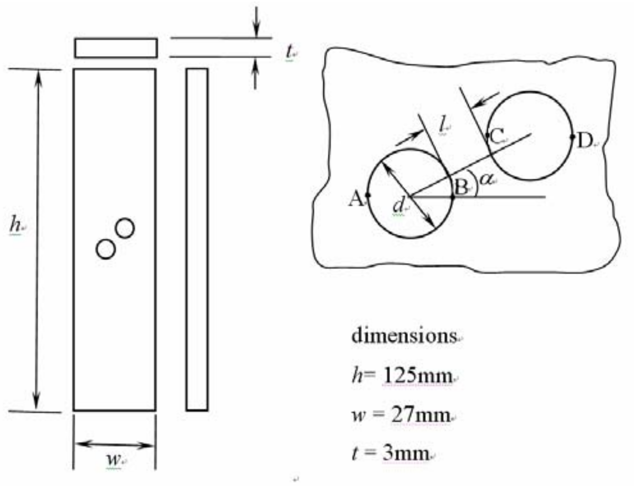


Fig.1, The sample of the fatigue test

## 2.2. Test

The fatigue test is implemented using a testing machine MTS-858, with the loading amplitude 0-16KN and loading frequency 1Hz (Fig.2). As the loading circle increased, the cracks will initiate from the edge of some hole early or late. Simultaneously, a CCD camera is used to record the process of the crack initiation and extension.

## 2.3. Result

It is widely believed that the first initiated crack should start from the inner edge of two holes, such as points B and C; owing to the circle stress in these points is larger than that in the outer points A and D. It is surprised that, for α equals 0, the first crack occurs from one point of the outer edge, the point A, see Fig.3. As the loading circle increases, the crack initiation has a little complicated. For example, in one sample, the second crack starts from the inner point of the other hole edge, the point C (see Fig.4), whereas, in another sample, we also find that the second crack starts form the outer edge of the other hole, the point D. As the cracks started from the outer side of the hole pair


Fig.2, The fatigue test using MTS

propagate to a distance, such as 0.8mm in our experiment, the first two crack stop, meanwhile, the new crack initiated from the inner edge of the holes. It is the inner edge cracks link each other, the two holes connected (see Fig.5). Thus, as the loading circle increases continually, the outer edge cracks restart propagate till arrive to the edge of the sample.

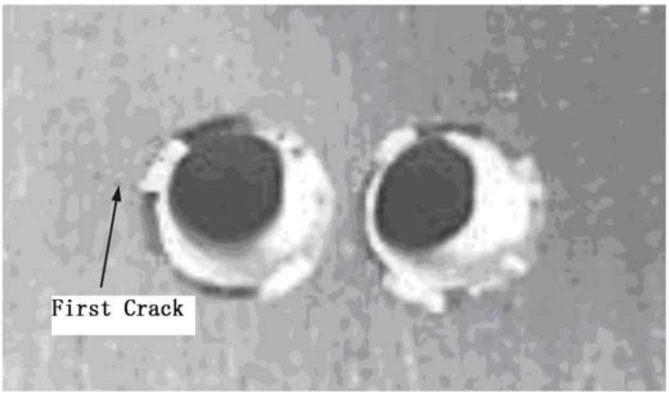

Fig.3, The first crack initiated from the outer edge of the hole

This phenomenon also occurs in the test of the sample with α equals 15 degree. As the loading circle increases, the first crack initiates from the outer edge of two holes, i.e., the point A or D. As the first two cracks propagate to some length, the crack driving force disappears, thus the cracks stop (Fig.6). Next, the inner edge cracks initiate (Fig.7) and propagate to each other till two holes connecte (Fig.8).

It is obvious that this phenomenon (the crack initiates from the outer edge firstly and after it propagates to some length, the inner crack initiates and propagates.) cannot be explained by the theories proposed till today, such as the maximum tangential stress theory, the maximum tension stress theory, and so on. Thus, in the next section, the crack initiated feature and the stress field around the holes will be investigated in detail to describe the phenomena, and a new theory based on the equivalent plastic strain field will be proposed.

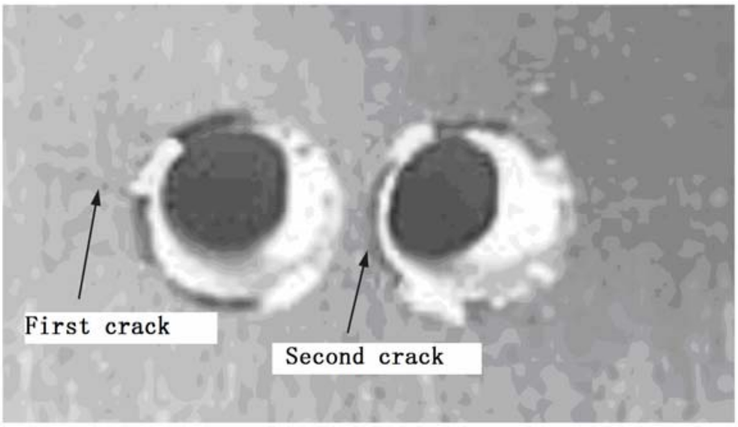


Fig.4, the second crack initiates from the inner edge of the hole in one sample

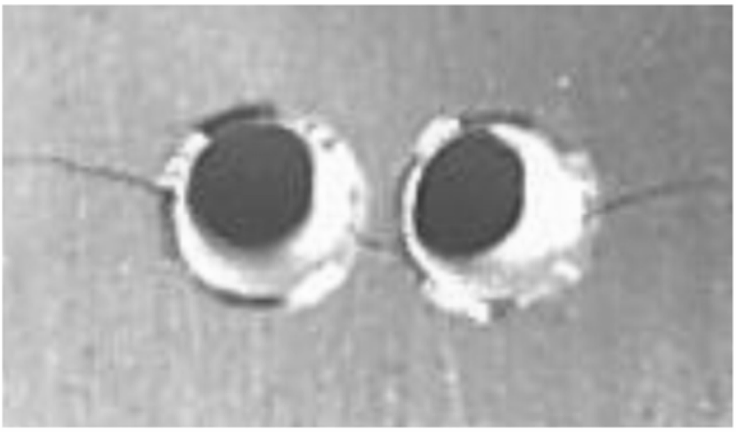

Fig.5, the inner edge cracks link each other and induce the two holes connected

# 3. Numerical analysis

## 3.1. The model of the numerical simulation

The stress strain field around the holes in the test sample is obtained by means of the elastic plastic finite element analysis. In this simulation, the numerical modal is the same as that in the fatigue test in shape and size. The mechanical properties of the material are the same as in the test. The stress strain curve of the material is shown in Figure 9. Different in the fatigue loading in the test, during the numerical simulation, a constant value of load is applied as the boundary condition for the numerical model. Thus the stress strain field obtained from the numerical simulation is the peak value stress strain field of the fatigue loading.

Fig.6, the first crack initiates from the outer edge of the hole

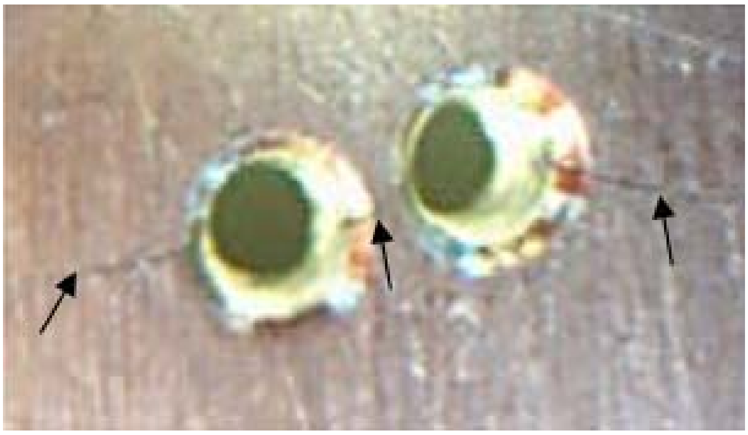

Fig.7, the inner crack initiates after the first crack stop to propagate.

Fig.8, the first crack initiates from the outer edge of the hole

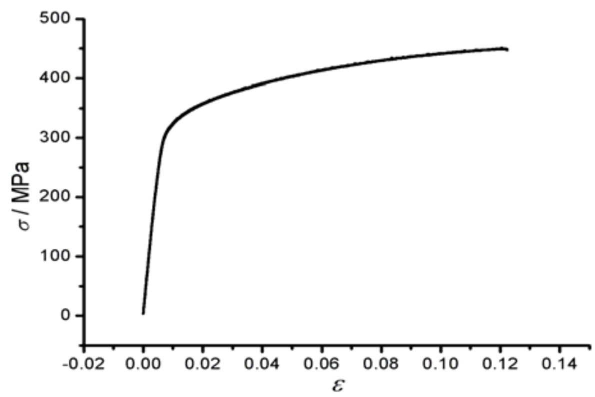


Fig.9, the stress strain curve of the material.

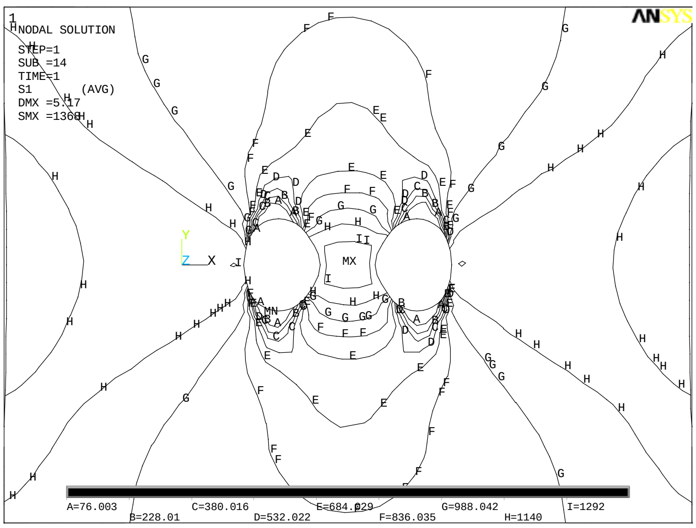


Fig.10, the principal stress distribution around the hole for α=0.

### 3.2. Stress strain field

For α equals 0, the property of contour line of the principal stress and the equivalent stress around the holes are shown in Figure 10 and 11 respectively. It is found from the figures that the principal stress and the equivalent stress in the first crack initiated point and the extended line is reached its maximum, However, if we compare the stress value of the initiated point with the adjacent point on the edge of the hole, we can find that there does not exist an obvious up rush for these stress compared to the surrounding edge point.

For α equals 15 degree, the contour line of the principal stress and the equivalent stress around the holes are shown in Figure 12 and 13 respectively. From these figures, we also find that there does not exist an obvious up rush for the principal stress and the equivalent stress in the initiated point from the fatigue test in the previous section compared with the surrounding edge point. Thus, it seems that the stress parameter is unsuitable to describe the crack initiation and propagation in this condition.

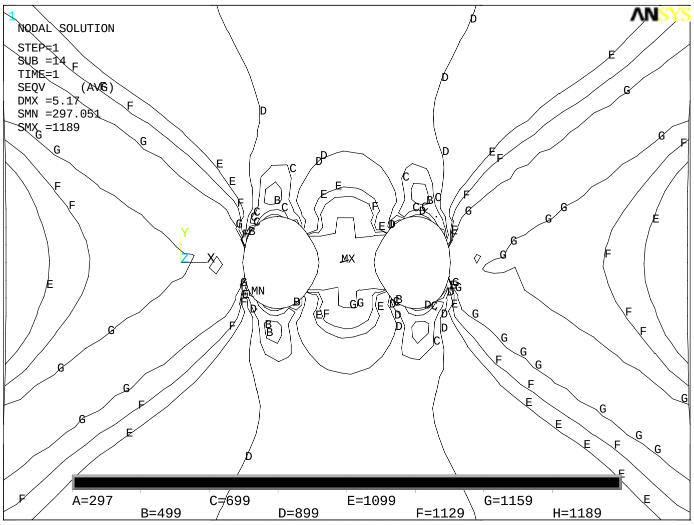


Fig.11, the equivalent stress distribution around the hole for α=0.

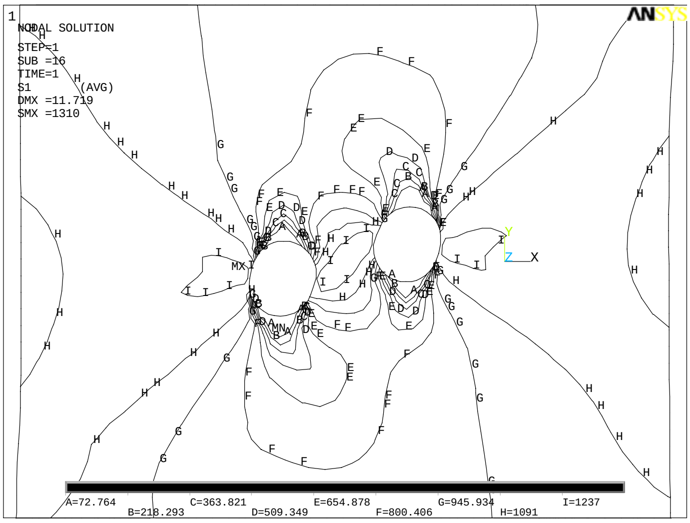


Fig.12, the principal stress distribution around the hole for α=15.

On the other hand, if we compare the equivalent strain distribution in the outer point, inner point and the adjacent point along the edge of the holes for the two sample (see Fig.14 and 15), we can find that there does exist an obvious up rush for the equivalent strain in the initiated point from the fatigue test in the previous section compared to the surrounding edge point. Thus, it seems that the parameter, equivalent strain, can be considered to describe the new phenomena we found in the test. Furthermore, comparing the first crack propagated direction with the equivalent strain field in the outer edge of the hole; it can be found that on this direction, the equivalent strain has a maximum decrease started from the outer edge point along the direction of the crack propagation, comparing with other direction.

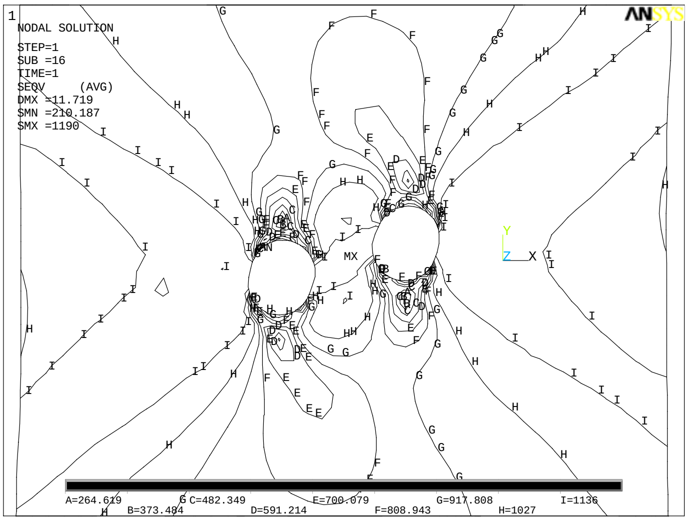


Fig.13, the equivalent stress distribution around the hole for α=15.

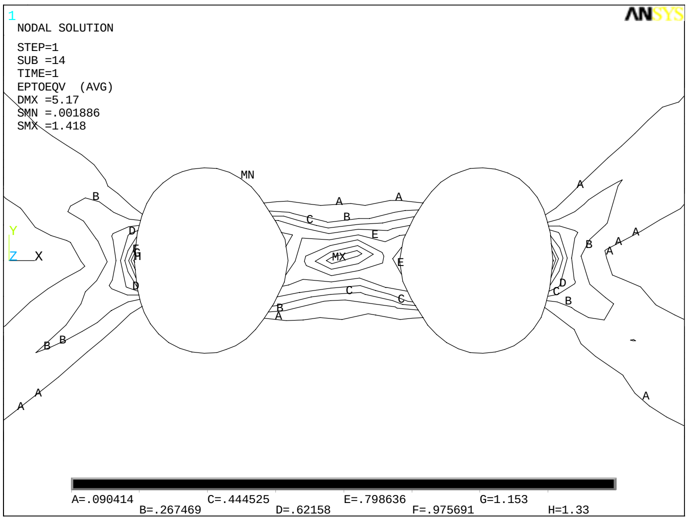


Fig.14, the equivalent strain distribution around the hole for α=0.

## 4. The theory of equivalent strain gradient

Based on the analysis in the previous section, a damage evolution theory based on the equivalent strain gradient is suggested in this article. The suggested criterion states that: the initiation of the crack is occurred when the equivalent strain reaches its critical value. The propagation direction of the initiated crack is coincided with the direction where the equivalent strain decrease gradient reaches its maximum.

The gradient of equivalent strain decrease with respect to the radial coordinate originating from the center point of the hole can be expressed as:

$$E_r = -\frac{\partial \varepsilon_{eq}}{\partial r} \quad , \qquad (1)$$

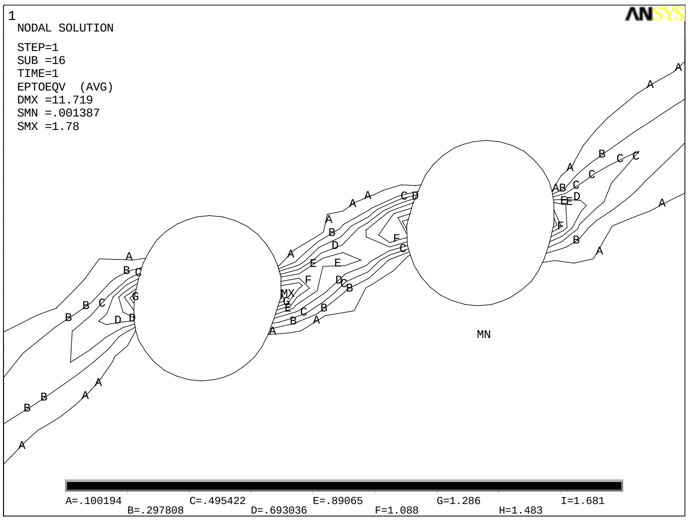


Fig.15, the equivalent strain distribution around the hole for α=15.

Thus, the maximum gradient is determined by:

$$\frac{\partial E_r}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial^2 E_r}{\partial \theta^2} \le 0 \quad , \qquad (2)$$

According to this theory, the crack initiated and propagated process on the void material can be described correctly.

## 5. Discussion

By means of the proposed theory, we can explain the new feature of the crack initiated and propagated in the fatigue test. Analyzing the equivalent strain decreasing from the outer edge point and the inner edge point in one hole (such as the left hole in the Fig.14 and 15) and along the path on the radial direction, it is obvious that the gradient of the equivalent strain decrease in the outer point is larger than that in the inner point, although the absolutely equivalent strain value in this two point is almost the same. It can be found that the inner part material between two holes has a larger elastic or plastic deformation than the other part of material. Here, the larger plastic deformation releases large part of the peak stress near the inner point, and postpones the crack initiated in this point. As the first initiated crack startes from the outer point and propagates to some length, the gradient of the equivalent strain decreased, while the deformation around the inner edge point increased to induce the gradient of the equivalent strain decreased in this point increased till larger than that in the outer propagated crack tip. Thus the outer propagated crack stops, and the inner edge crack initiates and propagates. Thereby, the two inner cracks links and the sample separates.

## 6. Conclusions

According to the experimental and numerical research, it is concluded that:
(1), the stress parameter, such as the principal stress or equivalent stress, can not appropriately describe the crack initiateand propagate from the edge of a void in porous material. Whereas, the strain parameter, such as the equivalent strain, can be used to describe this phenomena.
(2), based on the detailed analysis, a theory of gradient ot the equivalent strain is suggested. According to this theory, the crack initiated point and the path of the crack from the void edge can be predicted appropriately.

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