Finite element modeling of the coupling between thermal dissipation and fish-eye crack growth in very high cycle fatigue regime

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Abstract The aim of the paper is to model the thermal dissipation associated to a fish-eye crack growth during an ultrasonic fatigue testing. We use a Paris-Hertzberg crack growth model to simulate the evolution of the crack and a perfectly elastic-plastic constitutive law to model the plastic dissipation per cycle. A finite element analysis is used to compute the evolution of the temperature field during the crack propagation. Numerical results are presented and they agree well with experimental results.

Keywords: very high fatigue cycle, ultrasonic testing, finite element modeling, crack growth, thermal dissipation

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1. Introduction

The considerable attention given, in the past ten years, to fatigue fracture of metallic material in very high cycle regime, is mainly due to the increasing use of engineering materials in applications with service lives reaching up to 10^{10} load cycles. It has been shown that very high cycles fatigue (VHCF) failures of steels are characterized by a fracture which does not occurs on the surface but rather internally in the material, especially in high strength steels [1-3], and leads to the so-called fish-eye. Ultrasonic fatigue testing is a powerful tool for evaluating VHCF properties as 10^9 cycles can be completed in a day [4]. During an ultrasonic fatigue testing, the crack propagation in the tested specimen leads to an important cyclic plastic dissipation. One way of assessing this dissipated energy, and thus the crack propagation, is to use an infrared camera to measure the temperature increase during the test since part of this dissipation will occur as heat [5]. As far we know few studies have been devoted to the numerical computation of the temperature field associated with the propagation of a fatigue crack during VHCF failure [6,7].

In this presentation, we study numerically the temperature field associated with the propagation of a fatigue crack in a very high cycle fatigue regime during ultrasonic fatigue testing. The crack propagation is modeled by a classical Paris-Hertzberg crack growth law, and the plastic dissipation per cycle is determined by 3D-elastic plastic finite element modeling of stationary mode I crack under constant amplitude loading. A fraction of this plastic dissipation is converted heat and used as a mobile thermal source to compute the temperature field evolution. The first part of the presentation is devoted to the Finite Element modeling of the problem to be solved: The fatigue crack law is described, as well as the thermal dissipation model and the computation of the computation of the plastic energy dissipation. The second part of the presentation is devoted to the numerical results: The proposed model is compared with experimental results and the distribution of the plastic dissipation around the crack is computed.

2. Finite Element modeling

2.1. Fatigue crack growth law

Paris and co-workers [4] have developed estimation for crack growth life for internal initiation, termed *fish-eye* using the Paris-Hertzberg crack growth rate law [8]. In the present work we use this law to describe the growth of the fatigue crack

$$\frac{da}{dN} = b\left(\frac{\Delta K_{eff}}{E\sqrt{b}}\right)^3 \quad (1)$$

where b is the Burger's vector modulus, E the elastic modulus and ΔK_{eff} the effective stress intensity factor range.

2.2. Thermal dissipation model

We consider a problem of a fatigue crack in an elastic-plastic material. The temperature field in the cylinder is evaluated by solving the heat transfer equation

$$\rho\, C\, \frac{\partial T}{\partial t} = \lambda \Delta T + \beta \dot{W}^{irr} \quad (2)$$

where ρ is the mass density, C the specific heat, λ the heat conduction coefficient, and $\dot{W}^{irr}$ the plastic dissipation. The multiplicative factor β is the so-called Taylor-Quinney factor, which takes into account the energy storage.

2.3 Computation of the temperature field during the fish eye crack growth

The temperature field is computed in two steps, in a first step we perform an elastic plastic finite element analysis to compute the plastic energy dissipation as a function of the crack growth, and in a second step we perform a thermal finite element analysis to compute the evolution of the temperature field.

2.3.1 Computation of the plastic energy dissipation

The plastic energy dissipation is obtained by 3D elastic plastic finite element analysis. It was shown that in VHCF regime, the crack growth is not a significant portion of life in VHCF fatigue with *fish-eye* failure [9]. However, the number of cycles involved is still important ($\approx 10^5$ cycles) and a direct nonlinear computation will lead to prohibitive computational time. In this respect we choose to compute the energy dissipation per cycle dW^{irr}/dN during a single load cycle on a stationary crack for different radius a_i ($a_{in} = a_0 < a_1 < \ldots < a_n = a_{fi}$), as proposed by Klingbeil [10]. The mean energy dissipation per cycle is computed

$$\dot{W}^{irr} = \Omega\, \frac{dW^{irr}}{dN} = \int_{cycle} \sigma : d\epsilon^p \quad (3)$$

where Ω is the frequency of the loading. To simulate the plastic fatigue regime, and not a monotonic loading, two cycles are computed for each radius a_i and only the last cycle is used to evaluate the plastic dissipation. Such an approach neglects the contribution of the actual crack extension during any given load cycle. However, for Paris-Regime crack growth, both the plastic work and the surface energy contributions associated with the actual crack extension in any given cycle are negligible compared to the total plastic dissipation. In VHCF regime the imposed load is small compared to the yield strength and the contribution of the plastic wake will be neglected in our work. Finally an elastic- perfectly plastic constitutive model is considered in this work.

2.3.1 Computation of the temperature field

In a second step, the plastic dissipation is used as a moving thermal source in a transient heat conduction problem. The thermal problem is solved with an implicit integration scheme. To define the time discretization ($t_0 < t_1 < \ldots < t_n$), we consider the following simplifications in the history of the crack propagation: the crack is supposed to remain circular during the crack growth process, the crack closure effect is neglected and we use the analytical formula for a circular crack in an infinite media

$$\Delta K_{eff} = \Delta K = \frac{2}{\Pi}\Delta\sigma\sqrt{\Pi a} \quad (4)$$

The loading is given by the energy dissipation per cycle computed in Equation (3). The meshes used are the meshes used during the mechanical computation to avoid errors due the data transfer on $\left(dW^{irr}/dN\right)(a_i)$ between different meshes.

3. Numerical results

The specimen is modeled by a cylinder *C* which has a circular cross-section of radius R_c and a height *2L*. A small circular crack perpendicular to the cylinder axis lies in the center of the cylinder, as shown in Fig. 1. The radius of the crack is denoted by *a(t)* and its eccentricity from the center of *C* is denoted by *e*. *C* is submitted to a cyclic loading: During each cycle the stress is varied linearly from an initial minimum value $\bar{\sigma}_{min}$ to a maximum value $\bar{\sigma}_{max}$ and back to the initial value $\bar{\sigma}_{min}$. An important parameter used to characterize the cyclic load is the so-called *load ratio,* defined as $R = \bar{\sigma}_{min}/\bar{\sigma}_{max}$. Because of symmetry conditions only one quarter of the cylinder is modeled.

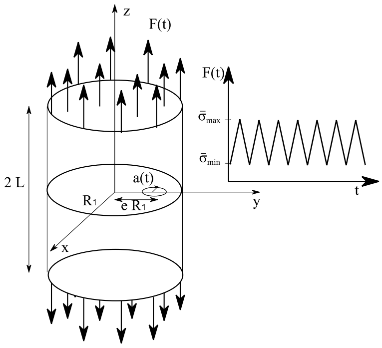

Figure 1. Specimen modeling

The tested material is a high-strength steel SAE 5120. Fatigue test are performed at ultrasonic fatigue frequency $\Omega = 20kHz$ with a stress ratio of $R = 0.1$ and stress amplitude $\Delta\sigma = 400Mpa$, using compressed air of $20^{o}C$ to cool the specimen. The material is approximated as elastic-perfectly plastic with thermomechanical properties $E = 200\,GPa$ (elastic modulus),

$\nu = 0.3$ (Poisson ratio), $\sigma_Y = 1067 Mpa$ (yield stress), $\rho = 7800 kg m^{-3}$ (mass density), $C = 460 J K^{-1} kg^{-1}$ (specific heat), $\lambda = 52 W K^{-1} m^{-2}$ (heat conduction coefficient). The Burger vector's modulus is taken such that $b = 1.8 \times 10^{-10} m$. The radius of the specimen is supposed to be $R_c = 1.5 mm$ and its height is defined by $L = 6 mm$. The finite element analysis is performed using Cast3M a general FE computer code developed by the CEA (French Alternative Energies and Atomic Energy Commission) with linear 3D elements.

3.1 Comparison with experimental data

The numerical results are compared with the experimental results obtained with two specimens. In Table 1 are given the number of cycles to failure N_f, the crack parameters measured after the failure a_{inc} and e, as well as the radius of the crack at $t = 0$ which is defined by $a_0 = a_{inc} / 0.94$ as proposed by Paris [20]

Table 1. Experimental parameters

Test	N_f (cycles)	a_{inc} (μm)	e	a_0 (μm)
1	$5,25 \times 10^7$	10.03	0.79	10.67
2	$4,24 \times 10^7$	13.80	0.83	14.68

The comparison between the measured and the computed temperature are plotted in Fig. 2 for test 1, and in Fig. 3 for test 2. These comparisons show a good agreement between the predicted and the measured values.

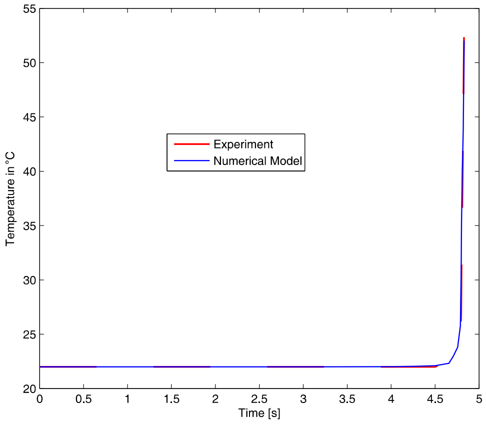

Figure 2. Test 1: Comparison between the modeled and the experimental temperature

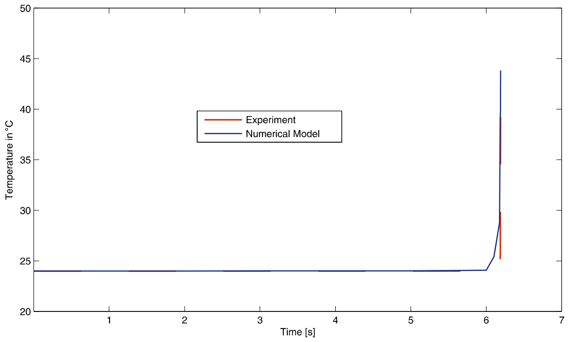

Figure 3. Test 2: Comparison between the modeled and the experimental temperature

3.2 Evolution of the plastic dissipation

The following results illustrate the evolution of the plastic dissipation per cycle, and of the temperature near the crack tip. The calculations are done with the geometric and materials values used for test 1. The crack tip line is divided in regular angular sections of 15°, and the evolution of the plastic dissipation evolution is computed in each section and shown in Fig 4. The plastic dissipation is not distributed regularly around the crack tip line as the crack progresses and is mainly concentrated in the two elementary volume which are the nearest to the surface.

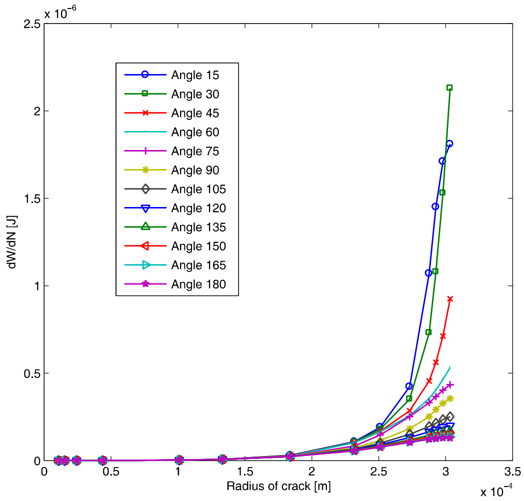

Fig 4. Evolution of the plastic dissipation as a function of the fish-eye radius

5. Conclusion

In this presentation, a weakly coupled thermo-mechanical finite element analysis of the propagation of a fatigue crack in VHCF fatigue regime has been proposed. The plastic dissipation per cycle has been computed from 3D finite element elastic perfectly plastic models of stationary crack in a cylinder under constant amplitude, mode I loading. The temperature rise during fatigue crack propagation is deduced from this plastic dissipation by the resolution of a transient heat conduction problem with a moving source. The numerical results are in a good agreement with experimental test, and show that the plastic dissipation, and hence the thermal source, is concentrated near the surface of the specimen.

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