ISSUES IN DYNAMIC CRACK PROPAGATION

F. NILSSON
Department of Solid Mechanics, Royal Institute of Technology, S-10044 Stockholm, Sweden

ABSTRACT

The goals of dynamic crack theory are discussed. Some important results for elastodynamic crack problems are briefly reviewed as well as some aspects of the elastic description of dynamic crack growth. Extensions of the elastic theory in form of visco-plastic, damage and cohesive zone modelling are discussed. As an example of possible applications of dynamic crack growth theory to micro-mechanics a particular problem with reference to cleavage fracture initiation is stated.

KEYWORDS
Dynamic, crack growth, elastodynamic, visco-plastic, damage mechanics

INTRODUCTION

The problem of how to understand dynamic crack propagation has over the years provided a challenge to researchers and practising engineers. The scientific approach to the problem area can be said to have started with the contribution by Mott (1948) who recognized the importance of accounting for the kinetic energy when analysing dynamic crack propagation. Numerous contributions to the subject have since appeared and the most comprehensive account up to date is the book by Freund (1990).

The general goals of dynamic crack growth theory are fairly easy to state. In a given geometry with a given initial crack the aim is to predict the time history of the crack growth after an initiation and in particular if the growth is arrested. The applications are mainly to situations where a complete failure may have very serious consequences and relying upon safety of crack growth initiation is not enough. Thus, engineering interest in the crack propagation and arrest problems has mainly been driven by the safety needs of certain branches of industry for instance the nuclear power industry. In the recent years other areas of application have emerged. It has for instance been recognized that crack growth arrest plays a crucial role for the initiation of cleavage fracture in ferritic steels. This is discussed somewhat further in this article. Other micromechanical applications are likely to appear in the near future.

SOME RESULTS FROM LINEAR ELASTIC CRACK GROWTH DYNAMICS

A substantial body of knowledge has been developed for dynamic crack growth in linearly elastic isotropic bodies. Specializing to the mode I case, the stress field in the vicinity of a dynamically growing crack admits the following representation

$$\sigma_{ij} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi}} r^{-1/2} \Sigma_{ij}(\varphi, \dot{a}/C_1, \dot{a}/C_2) + T\delta_{1i}\delta_{1j} + O(r^{1/2}) \ , r \to 0 . \quad (1)$$

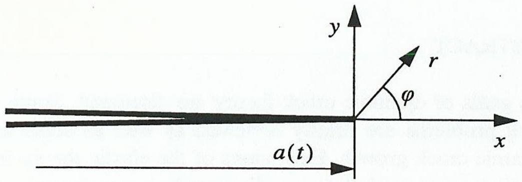

Fig. 1. Crack tip notation

Here r and φ denote polar coordinates (Fig.1) measured from the crack tip which is moving with the velocity $\dot{a}$, C_1 is the velocity of irrotational waves, C_2 the velocity of equivoluminal waves and δ_{ij} the Kronecker delta. Σ_{ij} are problem independent angle functions. The stress intensity factor K_{I} and the constant stress term T depend on the solution of the entire problem. Explicit expressions exist for the $r^{1/2}$-term (Freund, 1990; Freund and Rosakis, 1992; Nishioka, 1995) but these are too complicated to be shown here. The fully extended forms can be found explicitly in Nishioka (1995). The angle functions corresponding to this term depend both on the crack acceleration and the time derivative of the stress intensity factor in addition to the dependence of the crack velocity.

Dynamic crack problems are difficult to solve and there are few analytical results with general applicability. One is the relation between the energy transport to the crack tip and the stress intensity factor (*cf.* Freund, 1990). Since this unique relation exists there is no particular reason to consider crack growth in linear elastic materials in terms of the energy balance since the stress intensity factor approach is completely equivalent.

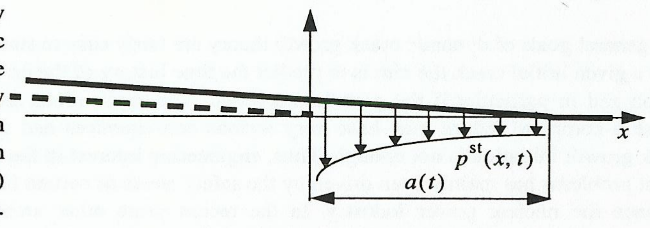

Fig. 2. A stopping and further propagating crack.

The most general result for the stress intensity factor for dynamic growth under plane mode I conditions is probably the following one, which can be deduced from works by Kostrov (1975) and Burridge (1976). Consider a crack growing in an arbitrary body under arbitrary symmetric dynamic loads. Imagine that the crack stops (see Fig. 2) at a point $x = 0$ along its path at the time $t = 0$ and that $p^{st}(x, t)$ is the normal stress that if this were to happen would act in the plane ahead of the tip. If instead of stopping the crack continues to propagate the stress intensity factor for the subsequent growth is for a limited time interval given by the expression

$$K_{\mathrm{I}}(t) = -f_{\mathrm{F}}(\dot{a}) \iint_D p^{st}(x, \tau)\, g(a(t) - x, t - \tau)\, dx d\tau , \quad (2)$$

where

$$D{:}\ \ 0 < \tau < t,\ \ a(t) - (t - \tau)C_1 < x < a(t) - (t - \tau)C_{\mathrm{R}} \quad (3)$$

$g(x, t)$ is a complex function given by Burridge (1976) and its particular form is not needed here. $f_{\mathrm{F}}(\dot{a})$ is a velocity dependent function first derived by Freund (1972a) and C_{R} denotes the velocity of Rayleigh surface waves. From eq. (2) a number of interesting results follow. Given the state at a certain time instant, say t_1, K_{I} will during a forward time interval not depend on the actual motion except for the tip position and speed. This time interval ends when a wave emitted from the tip at t_1 overtakes tip again after have being reflected at some boundary. Thus for instance the stress intensity factors before and after, respectively, a discontinous velocity jump from $\dot{a}^-$ to $\dot{a}^+$ are related as

$$K_{\mathrm{I}}^+ f_{\mathrm{F}}(\dot{a}^-) = K_{\mathrm{I}}^- f_{\mathrm{F}}(\dot{a}^+) . \quad (4)$$

This result can be used directly to determine a condition for arrest at an interface in a structure where the crack growth properties change discontinuously but the elastic properties remain continuous.

If the state at $t = 0$ is quasi-static, eq. (2) reduces to the celebrated result by Freund (1972b)

$$K_{\mathrm{I}} = f_{\mathrm{F}}(\dot{a}) \int_0^{a(t))} \sqrt{\frac{2}{\pi}} \frac{p^{st}(x)}{\sqrt{a(t) - x}} dx \quad (5)$$

It is to be noted that for a finite body the integral in (5) is not in general related to the quasi-static stress intensity factor.

In recent years significant steps have been taken towards an analytical understanding of three-dimensional elastodynamic crack growth problems (*cf.* Willis and Movchan, 1995; Geubelle and Rice, 1995). Such solutions for obvious reasons exhibit even greater complexity than the two-dimensional ones. Their value lies mainly in that they can form a basis for efficient numerical treatment. In fact even for two-dimensional problems the analytical solutions themselves are of limited value for practical situations and one must resort to numerical methods.

Other extensions of the simple isotropic linearly elastic theory have emerged during the last decades. The interest in polymeric composite materials which are inherently anisotropic has inspired research of dynamic crack growth in anisotropic elastic materials. The development is very similar to that of the isotropic theory although for obvious reasons considerably more complex. In contrast crack growth along an interface between two different elastic materials exhibits new features compared to the homogenous case. Most notable is the existence of a complex crack tip singularity which complicates the question of suitable crack growth criteria. Consideration of such cases is hovewer outside the scope of the present article.

CRACK PROPAGATION CHARACTERIZATION USING LINEAR ELASTIC ISOTROPIC THEORY

The most commonly used assumption for description of crack growth in isotropic linear elastic solids is that the crack velocity depends only on the stress intensity factor and thus that

$$K_{\mathrm{I}}(t) = K_{\mathrm{pc}}(\dot{a}) \text{, during growth,} \tag{6}$$

where the quantity in the right hand side (here termed the crack propagation toughness) depends only on the material. Deviations from this simple assumption have been observed in many experimental investigations (Dahlberg *et al.*, 1980, Ravi-Chandar and Knauss, 1982, among others) and there is in fact fairly little experimental support for the assumption implied by eq. (6). Different explanations for the deviations from this simple description have been advanced. Freund and Rosakis (1992) for instance attribute the loss of uniqueness to the influence of higher order terms in eq. (1). If the region where the material behaviour differs from that of an elastic material is larger than the singularity dominated zone, these terms might be expected to influence the crack growth processes. In addition the analyses by among others Freund and Rosakis (1992) and Nishioka (1995) show that crack acceleration and time derivatives of the stress intensity factor influence the form of the stress field for terms of order 1/2 and higher, so that such quantities may also affect the crack growth behaviour.

The other main group of attempts to explain deviations from eq. (6) is to analyse the crack growth with more elaborate (non-linear) constitutive models. Thus, for instance Brickstad (1983) applied elastic visco-plastic constitutive modelling of the Perzyna type to analyse experiments performed on thin plate specimens of a high strength steel. The adoption of this visco-plastic model has the advantage of resulting in a linearly elastic tip singularity and thus a non-zero uniquely defined energy flow to the tip. Brickstad was able to correlate the thus obtained energy flow to form a reasonable unique function of the crack velocity, while purely elastic analyses gave substantial deviations from eq. (6).

In a sense the two different approaches, multi-parameter elastic modelling or non-linear modelling, are but two different routes to achieve the same thing. In both cases it is the non-elastic deformation in the crack tip vicinity that requires an extension of eq. (6). When higher order terms of the elastic solution are used this implies that more detailed information about the field surrounding the non-elastic zone is utilized. Modelling of the non-elastic zone is avoided at the expense of keeping more parameters in the crack grow condition. In contrast it may be possible by more elaborate constitutive modelling to avoid including many parameters into the crack growth condition. It is however well known from quasi static fracture mechanics that elastic-plastic modelling is not necessarily sufficient to maintain a one parameter criterion but auxiliary parameters (for instance the so called Q-stress) may be needed. Thus it can be concluded that more complicated constitutive modelling should at least potentially reduce the number of parameters in the crack growth criterion. This is of great practical importance. The need to keep track of many parameters in a criterion complicates experimental work tremendously. In the present author's opinion this aspect is highly in favour of using more complex constitutive modelling and this is to be discussed in subsequent sections.

VISCO-PLASTIC MODELLING OF CRACK GROWTH

Parallelling the development in quasi-static fracture mechanics, the linearly elastic theory has been extended by incorporating plastic deformation. Since such deformation is clearly rate dependent in many materials it was early realized that so called visco-plastic theories were possible alternatives. Especially popular is the following constitutive theory due to Perzyna (1963).

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{(e)} + \lambda\left[H\left(\frac{\sigma_e}{\sigma_f(\varepsilon_e^{(p)})} - 1\right)\right]^m \frac{s_{ij}}{\sigma_e}. \tag{7}$$

Here, H denotes the Heaviside step function, $\dot{\varepsilon}_{ij}^{(e)}$ is the elastic strain rate and λ a fluidity material parameter. σ_e is the effective stress according to the von Mises yield condition, $\varepsilon_e^{(p)}$ the associated effective plastic strain, s_{ij} the stress deviator and m a material constant. The hardening behaviour is given by the dependence of the flow stress σ_f on the effective plastic strain. For very slow deformation eq. (7) reduces to conventional plastic theory. For the very (infinite) high strain rates in the vicinity of the crack tip the constitutive equation (7) has the property that if

$$m < \frac{3}{1 - 1/n}. \tag{8}$$

the crack tip singularity will be identical the elastic one in eq (1). The parameter n is here the exponent in a power law relationship for the quasi-static behaviour i.e.

$$\sigma_f \propto \varepsilon_e^{(p)1/n}. \tag{9}$$

The property that follows if eq. (8) is satisfied has the advantage that there is no ambiguity in the crack growth criterion. An equation of the same form as eq. (6) can be used where $K_{\mathrm{I}}(t)$ now is the strength parameter related to the inner elastic singularity. As mentioned previously Brickstad (1983) was successful in applying these concepts to experimental results. Later analytical and numerical studies (for instance Östlund, 1990, 1991) have however revealed that the spatial extent of the inner elastic singularity may be extremely small and in many cases it is doubtful if it is even comparable in size to the process region. Still, Brickstad's results were encouraging and it may well be that his visco-plastic analysis was not particularly accurate in the near vicinity of the crack tip. It may, however, have been reasonably accurate at some distance from the tip so that all important information about the state in rest of the body is contained in the near tip solution. Then it does not really matter which kind of singularity which arises as a consequence of the particular constitutive assumption provided that its structure is known and that its strength can be evaluated with sufficient accuracy.

There are fairly few experimental investigations, if any, that have been evaluated as systematically and successfully as Brickstad's. Perhaps the most ambitious experimental programme carried out to study dynamic crack growth is the wide plate test program (cf. Naus *et al.*, 1987). Here, crack propagation and arrest experiments were carried out on very large single edged notched specimens subjected to tensile loading and a thermal gradient along the prospective crack path. Several attempts including visco-plastic modelling of the previously described nature have been made in order to understand the experimental outcome, but no common view is yet in existence.

DAMAGE MECHANICS MODELLING

A strong trend in quasi-static as well as in dynamic fracture mechanics is to employ so called damage mechanics constitutive models in order to better simulate the behaviour at growing crack tips. A recent example of such calculations is the article by Mathur *et al.* (1996) where a three dimensional analysis of dynamic ductile crack growth in a thin plate is performed. Such analyses are nowadays capable of capturing at least qualitatively many features of observed crack growth processes. The analysis in the cited article and similar ones can for instance be regarded as extensions of the visco-plastic modelling discussed above. The extension is mainly that the decohesive processes are more explicitly taken into account by introduction of softening behaviour through the damage variable. Unfortunately the analyses are so complex that it is difficult to discern the characteristic features. Analytical results are scarce and here a very simple example of a complete solution using a type of damage formulation considered by Bui and Ehrlacher (1981) is to be commented upon.

A semi infinite crack under anti-plane strain (mode III) conditions moves steadily along the x-axis with a velocity $\dot{a}$ in an infinite medium. The boundary conditions at infinity are those of the singular linearly elastic solution governed by the stress intensity factor K_{III}. The material is linearly elastic as long as the effective stress $\tau_e = (\tau_x^2 + \tau_y^2)^{1/2}$ is less than a critical stress τ_c and in this case the governing equation is well known i. e.

$$\left(1 - \left(\frac{\dot{a}}{C_2}\right)^2\right) w_{,x'x'} + w_{,y'y'} = 0 \qquad (10)$$

x' and y' here refer to a coordinate system attached to and moving with crack. When the critical stress is exceeded the material ceases to be load bearing and thus the boundary conditions along the moving damage front surfaces become

$$\tau_n = \mu \frac{\partial w}{\partial n} = 0 , \qquad (11)$$

$$\tau_t = \mu \frac{\partial w}{\partial t} = \tau_c . \qquad (12)$$

Here n and t denote the normal and the tangential direction, respectively. w is the displacement and μ is the shear modulus. It is clear by the definition of the problem that all lengths can be scaled by $(K_{III}/\tau_c)^2$. In Fig. 3 the crack surface displacement is plotted for $\dot{a}/C_2$ near zero as function of the non-dimensional coordinates. The solution for higher values of $\dot{a}/C_2$ is different but possesses the same general features and is therefore not shown. One puzzling feature of the solution of the governing equations is that there is no difference whatsoever between the case of a stationary crack and the case of a crack moving with a low velocity. The shape of the damage zone shown in Fig. 4 is reasonable for a steadily moving crack but clearly not for a stationary crack loaded up to some level. An attempt of interpretation is that as soon as load is applied, however small, that damage criterion is reached and the crack starts to propagate. Thus models of the present type can not be used to predict initiation of crack growth. It is further noted that there is no possibility to predict the crack growth velocity from the obtained solution. Some additional criterion is needed and Bui and Ehrlacher (1981) suggested the height of the damage zone which in essence is equivalent to specifying a characteristic material length.

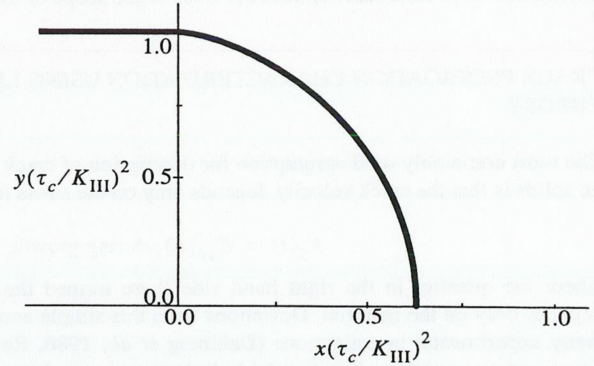

Fig. 3. Shape of the damage front.

The need to specify a critical material length seems to be common to all kinds of damage mechanics modelling for description of crack growth. In many of these calculations this fact has been overlooked and the material length that is specified becomes the characteristic size of the near tip mesh. This is itself a viable procedure as long as the analyst is conscious of the fact that the chosen element mesh is in fact part of the constitutive assumptions. The problem is by no means unique to crack analysis. Exactly the same difficulties are encountered for instance when analysing shear band development and does arise as soon as the material model admits softening behaviour.

There are different alternatives to overcome this problem. Lately interest has focused towards non-local constitutive models such as for instance strain gradient plasticity. In such models the critical material length has to be specified already in the constitutive model and is thus removed from the computational process. Another way is to model the internal material sizes explicitly by micro mechanical modelling. Both of these two alternatives probably require huge computational efforts.

A simpler alternative which has been used for a long time and been revived in recent years is to use cohesive zone modelling. In essence this is equivalent to restrict the softening behaviour to a single line (surface in the three dimensional case) and replace the relation between stresses and strains by a relation between traction and material separation. The critical length is here introduced by specifying a force versus separation law.

AN EXAMPLE OF A POSSIBLE MICRO MECHANICAL APPLICATION OF DYNAMIC CRACK GROWTH THEORY

In order to illustrate the possible use of crack growth theory to micromechanical applications an example for initiation of cleavage fracture is stated below. It is fairly commonly accepted that cleavage fracture in ferritic steels is mostly initiated from brittle inclusions such as carbides and sulphides. Such an inclusion may be irregularly formed and vary considerably between different particles. When the stresses become large enough at a particular inclusion, dynamic crack growth in the particle may be induced and a crack grows rapidly across the particle according to the sketch in Fig. 4. This is usually a very brittle process and the crack may or may not penetrate into the surrounding matrix material. This is usually considerably tougher than the inclusion material and it is possible that the crack may arrest after some amount of growth. Should it not become arrested the crack may continue to grow and cause a total cleavage fracture. Thus two requisites are thus needed for a complete failure to occur. The crack growth must initiate and it must not be arrested. The first one is fairly difficult to predict since it will depend on the details of the inclusion shape. The arrest question may be more well defined and therefore more stable to analyse. We shall here not attempt to solve the problem but rather discuss possible aspects that need to be taken into account.

First of all it has to be realized that the real case is always fully three dimensional. Qualitative understanding can probably be achieved by two dimensional modelling, either plane or axisymmetric. This is necessary since fully three dimensional modelling is extremely complex and presumably not possible to perform with contemporary numerical methods and computers.

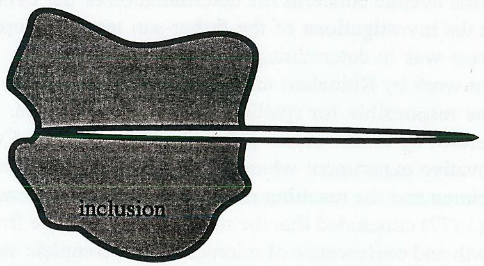

Fig. 4. A crack traversing an inclusion and propagating into the matrix.

The crack growth within the inclusion can presumably be modelled by linear theory according to eq. (6), while for the matrix material some non-linear model is needed, since in many occasions cleavage fracture is preceded by plastic deformation. A possible choice could be the visco-plastic models discussed above which have the advantage of a well known type of crack tip singularity. A particular obstacle to the analysis is the interface. Very little fundamental understanding exists regarding rapid crack growth over a material interface where the stiffness properties are so different as say for a carbide inclusion in a ferritic matrix.

A considerable difficulty is to assign proper crack growth properties to the respective materials. Thus for instance the crack propagation toughness obtained for a ferritic steel is necessarily the appropriate one for analysis of the matrix material. A macroscopic crack propagation experiment on a structural steel will for instance also depend the inclusions present in the material.

What can be gained from a hopefully successful analysis of the stated problem? In addition to a fuller understanding of cleavage fracture, at least partial answers to such important questions as a macroscopic criterion for cleavage fracture can be obtained. It is commonly assumed that cleavage fracture initiates if the maximum principal stress exceeds a critical value which in itself is taken as a random variable. It is easily realized from consideration of the stated example that other variables such as for instance the amount of plastic deformation and also stresses in other directions may be of importance. The statistical aspects can also be taken a step further and be coupled to the inclusion size and shape.

REFERENCES

Brickstad, B. (1983). A visco-plastic analysis of rapid crack propagation in steel. *J. Mech. Phys. Solids* **31**, 307-327.

Bui H. D. and Ehrlacher A. (1981). Propagation of damage in elastic and plastic solids. In: *Advances in Fracture Research.* 5th International Conference on Fracture (D. Francois ed.), pp. 533-541, Pergamon Press.

Burridge, R. (1976). An influence function for the intensity factor in tensile fracture. *Int. J. Engng Sci.* **14**, 725-730.

Dahlberg, L., Nilsson, F. and Brickstad, B. (1980). Influence of specimen geometry on crack propagation and arrest toughness. In: *Crack Arrest Methodology and Applications, ASTM STP 711* (G. T. Hahn and M. F. Kanninen eds.), pp. 89-108, American Society for Testing and Materials, Philadelphia Pa.

Geubelle, P. H. and Rice, J. R. (1995). A spectral method for three-dimensional elasto-dynamic fracture problems. *J. Mech. Phys. Solids* **43**, 1791-1824.

Freund, L. B. (1972a), Crack propagation in an elastic solid subjected to general loading-I. Constant rate of extension, *J. Mech. Phys. Solids* **20**, 129-140.

Freund, L. B. (1972b), Crack propagation in an elastic solid subjected to general loading-II. Non-uniform rate of extension, *J. Mech. Phys. Solids* **20**, 141-152.

Freund, L. B. (1990). *Dynamic fracture mechanics*. Cambridge University Press, Cambridge.

Freund, L. B. and Rosakis, A. J. (1992). The structure of near-tip field during elastodynamic crack growth. *J. Mech. Phys. Solids* **40**, 699-719.

Kostrov, B. V. (1975). On the crack propagation with variable velocity. *Int. J. Fracture* **11**, 47-56.

Mathur, K. K., Needleman A. and Tvergaard V. (1996), Three dimensional analysis of dynamic ductile crack growth in a thin plate. *J. Mech. Phys. Solids* **44**, 439-464.

Mott, N. F. (1948). Brittle fracture in mild steel plates. *Engineering* **175**, 16-18.

Naus, D. J., Nanstad, B. R., Bass, B. R., Merkle, J. G., Pugh, C. E., Corwin, W. R. and Robinson, G. C. (1987). Crack arrest behaviour in SEN wide plates of quenched and tempered A533B grade B steel tested under nonisothermal conditions. *NUREG/CR-49309*, U.S. Nuclear Regulatory Commission.

Nishioka, T. (1995). Recent developments in computational dynamic fracture mechanics. In: *Dynamic Fracture Mechanics* (M. H. Aliabadi ed.), pp. 1-10. Computational Mechanics Publications, Southampton.

Perzyna, P. (1963). The constitutive equations for rate sensitive materials. *Quarterly of Appl. Math.* **20**, 321.

Ravi-Chandar, K. and Knauss, W. G. (1982) Dynamic fracture under stress wave loading- A comparison of theory and experiment. *Int. J. Fracture* **20**, 209-222.

Willis, J. R. and Movchan A. B. (1995), Dynamic weight functions for a moving crack. I. Mode I loading. *J. Mech. Phys. Solids* **43**, 319-341.

Östlund, S. (1990). On numerical modeling and fracture criteria of dynamic elastic-plastic crack growth. *Int. J. Fracture* **44**, 283-299.

Östlund, S. (1991). Large scale yielding for dynamic crack growth in a strip geometry. *Int. J. Fracture* **49**, 219-237.