Scaling invariance of fatigue crack growth in gigacycle regime

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Abstract The role of the collective behavior of defect ensembles at the crack tip and the laws of fatigue crack propagation in R4 high-strength steel have been studied under conditions of symmetric tension–compression gigacycle loading at 20 kHz. At every stage of the fatigue crack growth, replicas from the sample side surface were taken and studied by the method of three-dimensional relief profilometry (using interferometer profilometer NewView 5010) so as to study the scaling-invariant laws of defect-related structure evolution.
Keywords gigacycle fatigue, scaling, crack, morphology.

1. Introduction

The task of assessing the working resource of important structures, in particular, those for aircraft engines, poses qualitatively new basic problem related to evaluation of the reliability of materials under conditions of cyclic loading in excess of 10^8–10^{10} cycles, which refer to the field of so-called gigacycle fatigue. This interest is related to the fact that the resource of loading for many important parts operating under conditions of cyclic loading exceeds the so-called multicycle range. The behavior of materials in the range of gigacycle fatigue reveals some qualitative changes in the laws governing both the nucleation of cracks (in the bulk of a sample) and their propagation, which are related to changes in the mechanisms of fatigue crack nucleation and propagation. In the range of gigacycle loading, the fatigue curve exhibits discontinuities and the behavior shows evidence of a significant increase in the role of environment, so that the problem acquires an interdisciplinary character.

The stages of material fracture in the range of gigacycle loading are classified based on the structural signs of damage related to a broad spectrum of spatial scales, including persistent slip bands (PSBs), fatigue striations, microcracks (formed as a result of PSB crossing), and grain-boundary defects. The main damage refers to the defect scales within 0.1 μm–1 mm, which are significantly smaller than those detected by the standard methods of nondestructive testing used for the conventional monitoring of reliability, in particular, during the exploitation of buildings.

An effective method for investigating the role of initial structural heterogeneity, monitoring the accumulation of defects on various scales (dislocation ensembles, micropores, microcracks), and determining critical conditions for the transition from dispersed to macroscopic fracture is offered by the quantitative fractography. This technique reveals the characteristic stages of fracture (crack nucleation and propagation), thus providing a base for evaluating the temporal resource of materials and structures under conditions of gigacycle loading.

The approach to characterization of the fracture surface morphology in terms of spatiotemporal invariants was originally proposed by Mandelbrot [1]. This method is based on an analysis of the relief of a fracture surface, which exhibits the property of self-affinity as manifested by the invariant characteristics of the surface relief (roughness) over a broad spectrum of spatial scales. On the other hand, these characteristics reflect a correlated behavior of defects on various scaling levels.

The universal character of kinetic laws establishing a relationship between the growth rate dl/dN of a fatigue cracks and a change in the stress intensity coefficient ΔK has been extensively studied both experimentally and theoretically. The power laws originally established by Paris [2] (and presently referred to as the Paris law) reflect the automodel (self-similar) nature of development of fatigue cracks. This law is related to a nonlinear character of damage development in the vicinity of the crack tip (called the "process zone" [2]):

$$\frac{dl}{dN} = A(\Delta K)^m, \tag{1}$$

where A and m are the experimentally determined constants. For a broad class of materials and wide range of crack propagation velocities under multicycle fatigue conditions, the exponent is typically close to $m = 2$–4.

The universal nature of the Paris law was interpreted [3] based on the self-similar laws of damage development and fracture focus formation in the form of "dissipative structures" representing ensembles of defects localized on the spectrum of spatial scales. The formation of these structures reveals a critical character of the transition from dispersed to macroscopic fracture as manifested by the structural-scaling transitions [4]. According to these notions, the propagation of cracks is related to establishing a long-range correlation interaction in multiscale ensembles of dissipative structures. This interaction which can be characterized by a certain correlation scale, above which the interaction proceeds to a scale that determines the subsequent increment in the length of the propagating crack (size of the "process zone"). This assumption concerning the critical conditions for the aforementioned transition was used in interpretation of the experimental data so as to explain the self-similar scenario of fatigue crack propagation in a steel sample under conditions of gigacycle loading.

2. Experimental conditions and materials

The samples (Fig. 1) of R4 high-strength steel (with a room-temperature fatigue limit of 600 MPa for 10^6 cycles at 10 Hz) were tested under fatigue loading conditions with symmetric tension–compression cycling at 20 kHz (gigacycle loading regime) on an original setup [5]. The testing machine consisted of the following main parts: (i) generator, which converted 50 Hz oscillations into an ultrasonic electric sinusoidal signal with a frequency of 20 kHz; (ii) piezoelectric transducer, which generated longitudinal ultrasonic waves and produced mechanical action at a frequency of 20 kHz; and (iii) ultrasonic waveguide, which increased the amplitude of mechanical stresses in the (working) central part of the sample.

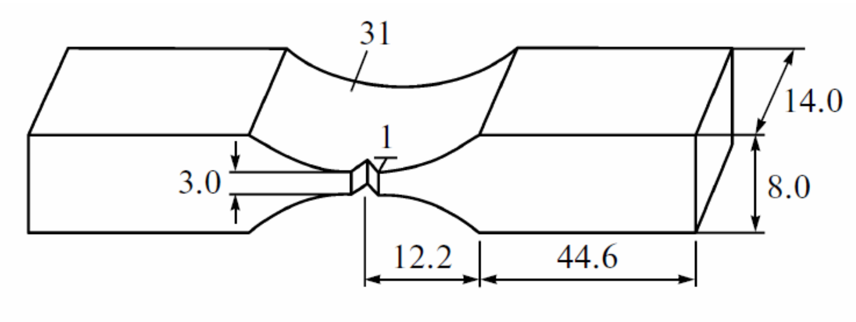

Figure 1. Schematic diagram and characteristic dimensions of the initial sample (in millimeters)

At the initial stage, a fatigue crack with a length of ~1.5 mm was nucleated, the subsequent growth of which was controlled by varying the amplitude of oscillations. The corresponding stress intensity coefficient ΔK was calculated by the following formula:

$$\Delta K = \frac{E}{1-\nu^2}\sqrt{\frac{\pi}{l}}U_0 Y(l/w), \quad (2)$$

where E is the Young's modulus, ν is the Poisson ratio, U_0 is the amplitude of oscillations, Y is the polynomial factor, and w is the sample width. For a given sample geometry (Fig. 1) the polynomial factor was as follows:

$$Y(l/w) = 0.635(l/w) + 1.731(l/w)^2 - 3.979(l/w)^3 + 1.963(l/w)^4. \quad (3)$$

Figure 2 shows the typical surface relief observed on a sample, which was obtained using a high-resolution NewView interferometer. This profilometer ensured a vertical resolution of 0.1 nm and a lateral resolution of 0.5 μm. These patterns were analyzed using the methods of correlation analysis for determining the conditions of scaling invariance in the ensembles of structures, which were assumed to govern the subsequent stage of crack development.

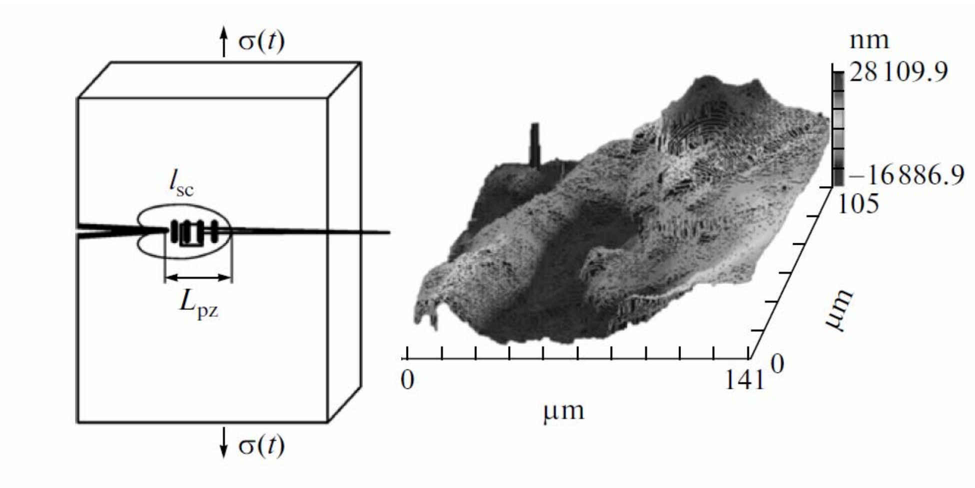

Figure 2. Schematic diagram showing a region at the crack tip and typical image of the surface relief

The structure scaling parameter of the measured surface profiles was determined based on the average difference $K(r)$ in relief heights z(x) on the fracture surface, which was defined and represented as follows:

$$K(r) = \left\langle \left(z(x+r) - z(x)\right)^2 \right\rangle_x^{1/2} \propto r^H, \quad (4)$$

where H is the index of surface roughness (Hurst exponent). By representing these data in logarithmic coordinates according to Eqs. (4), it is possible allows to evaluate the roughness index as a spatial invariant, which corresponds to a constant slope of the plot of ln$K(r)$ versus ln(r) for a definite scale on which it is observed. The linearity of ln$K(r)$ versus ln(r) plots allowed the roughness parameter to be evaluated as invariant in the given interval of scales r. The resolving power of the interferometer employed ensures determination of the upper and lower boundaries of scaling between K and r.

The minimal value of the scales r was assumed to determine the critical scale l_{sc} for establishing long-range correlation interactions in the ensembles of defects formed in a process zone scale L_{pz} (Fig. 2). The values of the critical scale l_{sc} for various stages of crack propagation are presented in the Table 1.

Table 1. Values of parameters at various stages of fatigue crack development

ΔK, $MPa\sqrt{m}$	Δl, m	ΔN ,cycle	dl/dN, m/cycle	l_{sc}, μm	H
6,200	$6{,}80 \cdot 10^{-4}$	$1{,}01 \cdot 10^{+6}$	$6{,}73 \cdot 10^{-10}$	2,6±0,2	0,57±0,02
5,890	$2{,}00 \cdot 10^{-4}$	$2{,}82 \cdot 10^{+5}$	$7{,}09 \cdot 10^{-10}$	2,5±0,1	0,56±0,03
5,596	$3{,}20 \cdot 10^{-4}$	$4{,}74 \cdot 10^{+5}$	$6{,}75 \cdot 10^{-10}$	3,4±0,1	0,54±0,03
5,316	$2{,}20 \cdot 10^{-4}$	$4{,}22 \cdot 10^{+5}$	$5{,}21 \cdot 10^{-10}$	1,7±0,3	0,52±0,02
5,050	$2{,}60 \cdot 10^{-4}$	$7{,}37 \cdot 10^{+5}$	$3{,}53 \cdot 10^{-10}$	3,7±0,2	0,49±0,03
4,797	$1{,}60 \cdot 10^{-4}$	$2{,}75 \cdot 10^{+5}$	$5{,}82 \cdot 10^{-10}$	2,2±0,2	0,47±0,05
4,558	$3{,}20 \cdot 10^{-4}$	$7{,}13 \cdot 10^{+5}$	$4{,}48 \cdot 10^{-10}$	4,1±0,3	0,59±0,03
4,330	$4{,}93 \cdot 10^{-4}$	$2{,}03 \cdot 10^{+6}$	$2{,}42 \cdot 10^{-10}$	3,6±0,2	0,49±0,03

Manifestations of the self-similar nature of the fatigue crack growth were studied using methods of the theory of similarity and dimensionality [6, 7]. The crack growth rate was defined as $a = dl/dN$ (where l is the crack length and N is the number of cycles) and studied as for correlation with the following parameters: $a_1 = \Delta K$, stress intensity coefficient; $a_2 = E$, Young's modulus; $a_3 = l_{sc}$, correlation scale in the ensemble of defects; $a_4 = L_{pz}$, the scale related to the process zone.

3D New View high resolution data of roughness in the crack process zone (Fig.1) revealed the existence of two characteristic scales: the scale of process zone L_{pz} and correlation length l_{sc} that is the scale when correlated behavior of defect induced roughness has started [1].

Using the Π-theorem and taking into account the dimensionality of variables $[dl/dN] = L$, $[\Delta K]$ $=FL^{-3/2}$, $[l_{sc}] = [L_{pz}] = L$, and $[E] = FL^{-2}$, the kinetic equation for the crack growth:

$$dl/dN = \Phi(\Delta K, E, l_{sc}, L_{pz}), \tag{5}$$

can be rewritten as follows:

$$\frac{dl}{dN}\frac{1}{l_{sc}} = \overline{\Phi}\left(\frac{\Delta K}{E\sqrt{l_{sc}}}, \frac{L_{pz}}{l_{sc}}\right). \tag{6}$$

Estimation of the values $\Delta K/(E\sqrt{l_{sc}}) << 1$ and $L_{pz}/l_{sc} >> 1$ suggest an intermediate-asymptotic character of the crack growth kinetics for Eq. (6) in the following form:

$$\frac{d\bar{l}}{dN} = \left(\frac{\Delta K}{E\sqrt{l_{sc}}}\right)^{\alpha}\left(\frac{L_{pz}}{l_{sc}}\right)^{\beta}, \tag{7}$$

where $\bar{l} = l / l_{sc}$. Introducing the parameter $C = \left(L_{pz} / l_{sc}\right)^{\beta}$, we can reduce the scaling relation (7) to the following form analogous to the Paris law:

$$\frac{d\bar{l}}{dN} = \mathrm{C}\left(\frac{\Delta K}{E\sqrt{l_{sc}}}\right)^{\alpha}, \tag{8}$$

where α is a universal exponent. This form is similar to the equation proposed by Hertzberg for $l_{sc} \to b$, where b is the Burgers vector. In the limit of small scales $l_{sc} \approx b$ the application of stress intensity factor conception is problematic and corresponding scaling laws can be introduced [2].

Using relation (8), which constructed based on the results experimental investigation of the fatigue crack growth kinetics with allowance for the calculated l_{sc} values, it is possible to estimate the exponent as α ~2.3, which corresponds to the slope of the straight line in the rectifying coordinates (Fig. 3).

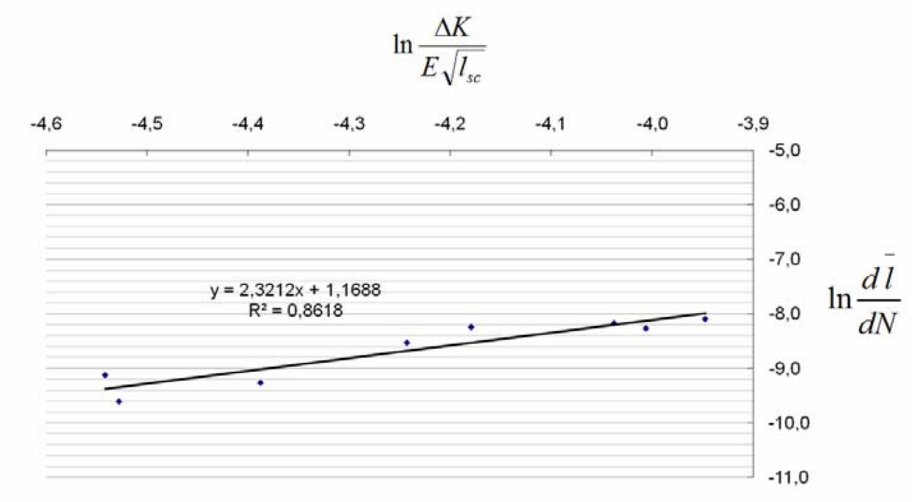

Figure 3. Rectified plot of relation (8)

A difference of the exponent α ~ 2.3 from values obtained in the regimes of multicycle fatigue testing suggests that there are certain specific features in the formation of fracture regions in the vicinity of crack tip under conditions of gigacycle loading.

3. Summary

The constancy of the scaling index (Hurst exponent) in a broad interval of spatial scales, which includes the scales of evolution of the typical defect substructures, leads to a conclusion that the kinetics of crack propagation can be considered within the framework of a broad class of critical phenomena, namely, structure–scaling transitions [3, 4] that describe the evolution of defects on various scaling levels. Determination of the scaling index of deformation induced defect structures can provide a physical explanation of the universality of this class of physical phenomena with respect to the scenarios of fracture in materials of various classes and the influence of structural states (including those formed by accidental dynamic impacts) on the “threshold” characteristics of the transition of a loaded material from plastic deformation to fracture.

Acknowledgments

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References

[1] B.B. Mandelbrot, The fractal geometry of nature, W.H. Freeman, New York, 1983.

[2] P. Paris, D. Lados, H. Tad, Reflections on identifying the real $\Delta K_{effective}$ in the threshold region and beyond. Engineering Fracture Mechanics, 75 (2008) 299–305.

[3] J.L. Lataillade, O.B. Naimark, Mesoscopic and nonlinear aspects of dynamic and fatigue failure (experimental and theoretical results). Physical Mesomechanics, 7 (2004) 55–66.

[4] O.B. Naimark, Advances in Multifield Theories of Continua with Substructure, in: G. Capriz, P. Mariano (Eds.), Birkhauser, Boston, 2003, pp. 75–114.

[5] C. Bathias, Piezoelectric fatigue testing machines and devices. Int.J. of Fatigue, 28 (2006) 1438–1445.

[6] G.I. Barenblatt, Scaling phenomena in fatigue and fracture. Int.J. of Fracture, 138 (2006) 19–35.

[7] R.O. Ritchie, Incomplete self-similarity and fatigue-crack growth. Int.J. of Fracture, 132 (2005) 197–203.