DYNAMIC ANALYSIS OF A PROPAGATING INTERFACE CRACK

Y. S. ING and C. C. MA
Department of Mechanical Engineering, National Taiwan University,
Taipei, Taiwan 10617, Republic of China

ABSTRACT

In this study, the transient response of a propagating interface crack between two different media is investigated. For time $t<0$, the crack is stress free and at rest. At $t=0$,a pair of concentrated anti-plane dynamic loadings are applied at the stationary crack faces. We assume that the stationary crack will begin to propagate along the interface with a subsonic speed as the incident wave in the upper medium or in the lower one arrives at the crack tip. A new fundamental solution is proposed in this study and the solution is determined by superposition of the fundamental solution in the Laplace transform domain. The Cagniard-de Hoop method (de Hoop (1958)) of Laplace inversion is used to obtained the transient solution in time domain. Numerical calculations of dynamic stress intensity factors are evaluated and discussed in detail.

KEYWORDS

Propagating interface crack, bimaterial, stress intensity factor.

INTRODUCTION

For the last two decades, the importance of composite materials has increased very rapidly in engineering applications because of their high strength and light weight. However, flaws contained at the interfaces of composite bodies due to improper adhesion may lead to serious danger, and a better understanding of interface fracture mechanics is needed. A stationary crack lying along the interface between dissimilar isotropic materials subjected to static loading was first considered by Williams (1959) for plane strain condition. A number of solutions for the stress and the displacement field near the crack tip are obtained by England (1965), Erdogan (1965) and Rice and Sih (1965).

In the field of propagating interface cracks, Brock and Achenbach (1973) analyzed the extension of an interface crack under the influence of transient horizontally polarized shear wave. Willis (1971) investigated the energy release rate of a steadily extending interface crack by means of the local form of the Griffith virtual work argument. In the recent years, Wu (1991) treated the similar but anisotropic problem and derived the crack-tip fields and energy release rate successfully by employing the Stroh formalism for anisotropic elasticity. Deng (1992) used the Radok's complex function formulation with a two-term complex eigen-expansion technique to analyze the near-tip fields for steadily growing interface cracks in dissimilar isotropic materials. The stress singularities and the angular stress distributions near a propagating interface crack in different transonic regimes for both anti-plane and in-plane cases are determined by Yu and Yang (1994,1995).

In this paper, the transient problem of an interface crack propagating with a subsonic speed in an infinite bimaterial is considered. At time $t=0$, the crack is at rest and a pair of anti-plane concentrated loadings act at stationary crack faces. After some delay time t_f, the crack begins to running along the interface with a constant velocity v as shown in Fig. 1. A new fundamental solution is proposed and it is successfully applied towards solving the problem. The alternative superposition scheme has been used to solve many transient problems for a homogeneous medium successfully, e.g., Tsai and Ma (1992) for a stationary crack and Ma and Ing (1995a) for a propagating crack.

REQUIRED FUNDAMENTAL SOLUTIONS

Consider a fundamental problem of anti-plane deformation for an extending interface crack in dissimilar materials. The crack propagates with a constant velocity v which is less than the minimum of the shear wave speed of these two materials. Figure 1 shows the interface crack geometry and the coordinate systems. The coordinate ξ defined by $\xi = x - vt$ is fixed with respect to the moving crack tip.

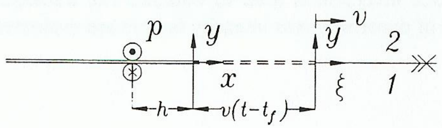

Fig. 1 The configuration and coordinate System of a propagating interface crack in a bimaterial.

In analyzing this problem, it is convenient to express the governing equations of wave motions in the moving coordinates $\xi - y$ as follows

$$\left(1-b_j^2v^2\right)\frac{\partial^2 w_j}{\partial \xi^2}+\frac{\partial^2 w_j}{\partial y^2}+2b_j^2 v\frac{\partial^2 w_j}{\partial \xi \partial t}-b_j^2\frac{\partial^2 w_j}{\partial t^2}=0, \qquad j=1,2 \tag{1}$$

where the subscript j ($j=1,2$) refers to the lower and upper media, respectively; w_j are the out-of-plane displacements, and b_j are the slownesses of the shear waves given by

$$b_j = 1/c_{sj} = \sqrt{\rho_j/\mu_j},$$

in which c_{sj} are the shear wave speeds, μ_j and ρ_j are the respective shear moduli and the mass densities of two materials. It is assumed that the shear wave speed in the lower material is less than that in the upper material($b_1 > b_2$). The nonvanishing shear stresses are

$$\tau_{yzj} = \mu_j\frac{\partial w_j}{\partial y}, \qquad \tau_{xzj} = \mu_j\frac{\partial w_j}{\partial x}. \tag{2}$$

The boundary conditions on the crack surfaces expressed in the Laplace transform domain can be described as follows

$$\bar{\tau}_{yz1}(\xi,0,s)=\bar{\tau}_{yz2}(\xi,0,s)=e^{s\eta\xi}, \qquad -\infty<\xi<0 \tag{3}$$

where s is the Laplace transform parameter and η is a constant. The overbar symbol is used for denoting the transform on time t. The displacements and shear stresses must be continuous on the interface which gives the following conditions on the interface

$$\bar{\tau}_{yz1}(\xi,0,s)=\bar{\tau}_{yz2}(\xi,0,s), \qquad 0<\xi<\infty \tag{4}$$

$$\bar{w}_1(\xi,0,s)=\bar{w}_2(\xi,0,s). \qquad 0<\xi<\infty \tag{5}$$

The solution of the proposed fundamental problem can be obtained by making use of integral transform methods. The final results expressed in the Laplace transform domain are

$$\bar{\tau}_{yz1}(\xi,y,s)=\frac{1}{2\pi i}\int\frac{Q_+^*(\eta)\alpha_{2+}^*(\lambda)}{\alpha_{2+}^*(\eta)(\eta-\lambda)Q_+^*(\lambda)}e^{s\alpha_1^*(\lambda)y+s\lambda\xi}d\lambda, \tag{6}$$

$$\bar{\tau}_{xz1}(\xi,y,s)=\frac{1}{2\pi i}\int\frac{\lambda Q_+^*(\eta)\alpha_{2+}^*(\lambda)}{\alpha_{2+}^*(\eta)(\eta-\lambda)\alpha_1^*(\lambda)Q_+^*(\lambda)}e^{s\alpha_1^*(\lambda)y+s\lambda\xi}d\lambda, \tag{7}$$

$$\bar{w}_1(\xi,y,s)=\frac{1}{2\pi i}\int\frac{Q_+^*(\eta)\alpha_{2+}^*(\lambda)}{s\mu_1\alpha_{2+}^*(\eta)(\eta-\lambda)\alpha_1^*(\lambda)Q_+^*(\lambda)}e^{s\alpha_1^*(\lambda)y+s\lambda\xi}d\lambda, \tag{8}$$

$$\bar{\tau}_{yz2}(\xi,y,s)=\frac{1}{2\pi i}\int\frac{Q_+^*(\eta)\alpha_{2+}^*(\lambda)}{\alpha_{2+}^*(\eta)(\eta-\lambda)Q_+^*(\lambda)}e^{-s\alpha_2^*(\lambda)y+s\lambda\xi}d\lambda, \tag{9}$$

$$\bar{\tau}_{xz2}(\xi,y,s)=\frac{1}{2\pi i}\int\frac{-\lambda Q_+^*(\eta)}{\alpha_{2+}^*(\eta)(\eta-\lambda)\alpha_{2-}^*(\lambda)Q_+^*(\lambda)}e^{-s\alpha_2^*(\lambda)y+s\lambda\xi}d\lambda, \tag{10}$$

$$\bar{w}_2(\xi,y,s)=\frac{1}{2\pi i}\int\frac{-Q_+^*(\eta)}{s\mu_2\alpha_{2+}^*(\eta)(\eta-\lambda)\alpha_{2-}^*(\lambda)Q_+^*(\lambda)}e^{-s\alpha_2^*(\lambda)y+s\lambda\xi}d\lambda, \tag{11}$$

in which

$$Q_+^*(\lambda)=\exp\left\{\frac{-1}{\pi}\int_{b_{2,2}}^{b_{1,2}}\tan^{-1}\left[\frac{\mu_2\left|\alpha_2^*(-z)\right|}{\mu_1\alpha_1^*(-z)}\right]\frac{dz}{z+\lambda}\right\}, \tag{12}$$

$$Q_-^*(\lambda)=\exp\left\{\frac{-1}{\pi}\int_{b_{2,1}}^{b_{1,1}}\tan^{-1}\left[\frac{\mu_2\left|\alpha_2^*(z)\right|}{\mu_1\alpha_1^*(z)}\right]\frac{dz}{z-\lambda}\right\}, \tag{13}$$

$$\alpha_j^*(\lambda)=\sqrt{b_j+\lambda\left(1-b_jv\right)}\sqrt{b_j-\lambda\left(1+b_jv\right)}=\alpha_{j+}^*(\lambda)\alpha_{j-}^*(\lambda), \qquad j=1,2, \tag{14}$$

$$b_{j,1}=b_j/(1+b_j v), \qquad b_{j,2}=b_j/(1-b_j v).$$

The corresponding result of the dynamic stress intensity factor in the Laplace transform domain is

$$\overline{K}(s)=\frac{-\sqrt{2(1-b_2 v)}Q_+^*(\eta)}{\sqrt{s}\alpha_{2+}^*(\eta)}. \tag{15}$$

DYNAMIC STRESS INTENSITY FACTOR OF A PROPAGATING CRACK

As shown in Fig. 1, a bimaterial medium is composed of two homogeneous, isotropic, and linearly elastic solids. Without loss of generality, it is assumed that $b_1>b_2$. A semi-infinite crack lying along the interface of the bimaterial is initially stress-free and at rest. At time $t=0$, a pair of equal and opposite concentrated anti-plane dynamic loadings with magnitude p are applied at the crack faces with a distance h from the tip. The time dependence of the loading is represented by the Heaviside step function $\mathrm{H}(t)$. After some delay time $t=t_f$, the dynamic stress intensity factor reaches its critical value and then the crack starts to propagate with a constant subsonic speed v ($v<b_1^{-1}<b_2^{-1}$) along $y=0$.

First we consider the analysis for the stationary crack for $t<t_f$. The incident field of the cylindrical wave generated by the concentrated loading expressed in the Laplace transform domain can be represented as follow

$$\overline{\tau}_{yz}^{i}(x,0,s)=\frac{1}{2\pi i}\int -pe^{s\lambda(x+h)}d\lambda. \tag{16}$$

The applied traction on the crack faces as indicated in (16), has the functional form $e^{s\lambda x}$. Since the solutions of applying traction $e^{s\eta x}$ on crack faces have been solved in the previous section (by setting $v=0$), the dynamic stress intensity factor expressed in the Laplace transform domain can be constructed by (15) and (16) as follow

$$\overline{K}^d(s)=\frac{1}{2\pi i}\int -pe^{s\lambda h}\left\{\frac{-\sqrt{2}Q_+(\lambda)}{\sqrt{s}\alpha_{2+}(\lambda)}\right\}d\lambda$$

$$=\frac{1}{2\pi i}\int\frac{\sqrt{2}pQ_+(\lambda)}{\sqrt{s}\alpha_{2+}(\lambda)}e^{s\lambda h}d\lambda, \tag{17}$$

in which

$$Q(\lambda)=Q_+(\lambda)Q_-(\lambda)=Q_+^*(\lambda)\big|_{v=0}Q_-^*(\lambda)\big|_{v=0},$$

$$\alpha_j(\lambda)=\alpha_{j+}(\lambda)\alpha_{j-}(\lambda)=\alpha_{j+}^*(\lambda)\big|_{v=0}\alpha_{j-}^*(\lambda)\big|_{v=0}=\sqrt{b_j+\lambda}\sqrt{b_j-\lambda}.$$

The dynamic stress intensity factor of the stationary interface crack expressed in time domain is

$$K^d(t)=\sqrt{\frac{2}{h}}\frac{p}{\pi^{3/2}}\int_{b_2}^{t/h}\frac{\mathrm{Re}[Q_-(\eta)]}{\sqrt{t/h-\eta}\sqrt{\eta-b_2}}d\eta. \tag{18}$$

For time $t>b_1 h$, the integral in (18) can be evaluated by using contour integration and yields

$$K^d(t)=K^s\mathrm{H}(t-b_1 h), \tag{19}$$

in which

$$K^s=p\sqrt{\frac{2}{\pi h}} \tag{20}$$

is the corresponding static solution in a homogeneous medium. It can be seen that the dynamic stress intensity factor of the stationary crack in a bimaterial is the same as the corresponding static value K^s in a homogeneous medium after the slower shear wave passed the crack tip. If $b_1=b_2=b$, for the homogeneous case, eq. (20) can be evaluated by letting $Q_+(\eta)=1$ and yields

$$K_h^d(t)=p\sqrt{\frac{2}{\pi h}}\mathrm{H}(t-bh). \tag{21}$$

In the previous discussion, it is known that the dynamic stress intensity factor will reach its corresponding static value immediately after the slower incident wave passed the stationary crack tip. It means that a stationary interface crack subjected to a pair of concentrated forces on its faces can begin to propagate only at time $b_2 h\le t\le b_1 h$. There are two special propagating cases to be considered here. The first one is that the crack starts to propagate at once when the incident wave with higher speed arrives at the stationary crack tip ($t_f=b_2 h$). The second one is that the crack starts to propagate when the slower incident wave arrives at the crack tip ($t_f=b_1 h$).

(1) Case 1 : delay time $t_f=b_2 h$

The applied concentrated loading on the interfacial crack faces written in the Laplace transform domain for the moving coordinate system will have the following form

$$\overline{\tau}_{yz}^{i}(\xi_1,0,s)=\frac{1}{2\pi i}\int\frac{pd}{\lambda-d}e^{sh(1-b_2 v)\lambda+s\lambda\xi_1}d\lambda, \tag{22}$$

in which $d=1/v$ is the slowness of the crack velocity and $\xi_1=x-v(t-b_2 h)$. From the combination of (15) and (22). The result of dynamic stress intensity factor expressed in the Laplace transform domain will be

$$\overline{K}^{v,1}(s)=\frac{-1}{2\pi i}\int\frac{pd\sqrt{2(1-b_2 v)}Q_+^*(\lambda)}{\sqrt{s}(\lambda-d)\alpha_{2+}^*(\lambda)}e^{sh(1-b_2 v)\lambda}d\lambda. \tag{23}$$

The dynamic stress intensity factor in time domain can be obtained as follow

$$K^{v,1}(t)=\frac{pd\sqrt{2h(1-b_2 v)}}{\pi^{3/2}}\int_{b_2 h}^{t}\frac{\mathrm{Re}\left[Q_+^*(-\tau/(1-b_2 v)/h)\right]}{\sqrt{t-\tau}\sqrt{\tau-b_2 h}\left[\tau+h(d-b_2)\right]}d\tau. \tag{24}$$

If $t>t_c$, which is the time the slower incident wave in material 1 catches up with the propagating crack tip, then the integration in (24) can be carried out and the final result is

$$K^{v,1}(t)=p\sqrt{\frac{2}{\pi\left[v(t-b_2 h)+h\right]}}Q_+^*(d)(1-b_2 v)^{1/2}\mathrm{H}(t-t_c), \tag{25}$$

where $t_c=b_1 h(1-b_2 v)/(1-b_1 v)$. The expression for $K^{v,1}(t)$ in (25) has the interesting form of the product of a function $Q_+^*(d)(1-b_2 v)^{1/2}$ and the corresponding static stress intensity factor K^s in (20) for applying a pair of concentrated loadings at crack faces with a distance $v(t-b_2 h)+h$ from the crack tip. The value $Q_+^*(d)(1-b_2 v)^{1/2}$ is an universal function which depends only on crack speed and material properties. If $b_1=b_2$ and $\mu_1=\mu_2$, we have

$Q_+^*(d)=1$ and the solution in (25) for the propagating interface crack in a bimaterial can be reduced to that obtained by Ma and Ing (1995b) in a homogeneous medium.

(2) Case 2 : delay time $t_f = b_1 h$

For the second case, the fracture toughness of the bimaterial is assumed to be equal to the corresponding static value in (21). For $b_2 h < t < b_1 h$, the crack is still at rest and the dynamic stress intensity factor for this stationary crack can be calculated by using the formulation in (24). Follow the similar procedure in case 1, the dynamic stress intensity factor for the propagating interface crack expressed in the Laplace transform domain can be obtained as follow

$$\overline{K}^{v,2}(s) = \frac{-1}{2\pi i}\int \frac{pd\sqrt{2(1-b_2 v)}Q_+^*(\lambda)}{\sqrt{s}(\lambda-d)\alpha_{2+}^*(\lambda)} e^{sh(1-b_1 v)\lambda} d\lambda. \qquad (26)$$

The dynamic stress intensity factor expressed in time domain is

$$K^{v,2}(t) = p\sqrt{\frac{2}{\pi[v(t-b_1 h)+h]}}Q_+^*(d)(1-b_2 v)^{1/2} \mathrm{H}(t-b_1 h). \qquad (27)$$

The same as $K^{v,1}(t)$ in (25), the expression for $K^{v,2}(t)$ in (27) has a interesting form of the product of a function $Q_+^*(d)(1-b_2 v)^{1/2}$ and the corresponding static stress intensity factor K^s in (21) with a distance $v(t-b_1 h)+h$ from the crack tip.

NUMERICAL RESULTS

The wave fronts for the propagating crack in a short time period are plotted in Fig. 2. In this figure, "1" and "2" indicate the direct waves produced by the applied forces in the material 1 and 2, respectively. The diffracted waves "ij" ($i,j=1,2$), denote the waves in medium i resulting from the diffraction of a disturbance induced by the applied loading in medium j.

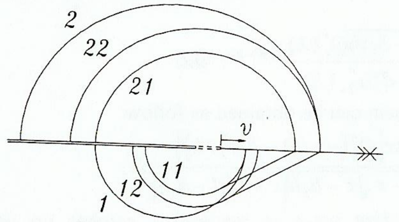

Fig. 2 Wave fronts of the incident and diffracted waves of case 2 situation for $t > t_f$.

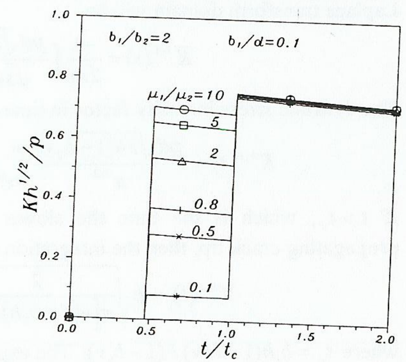

Fig. 3 Stress intensity factors of a propagating interface crack in case 1 for different values of μ_1/μ_2.

Figure 3 and 4 show the dimensionless stress intensity factors $Kh^{1/2}/p$ for the case 1 situation

of the propagating interface crack versus dimensionless time t/t_c for various values of μ_1/μ_2 and b_1/b_2. It is of interest to see that for $t > t_c$ the dynamic stress intensity factors are almost equal for this small crack velocity $v = 0.1c_{s1}$ under different material combination.

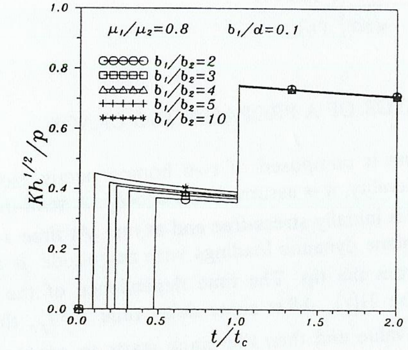

Fig. 4 Stress intensity factors of a propagating interface crack in case 1 for different values of b_1/b_2.

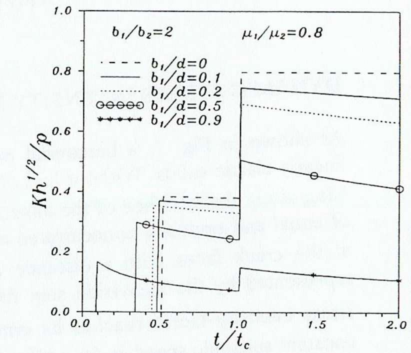

Fig. 5 Stress intensity factors of a propagating interface crack in case 1 for different values of crack velocity v.

The dynamic stress intensity factors for different values of b_1/d under constant b_1/b_2 and μ_1/μ_2 are shown in Fig. 5. It can be found that the higher crack velocity is, the smaller dynamic stress intensity factor. Hence, the stationary crack has the largest dynamic stress intensity factor among these different running cases.

Figure 6-7 show the dynamic stress intensity factors for the case 2 situation of the propagating interface crack.

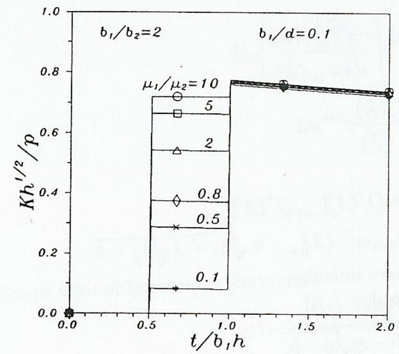

Fig. 6 Stress intensity factors of a propagating interface crack in case 2 for different values of μ_1/μ_2.

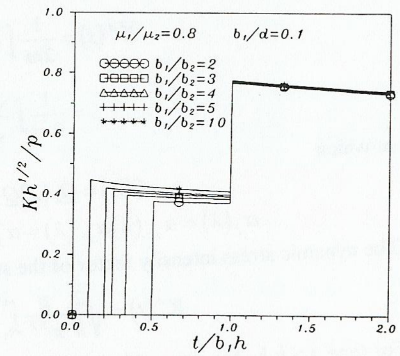

Fig. 7 Stress intensity factors of a propagating interface crack in case 2 for different values of b_1/b_2.

It can be also seen in Fig. 6-7 that dynamic stress intensity factors are almost equal for $t = b_1 h$.

CONCLUSIONS

Two propagating cases are considered and the obtained results have many particular phenomena. The expression of dynamic stress intensity factor of a propagating interface crack has the interesting form of the product of a universal function and the corresponding static stress intensity factor under the same boundary condition. Moreover, the dynamic stress intensity factor of a propagating interface crack is approximately equal to that in a homogeneous medium for small crack velocity ($v \le 0.1c_{s1}$).

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