

## Rock fracture toughness testing using SCB specimen

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**Abstract** There are various testing methods for measuring fracture toughness of rocks. The semi-circular bend (SCB) specimen has recently received much attention by researchers for testing mode I fracture toughness of rocks and other geo-materials. While the SCB specimen is often prepared by a straight crack, chevron notched semi-circular bend specimen (CCNSCB) has been rarely utilized. In this paper, using the analytical methods for evaluating the minimum dimensionless stress intensity factor ( $Y_{\min}^*$ ) of chevron notched specimens, the slice synthesis method is employed to obtain the dimensionless critical stress intensity factor of the specimen. Then, fracture tests are performed on a white crystalline rock under mode I loading using the CCNSCB specimen. The experimental results show very little scatter in the measured values of fracture toughness.

**Keywords** Rock, CCNSCB, Experiments, Finite element modeling

### 1. Introduction

Fracture mechanics provides an engineering description for the transformation of an intact structural component into a broken one through crack extension [1]. Physical operations like blasting and crushing which are used in practical applications such as tunneling, drilling and mining projects are often considered as a process of formation and growth of cracks in rock masses. Fracture toughness is a major parameter in fracture mechanics which represents the energy required to initiate brittle failure around the crack tip. Fracture experiments on full scale rock masses are difficult and expensive. Thus, researchers prefer to perform their fracture analyses utilizing various laboratory-scale cracked specimens. Three testing methods have been suggested by the International Society for Rock Mechanics (ISRM) for measuring rock fracture toughness: (i) Chevron Bend specimen method (CB) (ii) Short Rod specimen method (SR) [2], and (iii) Cracked Chevron Notched Brazilian Disc (CCNBD) specimen method [3]. The semi-circular bend (SCB) specimen proposed by Chong and Kuruppu [5] has also recently received much attention by researchers. Some advantages of the SCB specimen are its simplicity, minimal requirement of machining and ability of preparation from rock cores. The SCB specimen can also be used as an alternative to the ISRM standard specimens in determining fracture toughness in orthogonal directions of transversely isotropic materials or sedimentary rock, such as sandstone or oil shale [6]. In addition to the rocks, the SCB method has been applied to other core-based brittle materials, such as concrete and asphalt. Furthermore, SCB is a suitable specimen to rock fracture toughness test at high strain rates [6].

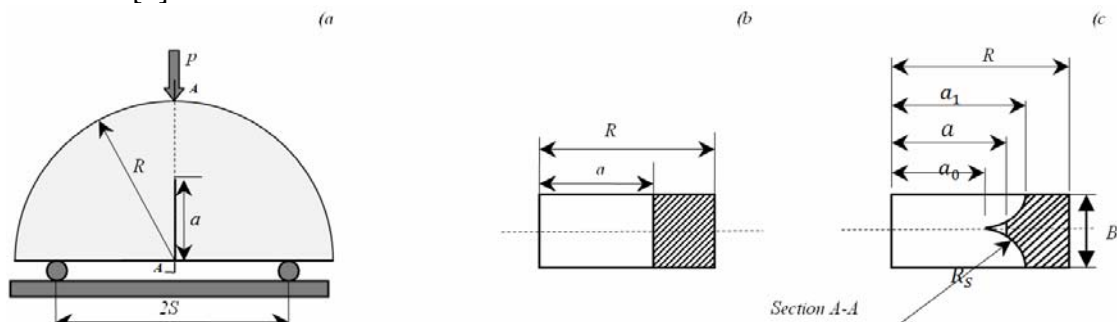


Figure 1. a) Schematic view of SCB specimen b) Straight crack c) Chevron notched

Crack in rock test specimens are usually created either with a straight front or a chevron shape, as shown in Figure 1. The chevron notched SCB specimen in the static mode has been rarely studied and only a limited SIF calibration was done by Kuruppu using a 3D finite element method [7–9]. The fracture toughness values of SCB with straight crack are calculated from [10]:

$$K_{IC} = \frac{P_f \sqrt{\pi a}}{2Rt} Y_I(a/R, S/R) \quad (1)$$

where  $P_f$  is the maximum failure load,  $Y_I$  is the dimensionless stress intensity factor,  $t$ ,  $R$  and  $a$  are the thickness, radius of SCB specimen and crack length, respectively.  $Y_I$ , also known as geometry factor, is a function of the ratio of crack length over the semi-disc radius ( $a/R$ ) and the ratio of half-distance between the two bottom supports over the semi-disc radius ( $S/R$ ) [11]. Lim et al. [10] found SIFs in terms of  $a/R$  and  $S/R$ . Their results can be summarized by the following relation:

$$Y_I = S/R \left( 2.91 + 54.39\alpha - 391.4\alpha^2 + 1210.6\alpha^3 - 1650\alpha^4 + 875.9\alpha^5 \right) \quad (2)$$

where  $\alpha$  is  $a/R$ .

If a standard CCNBD specimen is cut into two equal parts, two pieces of CCNSCB are obtained. Figure (1-c) shows the chevron notched SCB specimen and its geometrical coefficients.  $\alpha_0 (= a_0/R)$ ,  $\alpha_1 (= a_1/R)$  and  $\alpha_m (= a/R)$  are the dimensionless initial crack length, dimensionless final crack length and dimensionless critical crack length, respectively.  $\alpha_s$  is a coefficient obtained from dividing  $R_s$  (radius of rotary saw) by  $R$ . Also  $\alpha_B (= B/R)$  is the normalized thickness.

Similar to the equation suggested by ISRM for the CCNBD specimen [4], the initiation fracture toughness  $K_{IC}$  of CCNSCB specimen can be determined as:

$$K_{IC} = \frac{P_{\max}}{B\sqrt{D}} Y_{\min}^* \quad (3)$$

where  $P_{\max}$  is the measured maximum load,  $B$  and  $D$  are the thickness and diameter of the disc respectively,  $Y_{\min}^*$  is the minimum value of  $Y^*$ , which is the dimensionless stress intensity factor (SIF). It should be noted that all the restrictions and the geometrical relationships for the CCNBD is assumed to be applicable to CCNSCB too. So far, several approximate analytical methods have been used to determine  $Y^*$ , although applicability and accuracy of any of these methods have not been evaluated in CCNSCB. In this paper a slice synthesis method (SSM) is presented to evaluate the minimum dimensionless stress intensity factor in the CCNSCB specimen. Then, its accuracy is examined using both experimental test and finite element method. Experimental results show that the CCNSCB specimen can be employed for measuring rock fracture toughness.

## 2. Slice synthesis method in CCNSCB specimen

Slice synthesis method, proposed first by Bluhm [12], is a semi-analytical method for solving fracture problems with curved crack fronts. Wang et al. [13] used the SSM to calculate the stress intensity factor of CCNBD. In this method, the thickness of the sample is cut into a number of slices each with a thickness  $\Delta t$  as shown in Fig. 2. The analysis of a single slice is easier than analyzing the whole specimen. First, analytical equations are written for each slice. Then, by combining these equations, an equation for the entire sample can be achieved according to the

equilibrium principle. Analytical relations can be extracted in specimens of complex configuration using SSM method. No analytical relationship has been reported in papers for obtaining the stress intensity factors for the CCNSCB specimen and it is rather difficult using experimental or numerical methods. Every slice in CCNSCB is considered as a SCB with the straight crack. In fact, the central portions of CCNSCB with the straight crack front width  $b$  need not to be cut into thin slices. Since analytical solutions exist for the calculation of the stress intensity factor in the SCB specimen with a straight crack, an equation can be written for each portion and also for the middle section and finally with combining the stress intensity factors of these two sections, the formula for calculating the dimensionless stress intensity factor of the CCNSCB is obtained. More details on the procedure can be found in [13], but the general equation is written as:

$$Y^* = \left[ \frac{b/B}{Y(\alpha)} + \sum_{i=1}^N \frac{2 \cdot \Delta t / B}{\beta Y(\alpha_i)} \right]^{-1} \quad (4)$$

where  $\Delta t$  and  $N$  are the thickness of each slice and the number of slices, respectively.  $Y$  is the dimensionless stress intensity factor of SCB with straight crack and  $\alpha_i = a_i/R$  where  $a_i$  is the crack length of  $i$ th slice. Parameters  $\Delta t$ ,  $\alpha_i$  and  $b$  are related to the dimensionless crack length ( $\alpha$ ).

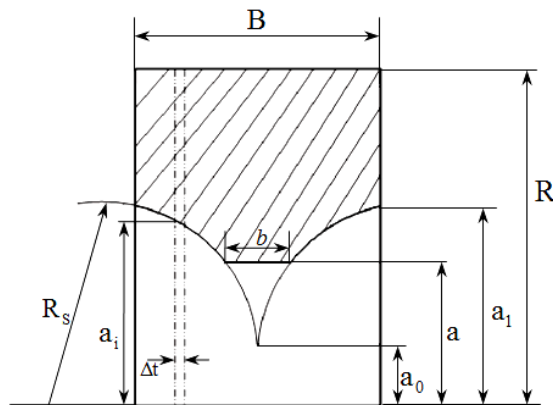


Figure 2. Slice synthesis method for CCNSCB specimen

$\beta$  reflects the difference between the stress intensity factor of the central part and that of the lateral part.  $\beta$  can be calculated as:

$$(5)$$

$$\beta = 1 + \gamma \frac{\alpha_1 - \alpha}{\alpha_B}$$

The important and difficult part of the SSM method is the calculation of  $\beta$ . By comparing the results of three-dimensional finite element analysis, Wang et al. [13] predicted the coefficient  $\gamma$  to be 0.9 in CCNBD specimens. Here, we employed SSM method in the CCNSCB specimen used previously by Kuruppu [8] and by comparing the results of SSM with the results of kuruppu [8],  $\gamma=0.5$  was found an appropriate value.

Thus, to find the minimum dimensionless stress intensity factor in CCNSCB, it is sufficient to minimize equation (4) as:

$$(6)$$

$$\frac{\partial Y^*}{\partial \alpha} = 0$$

The related numerical results will be presented and discussed later in section 4.

### 3. Sample preparation and fracture toughness tests

A crystalline rock was selected for fracture toughness tests. This type of rock contains very few discontinuities and can be assumed to be isotropic and homogeneous.

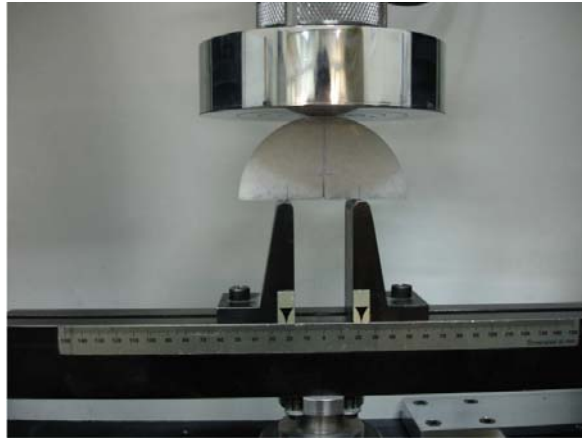


Figure 3. Loading set-up utilized for conducting fracture tests in CCNSCB specimen

The rock samples were prepared from a rock sheet of 20mm thick using water jet to form semi-circular disks of diameter 80mm. A rotary saw of radius  $R_s = 50\text{mm}$  and thickness 0.6mm was used to generate the chevron notch in the specimens. The penetration of rotary saw into specimens was considered 11mm. In the experiments, the half-distance between the two bottom supports ( $S$ ) was 20mm. The standard 3-point bend loading was used for fracturing the SCB specimens (see Fig. 3). After creating the chevron notch, the specimen was placed inside the loading set-up and the failure loads were recorded. SANTAM/STM-150 machine with a capacity of 15000N was utilized for conducting the fracture tests on the CCNSCB specimens.

## 4. Dimensionless stress intensity factor

### 4.1. Numerical and experimental results

It was observed in the entire experiments that when the load reached its critical level, sudden failure took place for the test samples. The load-displacement curve was almost linear up to the maximum load. Thus, using the critical failure load in each experiment, the fracture toughness values of CCNSCB specimen can be computed from Eq. 3. The details of experimental results obtained from each test including, the fracture loads are listed in Table 1.

Using the maximum load obtained from the experiment, the corresponding fracture toughness is calculated using finite element modeling. The singular elements were considered in the first ring of elements surrounding the crack tip for producing the square root singularity of stress/strain field. A  $J$ -integral based method was used for obtaining the stress intensity factors. Fig. 4 shows a typical mesh pattern generated for simulating the CCNSCB specimen.

The average value of maximum load was 1.877kN and using this value in the finite element model, mode I fracture toughness  $K_{IC}$  is obtained  $1.208 \text{MPa}\cdot\text{m}^{1/2}$  and by its substitution in Eq. (3), the dimensionless stress intensity factor in the CCNSCB specimen is found 3.64. During the simulation,

the critical crack length is derived from Eq. 6. Average  $K_{IC}$  values for 3D model of CCNSCB considered were confirmed to converge to constant values with Twenty-five concentric rings of crack front mesh elements. Here, fracture toughness is considered from the middle point of crack front.

Table 1. Summary of fracture tests conducted on CCNSCB specimens manufactured from the crystalline rock

Test No.	$p_{max} (kN)$
1	1.914
2	1.82
3	1.8
4	1.871
5	1.876
6	1.981
<b>Average</b>	<b>1.877</b>

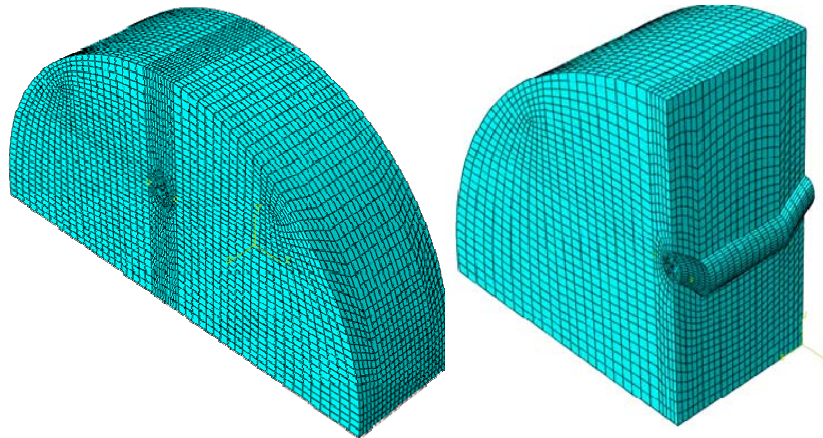


Figure 4. A typical finite element mesh used for simulating CCNSCB specimen.

The experiments conducted on CCNSCB showed that before the crack reaches its critical length, no significant resistance is observed with crack extension. Indeed, in the chevron-notched specimens, the crack is allowed to extend sub-critically, due to the high stress concentration. The chevron notch causes the crack propagation to initiate at the tip of the V alignment and to extend radially outwards in a stable fashion until the point where the fracture toughness is obtained. V or chevron shape generates a process of stable crack growth under increasing load, from the initial crack length ( $a_0$ ) to its critical length ( $a_m$ ). Beyond  $a_m$ , the crack extension takes place in an unstable fashion. At the critical length, the pre-crack has fully developed in the rock sample and the fracture toughness is evaluated. This procedure also provides a smaller fracture process zone (FPZ), compared to a straight crack. The experimental results also show that V-shaped grooves provide sharper critical cracks and lower scatter in fracture loads. Therefore, this specimen can be a good alternative for measuring fracture toughness of rock masses and for investigating the process of crack growth in brittle materials.

## 4.2. SSM method results

By using the dimensionless geometric parameter ( $\alpha_s, \alpha_0, \alpha_1$  and  $\alpha_B$ ) and substituting into equation (3), SSM model predicts the value of 3.541 for minimum dimensionless stress intensity factor, which is 2.7% lower than the value obtained from the experimental data (see Fig. 5). Also note that  $Y_I$  is the dimensionless stress intensity factor of the SCB specimen with a straight crack. Here, equation (2) was used for calculating  $Y_I$ .

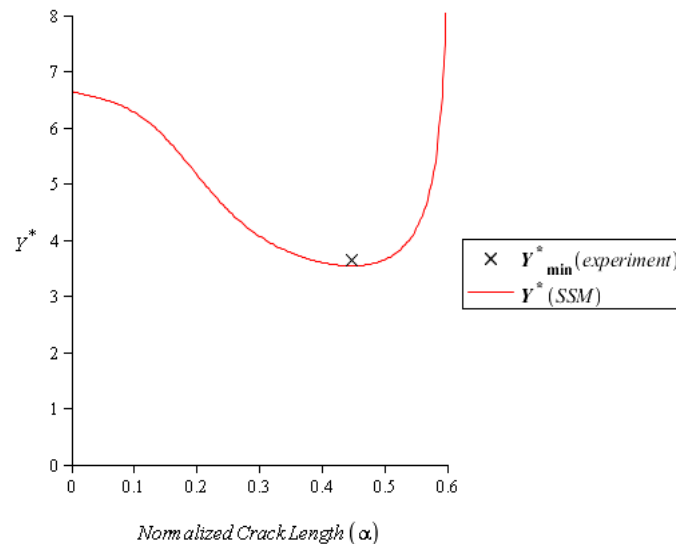


Figure 5. slice synthesis method utilized for estimating  $Y^*_{\min}$  in CCNSCB specimen

Because of the convenience and accuracy of SSM, this method can be suggested as a reliable method for evaluating the minimum dimensionless stress intensity factor ( $Y^*_{\min}$ ) in the CCNSCB specimens.

## 5. Conclusions

Although the SCB specimens with straight crack front have been used frequently by researchers, very few studies have been reported on the use of CCNSCB specimen in rock fracture toughness testing. The minimum dimensionless stress intensity factor of CCNSCB was calculated using SSM and its accuracy was assessed experimentally and by finite element method.



Figure 6. Fracture surface of CCNSCB specimen

Under mode I loading, a V-shaped (chevron) crack results in the automatic formation of a sharp and

natural crack in the specimen causing a lower scatter in the experimentally obtained fracture loads. Observed fracture surface shows that surface roughness of the cracked chevron notched specimens is relatively low for the tested white crystalline rock (see Fig. 6). Meanwhile, it is finally recommended that similar fracture tests are performed on the SCB specimens having straight cracks and the experimental results are compared when these two different methods are used for generating the artificial crack.

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