WAVE RESISTANCE IN CRACK PROPAGATION: LATTICES AND HOMOGENEOUS STRUCTURED MEDIA

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ABSTRACT

Wave resistance to dynamic crack propagation arises because of excitation of structure-associated waves which carry energy away from the crack tip. Two types of strucure are important: the initial, spatial structure and the surface structure (as damaged crack faces with their roughness, microcracking etc.) induced by the waves in the vicinity of the crack tip. In this context, discrete lattices and homogeneous media with a structure – media with internal degrees of freedom – are considered. In the latter case, the spatial structure is introduced by the strain-rate-dependent moduli and density. The surface structure is described by a specific dynamic condition at the crack faces. Energy release associated with the surface structure is determined. A general, structure- and crack-speed-dependent expression of the wave resistance is presented. The conditions of the existence of energy release in a 'refined' model of the medium are shown and a procedure for the proper homogenization of a structured material (to be adequate for fracture dynamics) is found.

KEYWORDS

Crack dynamics, wave resistance, lattices, strain-rate-dependent media, surface structure, homogenization, size effect.

INTRODUCTION

It is well-known that dynamic cracks do not obey the Griffith-Irwin energy-release-rate criterion. When the energy release rate increases the crack increases the energy consumption rather than its speed. Even in a purely elastic material, there is a mechanism of additional dissipation, apart from that spent on the surface energy: at the moving crack tip (as at a point of a dramatic jump in the stress field), there is a strong, crack-speed-dependent excitation of structure associated waves. These waves can carry away a lot of energy released on the macro-level, thus creating wave resistance to the crack motion. For steady-state crack propagation, the wave resistance can be described only in terms of a structured medium (a medium with internal degrees of freedom) because there are no such wave modes in the model of a non-structured material.

Two types of the material structure have importance: the initial material structure of the body (the *spatial structure*) and the *surface structure*, i.e. the damaged crack faces with their roughness, microcracking, etc., generated by the structure-associated waves. In a sense, the importance of both kinds of structure is similar to that of spatial and surface energies of the elastic body.

In view of these facts, one can see that a part of the total energy release (in addition to the energy spent on the surface energy itself) is spent on formation of the surface structure, another part is radiated by the spatial-structure-associated waves, and the remaining energy is carried away from the crack tip by the surface structure waves. It is essential that these energy fluxes be interdependent since the surface structure is created under the combined effect of all the waves. This makes it clear that in an adequate theory of crack dynamics, the initial, spatial structure, the induced surface structure and the process of its formation, the waves excitation and propagation, all have to be taken into account.

Discrete Lattices

Discrete lattice models were repeatedly used for determination of dynamic phenomena caused by a material structure. Note here the books by Kunin (1982, 1983) and Askar (1985) devoted to this subject.

A number of works exists, devoted to the analysis of dynamics of dislocations and cracks in discrete lattices as simple models of structured media: Atkinson and Cabrera (1965), Celli and Flytzanis (1970), Thomson *et al.* (1971), Weiner and Pear (1975), Ashurst and Hooper (1976), Maslov (1976, 1980), Sieradziki *et al.* (1988), Machova (1992).

Exact analytical solutions to such problems (2D-problems in fracture dynamics in lattices and 1D-problems in phase transformation dynamics in a chain of the simplest structure) were published in the 1980s. Investigations in this field were carried out to derive a relation between the structure and the speed of the crack or a phase separation interface, on the one hand, and the dissipation, on the other hand. In addition, these investigations made it possible to describe a number of phenomena which could not be discovered in the classical non-structured elastic medium.

Mode III of dynamic crack propagation in a square lattice was considered by Slepyan (1981a, 1981b, 1982a) for the sub-critical and super-critical crack speeds and for a general case, respectively. Modes I and II for a triangular lattice were studied by Kulakhmetova *et al.* (1984). The ratios, $r(v)$, of the "local" energy release rate, G_0 (the energy which is spent on the fracture itself – in accordance with the solution for the lattice), to the "global" energy release rate, G, on the macro-level – defined by the long wave approximation of this solution – were found for these three modes as functions of the ratios of the crack velocity, v, to the corresponding critical velocity (Slepyan, 1981a, 1993a; Kulakhmetova *et al.*, 1984). Similar relations were obtained by Marder and Gross (1995) for the lattice strips (also the lattice strip was considered by Slepyan, 1986). The same problems for anisotropic lattices (lattices that correspond to anisotropic elastic media) were solved by Kulakhmetova (1985a, 1985b).

Some general conclusions concerning the wave resistance to a propagating singular point are presented by Slepyan (1982b). Mikhailov and Slepyan (1986) used the same technique for the investigation of crack propagation in a model of a composite material; Slepyan and Kulakhmetova (1986) made use of this approach for a model of rock joints. Finally, the papers by Slepyan and Troyankina (1984, 1988) were devoted to fracture waves in the piece-wise-linear and piece-wise-linear-shifted chain structures (in the latter, the right stable branch of the stress-strain diagram is shifted with respect to the origin of the coordinates). These chain structures were the models for the phase transition dynamics in structured media, and all major phenomena that accompany the crack propagation in a lattice are presented in these more simple models which are susceptible to more detailed analysis. Many of the above mentioned results can be found in the book (Slepyan, 1990) and papers (Slepyan, 1993a, 1996).

Under the condition of crack velocity oscillations, the energy radiation can accompany the crack propagation in a non-structured homogeneous material too (Rice, 1978; Slepyan, 1978). In particular, such oscillations can be caused by a nonuniform, wavy-shaped distributed toughness (this phenomenon was considered by Das and Aki, 1977; Freund, 1987; Gilles and Rice, 1994).

The influence of the structure-associated waves depends on the crack speed; an extremal, minimal-wave-dissipation speed exists which required the minimal total (macro-level) energy release rate: almost all the released energy transforms into the surface energy. This extremum is clearly visible on the plots of dissipation versus crack speed (Slepyan, 1981a, 1993a; Kulakhmetova *et al.*, 1984). When the crack speed is less than the extremal one, the fracture does not coincide with the maximal stress intensity but, at least, takes place after the maximum. This is possible in the case of a delay in fracture. If this is not the case, the extremal speed must coincide with the lower limit of the crack speed. In accordance with this, the lower speed limits for cracks and failure waves in discrete lattices were found based on the analysis of the exact solutions (Slepyan, 1981b; Slepyan and Troyankina, 1984; Marder and Gross, 1995).

When the crack speed exceeds the extremal value, the fracture takes place before the maximum of the stress intensity. In this case however, the wave radiation from the propagating crack tip increases with the crack velocity. Indeed, the fracture can consume only a part of the energy release, and the ratio of the surface energy to the total energy release rate must tend to zero when the crack speed approaches the critical one (Slepyan, 1981a, 1993a; Kulakhmetova *et al.*, 1984). Therefore, under a constant surface energy, the wave resistance increases unboundedly when the crack speed approaches the critical wave speed. This high-amplitude wave motion leads to damage of the material in the vicinity of the crack tip. This phenomenon, which defines the upper crack speed limit, was noted in the book by Slepyan (1990) and investigated by Marder and Gross (1995).

Taking into account the fact that, in dynamics, the stress intensity for some inclined directions approaches that for the crack line continuation, one can see that the upper limit cannot be too far from the lower one. Note that in the case of "weak bonds", when the crack continuation line is much weaker than the surrounding material, crack speed can achieve the critical one (Lee and Knauss, 1989), and this does not contradict the above simple considerations.

The surface structure created during the crack propagation (see Ravi-Chandar and Knauss, 1984; Fineberg *et al.*, 1991, 1992; Marder, 1995) with roughness of the crack surfaces, micro-cracking and micro-branching are common for crack dynamics, and this increases energy dissipation dramatically. As a result, in dynamics, cracks do not obey any limited-energy-release-rate criterion, and under ordinary conditions crack propagation appears to correspond to a maximum dissipation rate per unit time (Slepyan, 1992, 1993b).

A micro-nonuniform crack propagation was studied theoretically, in particular, by Rice *et al.* (1994). Movchan *et al.* (1996) represent asymptotic analysis of out-of-plane perturbations of 2D and 3D cracks. Gilles and Rice (1994) studied oscillations in crack dynamics in a medium with nonuniform distribution of the surface energy. In a sense, this corresponds to a structured material. A specific case of the crack speed oscillations in a lattice was considered by Slepyan (1986). Irregular brittle fracture was considered by Frantziskonis (1994).

A number of works were devoted to stability of crack propagation in discrete lattice (Fineberg *et al.*, 1991, 1992; Marder, 1991; Marder and Xiangmin Liu, 1993; Marder and Gross, 1995; Langer, 1996; Ching *et al.*, 1996). Xu and Needleman (1994) considered crack dynamics in a homogeneous body but with a lattice of possible crack trajectories.

Lattices and Homogeneous Structured Media

A simple lattice model of an elastic or viscoelastic material gives an insight into what occurs during crack propagation. However, such a model has at least one disadvantage: it is strongly inhomogeneous and anisotropic on the 'microlevel' (on the level of the structure). This presents problems for a quantitative estimation of the role of the radiation in the formation of the surface structure. For instance, conditions of the microbranching were found to depend on the shape of the lattice of possible crack trajectories (Xu and Needleman, 1994). Therefore, the results of the crack dynamics analysis in a discrete lattice are lattice-structure-dependent, and seem to differ from those for a homogeneous material. At the same time, the crack dynamics phenomena considered are more likely to be associated with a macroscopic scale rather than the atomic one, and hence – with a continuous material, where the 'structure-associated waves' can be defined not by a discrete structure of the material but by its dynamic features. In this connection, it is of interest to consider homogeneous materials with an internal structure. Some principles of the mathematical description of such media (to be adequate for fracture dynamics) are presented below.

A SIMPLE EXAMPLE OF WAVE TRANSFORMATION

The simplest example of wave transformation in a moving interface was shown by Slepyan (1984). The collision of an inclined thread with a rigid foundation was considered. If no bending stiffness exists, all the kinetic energy of the thread dissipates into the moving point at the end of the contact region. However, in the case of positive bending stiffness, the energy does not dissipate into this point but radiates away along the moving part of the thread in the form of a bending wave. Note that the phase velocity of the bending wave which is equal to the impact point velocity, v, is half the group velocity, and this allows for the energy to outflow. The solution to the problem (in term of the distance, u, between the thread and the foundation) is as follows:

$$u = \frac{c}{v}\left[\eta - \frac{\sqrt{EI/\rho A}}{v}\sin(\sqrt{\rho A/EI}v\eta)\right]H(\eta), \quad \eta = x - vt, \tag{1}$$

where c, EI and ρA are the particle velocity of the impact, the bending stiffness and the mass per unit length, correspondingly, and $H(\eta)$ is the unit step function. Evidently, the sinusoidal wave disappears when the modulus, E, tends to zero (its amplitude tends to zero together with the modulus). At the same time, the energy flux borne by the wave remains constant independently of this modulus (the energy flux in the bending wave, propagating outward through the moving portion of the thread, is $[EI(u'')^2+\rho A(vu')^2]v/2 = \rho Ac^2v/2$). Note that $\rho Ac^2v/2$ is the same as the loss in initial kinetic energy of the thread per unit time.

Thus, introducing an arbitrary small bending stiffness (here the bending stiffness represents a structure) one dramatically changes the type of energy dissipation: the released kinetic energy of the thread becomes 'surface' energy if no bending stiffness exists, and no energy is consumed by the surface in the case of a positive bending stiffness. Such a phenomenon is typical for fracture as well, where a crack-speed-dependent portion of the released energy is carried away from the crack tip by the structure-associated waves. This shows a weakness in a limited-energy-release-rate criterion of fracture in which the influence of the real structure of the medium has not been taken into account.

This example also reveals a possibility of transformation of high-frequency waves into a uniformly distributed motion. Indeed, in the case $E > 0, v < 0$, the bending wave propagates to the left, and it delivers the energy which transforms into the kinetic energy of the uniform motion of the thread with velocity $c < 0$ up from the foundation (also, the inverse process corresponds to the reverse course of time). This is nothing else but the energy transition from the 'micro-level' (the level of structure) to the macro-level. The same phenomenon can be observed in crack propagation under high-frequency oscillations of a lattice (see Slepyan, 1981b).

In this example, dissipation into the moving singular point exists as far as there is no 'higher order' derivative in the dynamic equation (such a derivative is introduced by taking the bending stiffness into account). This result is very general and imposes a restriction in the formulation of adequate models of continuous structured media with a dynamic crack. The conditions of a proper formulation of a structured medium, in which an energy release is preserved, are shown below.

WAVE RESISTANCE IN CRACK PROPAGATION

In this section, a general expression for the wave resistance to the crack propagation (or moving of another singular point) is derived as a structure-dependent function of the crack speed (Slepyan, 1982b, 1990). Suppose that a component of the displacement distribution, $u_-(\eta)$, for $\eta = x - vt < 0$ (i.e. on the crack propagating with velocity v) and the corresponding component of the traction distribution, σ_+, on the crack continuation, $\eta > 0$, are connected by the relation

$$S_u(0+ikv,k)u_-^F(0+ikv,k) + S_\sigma(0+ikv,k)\sigma_+^F(0+ikv) = Q^F(0+ikv,k). \tag{2}$$

Here Q is an external force, the upper index, F, means the Fourier transform (with the exponent $ik\eta$) of the corresponding function of η, and $0+ikv = \lim \epsilon+ikv$ ($\epsilon \to +0$). This limit corresponds to the causality principle when the steady-state solution is considered as a limit ($t \to \infty$) of the corresponding transient problem solution with zero initial conditions. For example,

$$u_-^F(0+ikv,k) = u_-^{LF}(s,k) \quad (s = \epsilon + ikv, \ \epsilon \to +0) \tag{3}$$

where the right hand part of this relation represents the combined Laplace and Fourier transforms (with parameters s and k, respectively), and the left hand part is the limit ($t \to \infty$) of the Fourier transform in the moving coordinate system (with the origin at $x = vt$). This concerns all the functions in (3).

It is assumed that the factorization exists as the representation

$$S(0 + ikv, k) = S_\sigma/S_u = S_+(0 + ikv, k)S_-(0 + ikv, k), \tag{4}$$

where S_+ and S_- have no singular points and zeros in the upper (Im $k \geq 0$) and lower (Im $k \leq 0$) half-planes, respectively. Suppose that point $k = k_n$ is a zero point of order ω for the product S_-S_u. In this case, Eq. (2) can be represented in the form

$$\frac{u_-^F}{S_-} + S_+\sigma_+ = f = \sum_{m=0}^{p} a_m\{[0 - i(k - k_m)]^{-1-m} + [0 + i(k - k_m)]^{-1-m}\}, \tag{5}$$

where $p < \omega \leq p + 1$. This expression corresponds to the homogeneous problem in the sense that $Q^F = fS_-S_u = 0$. The solution to this equation is

$$u_-^F = S_-\{\sum_n \sum_{m=0}^{p} (-1)^m a_{mn}[0 + i(k - k_n)]^{-1-m} + \sum_{m=0}^{q} b_m k^m\}, \tag{6}$$

$$\sigma_- = S_+^{-1}\{\sum_n \sum_{m=0}^{p} (-1)^m a_{mn}[0 - i(k - k_n)]^{-1-m} - \sum_{m=0}^{q} b_m k^m\}, \tag{7}$$

where $q = 1, 2, \ldots$, and terms of the sums must provide the solution with a finite energy in a finite area. Assume that the function $S(0 + ikv, k)$ has the asymptotes

$$S = A(0 + ik)^\alpha(0 - ik)^\beta + O(k^{\alpha+\beta+\gamma}) \quad (k \to 0),$$
$$S = Be^{\pm i\pi d}|k|^\nu + O(k^{\nu-\gamma}) \quad (k \to \pm\infty) \tag{8}$$
$$A, B, \gamma = \text{const} > 0, \quad d = \text{Ind } S \equiv [\text{Arg } S(0 + i\infty, \infty) - \text{Arg } S(0 - i\infty, -\infty)]/(2\pi).$$

Let us introduce a function S_* according to the relation

$$S = BS_*(0 + ikv, k)(0 + ik)^\alpha(0 - ik)^\beta N_+N_- \tag{9}$$
$$N_+ = \left[\left(\frac{A}{B}\right)^{1/(2n_+)} + (0 - ik)^2\right]^{n_+}, N_- = \left[\left(\frac{A}{B}\right)^{1/(2n_-)} + (0 + ik)^2\right]^{n_-},$$
$$n_+ = -\frac{1}{2}\left(d + \beta - \frac{\nu}{2}\right), \quad n_- = \frac{1}{2}\left(d - \alpha + \frac{\nu}{2}\right).$$

Under this definition,

$$S_* = 1 + O(k^\gamma) \ (k \to 0), \ S_* = 1 + O(k^{-\gamma}) \ (k \to \pm\infty), \ \text{Ind } S_* = 0. \tag{10}$$

Now the function S can be expressed as follows

$$S = S_+S_-, \ S_+ = S_{*+}\sqrt{B}(0 - ik)^\beta N_+, \ S_- = S_{*-}\sqrt{B}(0 + ik)^\alpha N_- \tag{11}$$
$$S_{*\pm} = \exp\left(\pm\frac{1}{2\pi i}\int_0^\infty \frac{\ln S_*(0 + i\xi v, \xi)}{\xi - k}\,d\xi\right),$$

where Im$k > 0$ (Im$k < 0$) for S_+ (S_-), respectively.

Taking into account the fact that $\ln|S_*|$ is even and ArgS_* is an odd function of k, and that Arg$S_*(0) = 0$, one can see that

$$S_{*\pm} \to 1 \ (k \to \pm i\infty), \ S_{*\pm} \to \lambda^{\pm 1} \ (k \to 0),$$
$$\lambda = \exp\left(\frac{1}{\pi}\int_0^\infty \text{Arg } S_*(0 + ikv, k)\frac{dk}{k}\right);$$
$$S_+ \sim \sqrt{B}(-ik)^{(\nu/2)-d} \ (k \to i\infty), \quad S_+ \sim \sqrt{A}(0 - ik)^\beta\lambda \ (k \to 0),$$
$$S_- \sim \sqrt{B}(ik)^{(\nu/2)+d} \ (k \to -i\infty), \quad S_- \sim \sqrt{A}(0 + ik)^\alpha\lambda^{-1} \ (k \to 0). \tag{12}$$

Using the energy-release-rate relation in the form (Slepyan, 1984)

$$G = \lim_{s\to\infty} s^2\sigma_+^F(is)u_-^F(-is) \tag{13}$$

and expressions (6) and (7), one now can see that a finite, nonzero energy release rate exists only in the case of an integer index, $d < \omega$. In this case,

$$G = G_0 = (-1)^d(\sum_n a_{mn})^2 \ (d \geq 0), \quad G_0 = -b_{-d-1}^2 \ (d \leq -1). \tag{14}$$

At the same time, asymptotes (12), $k \to 0$, correspond to the long-wave (low-frequency) approximation which describes the corresponding non-structured medium and which can be called the *macro-level.* On the macro-level

$$G = (-1)^D\lambda^{-2}a_{D0}^2 \ (D \geq 0), \quad G = -\lambda^{-2}b_{-D-1}^2 \ (D \leq -1), \tag{15}$$

where D is the macro-level index, $D = (\alpha - \beta)/2 < \omega$.

Thus we reach the following conclusions. An energy release rate exists only if the index is integer; it is positive for a positive even index and it is negative for positive odd or negative index. Finite, nonzero energy release rates on both levels, G_0 and G, exist simultaneously if $d = D$ because the macro-level asymptote corresponds to the same term in the right hand part of Eq. (6).

For the 'classical' fracture when the energy flux from infinity exists in the form of a nonoscillating wave, $n = 0$, and if $d = D < \omega$ the ratio of the energy fluxes is

$$r(v) = \frac{G_0}{G} = \lambda^2 = \exp\left(\frac{2}{\pi}\int_0^\infty \text{Arg } S_*(0 + ikv, k)\frac{dk}{k}\right). \tag{16}$$

The wave resistance, $R(v)$, is the difference between the total energy release rate, G, and the energy release on the micro-level (as the level of the structure), G_0:

$$R = G - G_0 = [1 - r(v)]G. \tag{17}$$

Note that for the determination of the wave resistance, there is no need in solving a specific crack dynamic problem but only the function $S(k)$, which depends on the structure of the medium and the crack speed, is important.

Consider several examples in terms of the above-mentioned results. In a homogeneous elastic medium without a structure (see Freund, 1990; Slepyan, 1990), for a subcritical crack speed, $\alpha = \beta = -1/2, D = 0$, and a positive dissipation is possible. In the plane problem with $c_R < v < c_2, \alpha = -3/2, \beta = 1/2, D = -1$, and only negative dissipation is possible. In the range $c_2 < v$, the index is not integer, and the energy release cannot exist (c_R and c_2 are the Rayleigh wave and shear wave speeds, respectively). In the collision of a thread with a rigid foundation, considered above [see (1)], $D = 1$ $(EI = 0)$, and there is an energy release, but for the 'structured' thread $((EI > 0), d = 2 \neq D$, and $G_0 = 0$.

The derived results are essential for the problem of a proper homogenization of a structured material to be adequate for fracture dynamics. Since $D = 0$ for a subcritical crack speed, the index, d, must be equal to zero for a structured, 'refined' model of the medium as well. Note that for a discrete lattice, $d = 0$ (Slepyan, 1982b, 1990).

HOMOGENEOUS MEDIA WITH A STRUCTURE

To avoid difficulties in the formulation of the boundary conditions for a structured continuous medium, consider media with a structure introduced by strain-rate-dependent moduli or/and effective density. Such a medium can be conservative or nonconservative. In either case, the relation between the total energy release rate, G (defined for the corresponding non-structured elastic medium), and the fracture energy itself, G_0 (defined for the considered continuous medium with a structure) is expressed by the same formula (16), and the only condition $d = D = 0$ must be satisfied.

To find a proper way of introducing a structure into the structureless model of a medium (or a change the structure), we appeal to the relations (8) and (9) and consider the problem of the index-invariant transformation of the function S. Assume that the medium considered is stable in the sense that the function S has no zero or singular points in the right half-plane of s, that is for $\epsilon > 0$ [see the definition (3)]. In this case, Ind S is independent of ϵ $(\epsilon > 0)$. Suppose that this function depends on parameter $\phi : S = S(\epsilon + ikv, k, \phi)$ and has an invariable index if $\phi \in \Omega$, and $\phi = \phi_0$ is an internal point of Ω.

To change the model of the medium, substitute parameter ϕ by a function $\mathcal{R}(s)$, Re $s = \epsilon > 0$, Im $s = kv$. We require (a) that this transformation preserve stability of the medium and (b) that $\mathcal{R}(s)$ tend uniformly to ϕ_0 when $\epsilon \to \infty$. In this case, the indices are independent of ϵ (see Gakhov, 1966), Ind $\mathcal{R} = 0$ and Ind S remains invariable as required. Consider two examples.

A Viscoelastic Material

Consider first the standard solid model of a viscoelastic material (see Christensen, 1982). In such a model, the bulk, K, and shear, μ, elastic moduli are replaced by the operators

$$K = K_0 \frac{1 + \tau_1 s}{1 + \tau_2 s}, \qquad \mu = \mu_0 \frac{1 + \tau_3 s}{1 + \tau_4 s}, \tag{18}$$

where $K_0, \mu_0 =$ const (they represent the low-rate moduli), s is the time-derivative symbol, $0 < \tau_2 < \tau_1$ and $0 < \tau_4 < \tau_3$ ($K_0\tau_1/\tau_2$ and $\mu_0\tau_3/\tau_4$ are the high-rate moduli).

This representation satisfies the above-mentioned conditions. Therefore, both the energy criterion of fracture can be satisfied and the wave resistance can be obtained based on this model. For steady-state crack propagation, the elastic moduli, which appear in the function S (for the corresponding elastic body), can be substituted by the expressions (18) in which $s = 0 + ikv$. In this way, for a loading applied far from the crack tip, the total energy release corresponds to the elastic body, and the wave resistance corresponds to energy dissipated in the volume of the body.

Note that a size effect is introduced by the 'relaxation' parameters $\tau_1, ..., \tau_4$. In particular, this effect can manifest itself in a viscoelastic strip, as an influence of the ratio of the width of the strip to the characteristic length unit of the material. Note also that such a unit, as a product of a relaxation parameter and the wave speed, can be very large.

A Conservative, Homogeneous Dynamic System

Consider an elastic material which interacts with a medium of uniformly distributed 'spring-mass' elements. The dynamic equations are

$$\frac{\partial \sigma_{mn}}{\partial x_m} - \rho_0 \frac{\partial^2 u_n}{\partial t^2} = \gamma(u_n - w_n), \qquad \rho_1 \frac{\partial^2 w}{\partial t^2} = \gamma(u_n - w_n), \tag{19}$$

where u_n, w_n and ρ_0, ρ_1 are displacements of the elastic and spring-mass media, respectively, and γ is the spring rigidity. Note that a wave in such a one-dimensional system with a distribution of the spring-mass medium within a range of frequences was considered by Slepyan (1967). Using the Laplace transform, one obtains

$$\frac{\partial \sigma^L_{mn}}{\partial x_m} - \rho s^2 u^L = 0, \qquad \rho = \frac{(\rho_0 + \rho_1)\gamma + \rho_0 \rho_1 s^2}{\gamma + \rho_1 s^2}. \tag{20}$$

Here ρ is the density operator. For a low-rate process $(s \to 0)$, $\rho \sim \rho_0 + \rho_1$, and for a high-rate process $(s \to \infty)$, $\rho = \rho_0$. This representation also satisfies the conditions required.

SURFACE STRUCTURE

In the framework of a homogeneous material, surface structure, as an additional waveguide, can be introduced by a specific dynamic condition at the crack faces. In the relation

$$u^{LF}_- = -S(s, k)(\sigma^{LF}_+ + \sigma^{LF}_-), \tag{21}$$

assume that $\sigma^{LF}_- = g(s, k) u^{LF}_-$. One now has

$$[1 + g(s, k)S(s, k)]u^{LF}_- + S(s, k)\sigma^{LF}_+ = 0. \tag{22}$$

For the steady-state crack propagation, this equation coincides with (2). In addition to the conditions mentioned above, we now require that the product $g(s, k)S(s, k) \to 0$, when $s, k \to 0$. This provides the relation (22) to be surface-structure-free for a low-rate process. For example, all the conditions are satisfied by the function $g = qs^2/(\tau^{-2} + s^2)$, where the surface structure intensity $q > 0$ and the relaxation time τ are constants. One can find that energy release rate associated with the surface structure is

$$G_s = \int_{x/v}^{\infty} \sigma_- \frac{\partial u_-}{\partial t}\, dt = \frac{q}{2\tau^2 v^2} |u^F_-(k)|^2, \quad k = \frac{1}{\tau v}. \tag{23}$$

It would be reasonable to define the surface structure intensity, q, to correlate with energy flux density in the wave field at the crack faces.

An investigation of a micro-nonuniform crack tip motion in such a medium with the spatial and surface structures is exceptionally interesting. However, we now restrict ourselves to the steady-state crack propagation. Some dependences of the wave resistance,

$R/G = 1 - r(v)$, versus the crack speed for the Mode I crack propagation are shown in Fig. 1. These results are obtained using the general relations (16) and (17), the expression of the function S for the non-structured elastic medium (Freund, 1990, Slepyan, 1990) and the introduced rate-dependent moduli (18), density (20) and surface structure (22). (Poisson's ratio, $\nu = 1/4$, and Rayleigh wave speed, $c = c_R = \sqrt{2 - 2/\sqrt{3}}c_2$, corresponds to the low-rate moduli and density.)

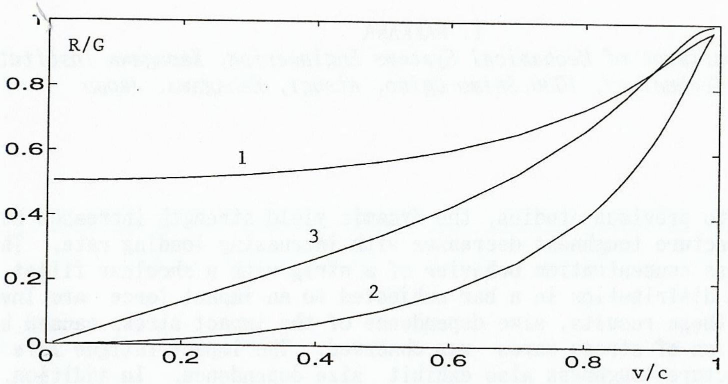

Fig. 1. Wave resistance.

1. Viscoelastic material (18); $\tau_1 = \tau_3 = 2, \tau_2 = \tau_4 = 1$. 2. Strain-rate-dependent material (20); $\rho_0 = 1, \rho_1 = 1, \gamma = 1$. 3. Material with the surface structure (22); $q = 1, \tau = 1$.

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