Separation of the Energy Release Rate of Fracture

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Abstract

Formally, the Griffith's energy release rate, the Irwin's integral expression of crack closure energy and the Rice's J-integral give same results for linear elastic material. But let us ask a question: do these three approaches really give an exact mathematical equality and an identical physical meaning? For this purpose, a uniform equation was therefore introduced into our new investigation. Thus, not only the Irwin's integral expression of crack closure energy and the Rice's J-integral can be derived from this uniform equation, but a very important result has been also obtained from them: the energy release rate is separable into two parts. The first part describes the extension of the crack front surface and the second the distortion.

Keywords the Griffith's energy release rate, the Irwin's integral expression of crack closure energy, the Rice's J-Integral and the new found S_k-Integral und T_k-Integral

1. Introduction

Cracks and fractures are known natural phenomena and occur everywhere in our daily lives. They are mostly associated with catastrophic consequences. Therefore attempts have been made for a long time to understand and finally to master them.

As a pioneer, Griffith (1920) [2] has laid a foundation through examining these phenomena energetically. Based on the Inglis (1913) [1] identified stress and displacement field for an elliptical crack-like hole in an infinite plate, he has considered the energy balance and introduced a new quantity "energy release rate" into the fracture mechanics. His mind was so fundamental that it has always been of great importance for the further development of fracture mechanics. But his theory was limited only on linear elastic material behavior and mode I loading case.

Irwin (1957, 1964) [3, 4] expanded the Griffith's thought on complicated load cases and established a relationship between the energy release rate and the stress intensity factor. So the stress intensity factor for linear elastic material has been widely used.

A path-independent integral, the well-known J-integral was introduced by Cherepanov (1967) [11] and Rice (1968a) [7] into the fracture mechanics. Rice in particular has derived a connection between the Griffith's energy release rate and the J-integral and interpreted the J-integral as an extended energy release rate. Since that time, the J-integral rapidly disseminated and was used for elastic-plastic material in the fracture mechanics.

All this shows a significant development in the classical fracture mechanics and is the foundation of the fracture mechanics. The classical fracture mechanics has found variety applications for many different areas. However, it must be clearly mentioned that it can only describe the crack problem in relatively simple cases and under certain conditions. It is not yet able to handle and exactly describe complicated crack problems in both natural events as well as in everyday life and eventually to solve.

This situation can certainly not satisfy us and the problems mentioned above let us thoroughly think whether the foundation of the fracture mechanics is consistent and where and what has not been considered.

The aim of this paper is to deal with such problems and to find a possible satisfactory solution. For this purpose it first has to deal with the existing theories systematically.

The investigation has the following assumptions: continuum mechanics of observation, stationary crack, quasi-static, small deformations, isotropic and linear elastic material behavior, elastic-plastic material behavior with power-law hardening and J_2 deformation theory.

There are other theories and attempts to deal with and to describe such problems. These are not part of this paper, and they are not discussed here.

2. The Classical Theories of Fracture Mechanics

2.1 Griffith's Theory

At first Griffith (1920) [2] published his fundamental work on the treatment of crack problems. He investigated an infinite plate with a crack-like elliptical hole under tension (Fig. 1) and gave an energy balance to this plate

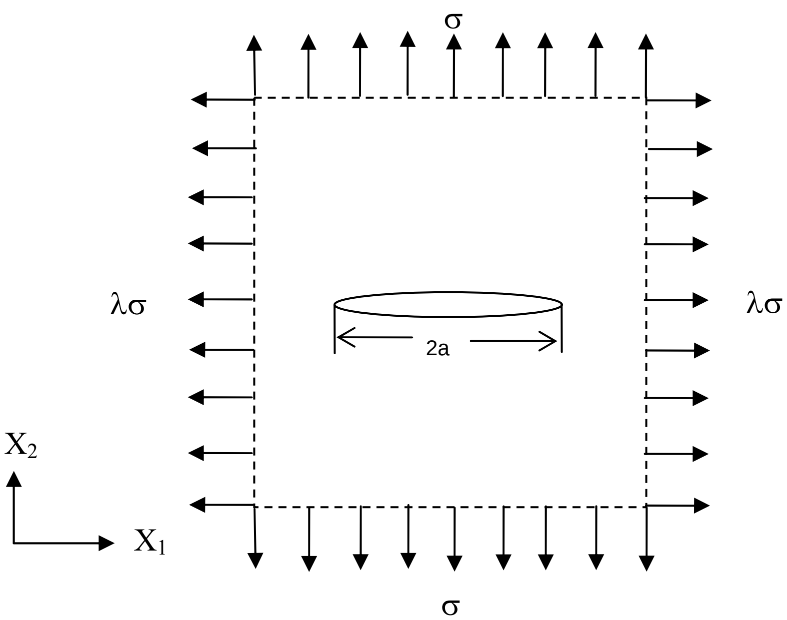

Fig. 1: an infinite plate with a crack-like elliptical hole under tension (Mode I)

$$\frac{\partial}{\partial a}(U - W - O) = \frac{\partial}{\partial a}(-\Pi - O) \geq 0\,, \tag{2.1}$$

where U is the work done by the external force, W the strain energy, $\Pi = -U + W$ the potential energy and O the surface energy.

Under consideration of the so-called “fixed-grips“ condition and with help of the stress and displacement field of Inglis (1913) [1] and by the vanish of the short axle of the ellipse he obtained following equation

$$\Pi = W = W_0 - \frac{\pi a^2 \sigma^2}{E'}\,, \tag{2.2}$$

where W_0 is the strain energy without crack, $E' = E$ for plane stress, $E' = E/(1-\nu^2)$ for plane strain, E is the elastic modulus and ν the Poisson's Ratio.

With Eq. (2.2) he got the energy release rate

$$G_I = -\frac{\partial \Pi}{\partial(2a)} = -\frac{\partial W}{\partial(2a)} = \frac{\sigma^2 a\pi}{E'} . \tag{2.3}$$

The index I denotes the mode I and the factor 2 means the whole elliptical crack length.

For the surface energy O he assumed the form

$$O = 4a\gamma , \tag{2.4}$$

where γ is the specific surface energy and should be a material constant. By substituting Eq. (2.3) and (2.4) into Eq. (2.1), the Eq. (2.1) becomes

$$G_I \geq 2\gamma . \tag{2.5}$$

With this, he predicted that the unstable crack growth occurs when the equation (2.5) is fulfilled.

From the Griffith's investigation it can be summarized:

His study is based on a two-dimensional, infinite plate with an elliptic crack-like hole under mode I loading case. He established the global energy balance for the whole body and recognized that under the fixed grips condition the strain energy change could only be considered and a residual amount of the strain energy change has to be given in order to proceed with the crack. This residual amount of energy divided by the crack change must have the same size or is larger than the surface energy, which then can create the new surface. From this energy, he has introduced a well-known fracture mechanics quantity which is referred to as "energy release rate".

It was unclear whether the Griffith's theory is applicable for general or complicated crack problems.

2.2 Irwin's Work

Irwin (1957, 1964) [3, 4] has attempted to answer the questions above. He extended Griffith's theory for mode I to mode II and III for linear elastic materials. Under consideration that the energy to close the crack (2.6)

$$I = \lim_{\Delta a \to 0} \frac{1}{\Delta a} \int_{A_1} \frac{1}{2} (\sigma_{12}^0 u_1^1 + \sigma_{22}^0 u_2^1 + \sigma_{23}^0 u_3^1) n_2^1 dA \tag{2.6}$$

has to be equal to the energy to extend the crack, he obtained a well-known equation

$$G = I = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{E}(1+\nu) , \tag{2.7}$$

where σ_{ij} is the stress tensor, u_i the displacement vector and n_i the normal on the crack front surface. $(\,)^0$ refers to the state of the time t and $(\,)^1$ the state of the time $t + \Delta t$, A_1 is the crack front surface to the state of time $t + \Delta t$ and K_I, K_{II}, K_{III} are the stress intensity factors for mode I, mode II and mode III.

So, a connection between the global quantity "energy release rate" G and the crack front field "stress intensity factor" K was given. Nevertheless the problem is that the equation (2.6) includes σ^0_{ij}, u^1_i und n^1_j which refer to the different states of time. This makes the equation (2.6) difficult to use for complicated problems.

2.3 The J-Integral

The path-independent J-integral

$$J = \int_A (wn_1 - \sigma_{ij} u_{i,1} n_j) dA \qquad (2.8)$$

was introduced by Cherepanov (1967) [11] und Rice (1968a) [7] into the fracture mechanics to determine some specific problems. The J-integral is not only applicable to linear elastic material but also used for hyper-elastic material. Specifically, Rice (1968b) [8] and Budiansky and Rice (1973) [9] have derived the relationship

$$-\frac{\partial \Pi}{\partial a} = \int_{A_0} wn_1 dA, \qquad (2.9)$$

and by consideration of the traction-free crack surface the equation (2.9) is finally equal to the J-integral

$$-\frac{\partial \Pi}{\partial a} = J. \qquad (2.10)$$

So the following Eq. (2.11)

$$J = G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{E}(1+\nu) \qquad (2.11)$$

is obtained, which is valid for linear elastic material, where w is the strain energy density, A_0 the crack front surface to the state of time t, A the area refers to any path in body, σ_{ij} the stress tensor, u_i the displacement vector and n_i the normal and $(\,)_{,1} = \partial(\,)/\partial x_1$.

It is seen above that for linear elastic material, the J-integral is both equal to the Griffith's energy release rate Eq. (2.10) and supplies the same result as from the Eq. (2.7) provided by Irwin. It is also applicable to elastic-plastic material. Thus, the J-integral became important in fracture mechanics and particularly for elastic-plastic material.

However, it can be recognized from the equations (2.6) and (2.8), that on the one hand the two equations Eq. (2.6) and Eq. (2.8) seem to be quite different, although they provide identical results (Eq. (2.7) and Eq. (2.11)), and on the other hand that the two integrals refer to different crack front surfaces, whereby the equation (2.6) to the crack front surface is linked to the time $t + \Delta t$, and the equation (2.8) is linked to the time t.

Due to the different formulations of the above theories we can provide the following questions:

- Do the Griffith's energy release rate G, the Irwin's I-integral expression of the crack closure energy and the Rice's J-integral really describe the same fact?
- Is there a uniform rule from which the different equations as Eq. (2.6) and Eq. (2.8) can be derived?
- Does new knowledge hide behind this difference?

To answer these questions requires us to conduct a thorough analysis in the next chapter.

3. Uniform Formulations of the Energy Release Rate

3.1 General Formulations

Let us first consider the change of the potential energy in a cracked elastic body shown in Figure 2(a), where the deformation of the crack front surface is adopted as shown in Figure 2(b),

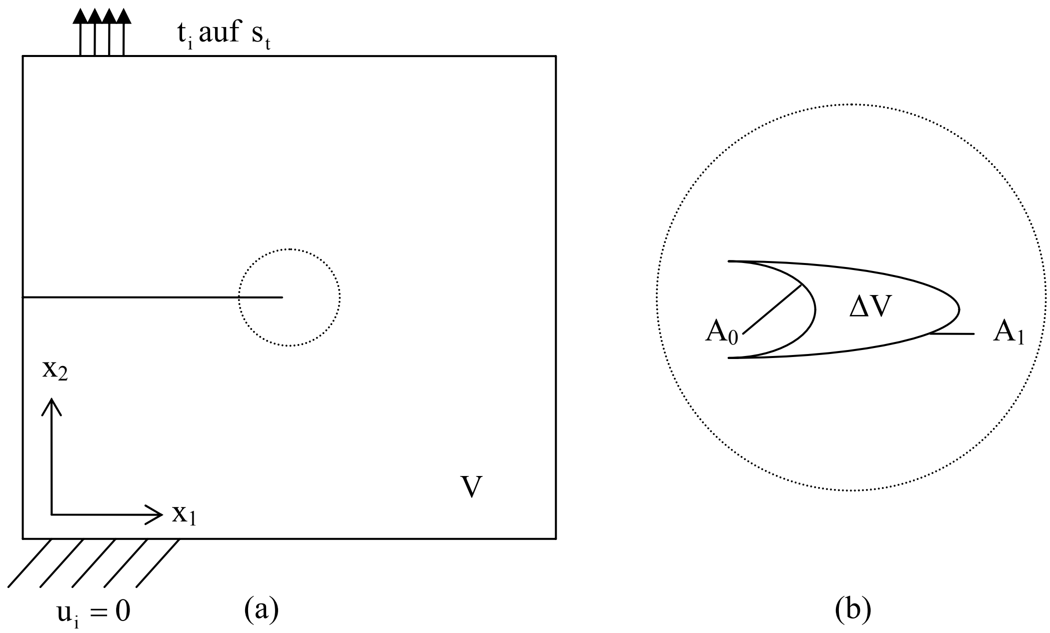

Fig. 2: (a) a cracked body, (b) the crack front field

$$\dot{\Pi} = (W - U)^{\bullet} = \frac{d}{dt}\int_V w dV - \frac{d}{dt}\int_{s_t} t_i u_i dA . \tag{3.1}$$

In the above equation Π is the potential energy, W is the strain energy, w is the specific strain energy, U is the work done by the external force, σ_{ij} is the stress tensor, ε_{ij} is the strain tensor, u_i is the displacement vector, t_i the force on the surface s_t and V the volume, $\dot{(\ )} = d/dt$ is the material time derivative and t the time-like parameter. It follows

$$\dot{\Pi} = \int_V \dot{w}\, dV - \int_{s_t} t_i \dot{u}_i\, dA + \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\Delta V} w(t + \Delta t) dV . \tag{3.2}$$

By looking at the first and the second term in the equation (3.2) and by the use of the Gaussian theorem, the terms cancel each other, so we get

$$\dot{\Pi} \;=\; \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\Delta V} w(t + \Delta t) dV . \tag{3.3}$$

This equation (3.3) is the basic equation from which we can derive different quantities.

3.2 Conventional Derivation

From the basic equation (3.3) Rice (1968b) [8], Eshelby (1970, 1956) [5, 6] and Budiansky & Rice received (1973) [9] a relation. For this, we give here the same formulation as by Budiansky & Rice (1973) [9] written:

$$\dot{\Pi} \quad = \quad \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\Delta V} w(t+\Delta t) dV = \int_{A_0} w\, v_i\, m_i dA\,, \tag{3.4}$$

where v_i is the velocity and m_i the normal outward. By substituting $v_i = \delta_{ik}$, $m_i = -n_i$ and $\xi_k \equiv -\dot{\Pi}$, we obtain

$$\xi_k = \int_{A_0} w\, n_k dA\,. \tag{3.5}$$

By consideration of traction-free crack surface it follows

$$\xi_k = J_k \tag{3.6}$$

and

$$J_k = \int_A (w\, n_k - \sigma_{ij} u_{i,k} n_j) dA\,, \tag{3.7}$$

where Eq. (3.6) is Eshelby's result [5] and J_k is the Integral derived by Knowles & Sternberg (1971/1972) [10] and with k = 1 J_k is the J-Integral

$$J_1 = J. \tag{3.8}$$

Thus, the connection between the J_k integral and the energy release rate has been established.

3.3 A New and Generalized Way to Derive the Irwin's I-integral

Let us now consider the specific strain energy w in Eq. (3.3) for linear elastic material

$$w(t+\Delta t) = w^1 \approx (\sigma_{ij}^0 \varepsilon_{ij}^1 + \Delta\sigma_{ij} \varepsilon_{ij}^0)/2\,. \tag{3.9}$$

The term $\Delta\sigma_{ij}\Delta\varepsilon_{ij}/2$ in the above equation has disappeared because of having an infinitesimal size of higher order. By substituting Eq. (3.9) into Eq. (3.3) and under consideration of the equilibrium condition $\sigma_{ij,j} = 0$ and the traction-free crack surface as well as with help of the Gaussian theorem, it follows

$$\dot{\Pi} \quad = \quad \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\Delta V} \frac{1}{2} \sigma_{ij}^0 \varepsilon_{ij}^1 dV = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{A_1} \frac{1}{2} \sigma_{ij}^0 u_i^1 m_j^1 dA\,. \tag{3.10}$$

In the Eq. (3.10) the expression $\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\Delta V} \frac{1}{2} \Delta\sigma_{ij} \varepsilon_{ij}^0 dV$ vanishes, which can be proved by using the mean value theorem of the integral

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\Delta V} \frac{1}{2} \Delta\sigma_{ij} \varepsilon_{ij}^0 dV = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \frac{1}{2} (\Delta\sigma_{ij} \varepsilon_{ij}^0)_M \Delta V = 0\,,$$

where M is a certain point in the volume ΔV.

By substituting $I^* = -\dot{\Pi}$ and $m_i = -n_i$ and by replacing t with the actual crack length a into (3.10), we get the generalized equation

$$I^* = \lim_{\Delta a \to 0} \frac{1}{\Delta a} \int_{A_1} \frac{1}{2} \sigma_{ij}^0 u_i^1 n_j^1 dA \,. \qquad (3.11)$$

For a special case of j = 2 it follows

$$I = I^*\big|_{j=2} = \lim_{\Delta a \to 0} \frac{1}{\Delta a} \int_{A_1} \frac{1}{2} (\sigma_{12}^0 u_1^1 + \sigma_{22}^0 u_2^1 + \sigma_{32}^0 u_3^1) n_2^1 dA \,. \qquad (3.12)$$

This is the Irwin's equation (2.6).

In the equation (3.11) it is seen that they have the quantities σ_{ij}^0, u_i^1 and n_j^1 which relate to the different times. As mentioned above, this leads to a difficult use of the equation (3.11) for complicated problems.

4. Separation of the Energy Release Rate and Introduction of Two New Quantities

4.1 A New Vector Quantity S_k for Describing the Crack Front Extension

The specific strain energy w in Eq. (3.3) for linear elastic material has an another form

$$w(t + \Delta t) = w^1 \approx (\sigma_{ij}^1 \varepsilon_{ij}^0 + \sigma_{ij}^0 \Delta\varepsilon_{ij}) / 2 \,. \qquad (4.1)$$

The term $\Delta\sigma_{ij}\Delta\varepsilon_{ij} / 2$ in the above equation has also disappeared because of having an infinitesimal size of higher order. By substituting the Eq. (4.1) into Eq. (3.3) and by considering the equilibrium condition $\sigma_{ij,j} = 0$ and the traction-free crack surface as well as by using the Gaussian theorem, it follows

$$\dot{\Pi} \;=\; \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\Delta V} \frac{1}{2} \sigma_{ij}^1 \varepsilon_{ij}^0 dV = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{A_0} \frac{1}{2} \sigma_{ij}^1 u_i^0 m_j^0 dA \,. \qquad (4.2)$$

In the Eq. (4.2) the expression $\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\Delta V} \frac{1}{2} \sigma_{ij}^0 \Delta\varepsilon_{ij} dV$ vanishes, which can be also proved by using the mean value theorem of the integral

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\Delta V} \frac{1}{2} \sigma_{ij}^0 \Delta\varepsilon_{ij} dV = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \frac{1}{2} (\sigma_{ij}^0 \Delta\varepsilon_{ij})_M \Delta V = 0 \,,$$

where M is a certain point in the volume ΔV.

The equation (4.2) is new and will therefore be used furthermore to derive a new quantity. Now we consider the above equation (4.2)

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{A_0} \frac{1}{2} \sigma_{ij}^1 u_i^0 m_j^0 dA = \int_{A_0} \frac{1}{2} \frac{\partial \sigma_{ij}}{\partial t} u_i m_j dA \,. \qquad (4.3)$$

Since t is a time-like parameter and can be replaced by the actual crack extension a for the quasi-static problem, it follows with help of $m_i = -n_i$ and by use of the directional derivative

$$-\frac{d\Pi}{da} = l_k \int_{A_0} \frac{1}{2} \sigma_{ij,k} u_i n_j dA \,, \qquad (4.4)$$

where l_k is the direction vector of the crack extension. Now we get a new quantity, which is denoted with S_k

$$S_k = \int_{A_0} \frac{1}{2} \sigma_{ij,k} u_i n_j dA . \tag{4.5}$$

Then the Eq. (4.4) can be written as follows

$$-\frac{d\Pi}{da} = l_k S_k . \tag{4.6}$$

By using the above equation (4.5) and by help of the Gaussian theorem as well as by considering $\sigma_{ij,j} = 0$ and $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$, we can derive a path-independent integral by converting

$$\oint_B \frac{1}{2} \sigma_{ij,k} u_i n_j dA = \int_V \frac{1}{2} \sigma_{ij,k} u_{i,j} dV = \oint_B \frac{1}{2} w^c n_k dA , \tag{4.7}$$

where w^c is the complementary energy density and B is an arbitrary closed area in the body. Since $w = w^c$ for linear elastic material, it follows

$$\oint_B \frac{1}{2} \sigma_{ij} u_{i,k} n_j dA = \oint_B \frac{1}{2} w n_k dA . \tag{4.8}$$

Finally, we can form a new quantity

$$\oint_B \frac{1}{2} (\sigma_{ij,k} u_i - \sigma_{ij} u_{i,k}) n_j dA , \tag{4.9}$$

which always vanishes. By consideration of the traction-free crack surface and $B=A+(-A_0)$ we can then write

$$S_k = \int_{A_0} \frac{1}{2} \sigma_{ij,k} u_i n_j dA = \int_A \frac{1}{2} (\sigma_{ij,k} u_i - \sigma_{ij} u_{i,k}) n_j dA . \tag{4.10}$$

Based on the Griffith's energy release rate we have derived a new vector quantity S_k from the basic equation (3.3), which has an integral form (4.5). This integral is path-independent (4.9) and describes the crack front extension.

However, it must be clearly stated that the two equations (3.11) and (4.4) are derived only under the condition that the deformation of the crack front surface is assumed as in Figure 2. Whether this assumption is true for the real deformation of the crack front surface has yet to be thoroughly analyzed and investigated. To answer this question we have to continue considering the basic equation (3.3) even more precisely.

4.2 A Second New Vector Quantity T_k for Describing the Crack Front Distortion

From the basic equation (3.3), we have handled the strain energy density and generated two important results: a) the first form (3.9) leads to a generalized Irwin's integral expression of crack closure energy, b) the second form (4.1) provides a new vector quantity S_k, which has an integral form, and this integral is path-independent.

Now the question arises whether S_k is a single quantity which can be obtained from the basic equation (3.3) and what and where has not been taken into account.

Therefore, we must again consider the basic equation (3.3) accurately. In the above consideration we have tacitly assumed, that the volume fraction of ΔV is like in Figure 2, without analyzing it. To check whether this assumption for the deformation of the actual crack front surface is true, the deformation of the crack front surface must be again examined carefully. Let us now consider the body with cracks (Fig. 1) again. Under loading, the crack will extend and at the same time it will open, as shown below in Figure 3. As already known, only a small crack front surface will help for crack growth. Thus, the volume fraction ΔV must be included by surfaces A_0, A_1 and A_β. Especially the surface A_β describes the crack opening and this has been neglected until now.

By using the Gaussian theorem the basic equation (3.3) can be written in the new volume fraction ΔV (Fig. 3) as follows

$$\dot{\Pi} = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{\Delta V}\frac{1}{2}\sigma^1_{ij}\varepsilon^0_{ij}dv = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{A_0}\frac{1}{2}\sigma^1_{ij}u^0_i m^0_j dA + \lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{A_\beta}\frac{1}{2}\sigma^1_{ij}u^0_i m^\beta_j dA\,. \qquad (4.11)$$

The first term of equation (4.10) has already been dealt above. The second term is new and we will study it more precisely now. By replacing the time t with the crack length a and by use of the Stockes's theorem, the second term can be reformulated as follows

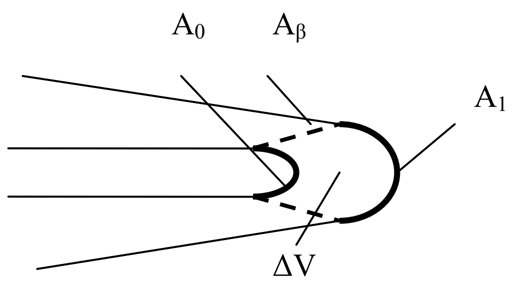

Fig. 3: real deformation of the crack front surface

$$\lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{A_\beta}\frac{1}{2}\sigma^1_{ij}u^0_i m^\beta_j dA = -l_k\int_{A_0}\frac{1}{2}\varepsilon_{qml}\varepsilon_{ljk}\partial_m\sigma_{ij}u_i n_q dA\,, \qquad (4.12)$$

where l_k is the direction vector of the crack extension, ε_{ijk} is the alternative tensor and ∂_i is the differential operator. Now we receive a second new quantity T_k

$$T_k = \int_{A_0}\frac{1}{2}\varepsilon_{qml}\varepsilon_{ljk}\partial_m\sigma_{ij}u_i n_q dA\,, \qquad (4.13)$$

which describes the distortion of the crack front surface.

It can easily be proved that the surface integral T_k above is path-independent by using the Gaussian theorem

$$\begin{aligned} T_k = \oint_B\frac{1}{2}\varepsilon_{qml}\varepsilon_{ljk}\partial_m\sigma_{ij}u_i n_q dA &= \oint_B\frac{1}{2}(\partial_k\sigma_{ij}u_i n_j - \partial_j\sigma_{ij}u_i n_k)dA \\ &= \int_V\frac{1}{2}(\partial_j\partial_k\sigma_{ij}u_i - \partial_k\partial_j\sigma_{ij}u_i)dV \\ &= 0 \end{aligned}$$

where B is an arbitrary closed area. It can be seen above, that

- the new vector quantity T_k has an integral form,
- it is path-independent,
- it describes the distortion of the crack front surface.

5. Two New Quantities S_k and T_k for Elastic-Plastic Material Behavior

Now we will extend the new found quantities S_k and T_k for linear elastic material behavior into elastic-plastic material behavior with power law hardening. So let us write the strain energy density w for this material as follows

$$w \approx \frac{n}{n+1}\sigma_{ij}\varepsilon_{ij}\,, \tag{5.1}$$

where n is the hardening parameter of material. In the above equation the part of elastic strain energy density is neglected, since the value of this part is very small in comparison with the plastic part.

It is shown that the Eq. (5.1) is a more generalized form of the strain energy density, which is valid not only for elastic-plastic material behavior but with n=1 also valid for linear elastic material behavior. Therefore, we can derive some quantities for this material as done in the last chapter 3 and 4.

In analogy to Eq. (3.11) we get the new generalized I^*-integral for elastic-plastic material behavior

$$I^* = \lim_{\Delta a\to 0}\frac{1}{\Delta a}\int_{A_1}\frac{n}{n+1}\sigma^0_{ij}u^1_i n^1_j dA\,. \tag{5.2}$$

In the same way we can still receive in analogy to Eq. (4.10)

$$S_k = \int_{A_0}\frac{n}{n+1}\sigma_{ij,k}u_i n_j dA = \int_{A}\left(\frac{n}{n+1}\sigma_{ij,k}u_i - \frac{1}{n+1}\sigma_{ij}u_{i,k}\right)n_j dA \tag{5.3}$$

and finally in analogy to Eq. (4.13)

$$T_k = \int_{A_0}\frac{n}{n+1}\varepsilon_{qml}\varepsilon_{ljk}\partial_m\sigma_{ij}u_i n_q dA\,. \tag{5.4}$$

The integrals of the new found quantities S_k and T_k in Eq. (5.3) and (5.4) for this material behavior are also path-independent.

6. Crack Driving Energy and Crack Force

In the above considerations, we have examined the basic equation (3.3) and could first derive the previously existing fracture mechanics parameters such as the generalized Irwin's I*-integral expression of the crack closure energy and the generalized Rice's J_k-integral. In particular we found the both new quantities S_k and T_k, where S_k describes the crack front extension and T_k the crack front distortion. Now, we can write the generalized energy release rate (4.11) as follows

$$-\frac{d\Pi}{da} = (S_k + T_k)l_k\,. \tag{6.1}$$

By substituting $P_k = S_k + T_k$ and by setting k = 1 for a special case "pure mode I", we obtain a very important result from the equation (6.1):

$$G = P_1 = S_1 + T_1, \qquad (6.2)$$

where G is the Griffith's energy release rate. This means:

the energy release rate is separable and can be separated into two parts.

By introduction of $W_F = -\Pi$ the equation (6.1) finally becomes

$$dW_F = P_k da_k, \qquad (6.3)$$

where W_F is the crack driving energy. The equations (6.1) and (6.3) show that the change of the crack driving energy consists of two vector quantities P_k and da_k, whereby da_k is the crack front deformation. Thus, the vector quantity P_k contains the force character and is therefore identified as the whole crack force. Since $P_k = S_k + T_k$, S_k or T_k represents a partial crack force. The energy balance of the cracked body can be reformulated as follows:

$$\frac{dU}{da} - \frac{dW}{da} - \frac{dW_F}{da} = 0, \qquad (6.4)$$

where U is the work done by the external force, W is the strain energy and W_F is the crack driving energy.

5. Summary and Conclusion

The questions that we set at the beginning of the article, are already covered in detail and answered. The important results can be summarized as follows:

- We have introduced a uniform equation (3.3) and derived the previously existing fracture mechanics parameters such as the generalized Irwin's I^*-integral expression of the crack closure energy and the generalized Rice's J_k-integral from it,
- based on the uniform equation (3.3), we have also found two new vector quantities S_k und T_k, which indicate that the energy release rate can be separated into the new found vector quantities S_k und T_k, where S_k provides the crack front extension, and the other T_k describes the crack front distortion,
- the two vector quantities S_k and T_k are formulated in an integral form and are path-independent,
- it has also given that the new quantities S_k and T_k as well as the generalized Irwin's I^*-integral expression of the crack closure energy are not only valid for linear elastic material behaviour but also for elastic-plastic material behavior,
- the two vector quantities S_k und T_k have the force character. So S_k is denoted as the partial crack force for crack front extension and T_k the partial force for crack front distortion.

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