TRUE SHEAR AND MIXED MODE FRACTURE TOUGHNESS TESTING OF MATERIALS

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ABSTRACT

The conventional approach to the shear and mixed mode fracture of materials is to adopt a criterion based on some postulated relationship between the corresponding mode of loading and mode I fracture toughness, irrespective of the actual fracture mechanism. Accordingly, it is generally accepted that it would only be necessary to develop standard tests to determine K_{IC}, for example in ASTM E399-83 (American Society for Testing and Materials, 1986) and BS 5447 (British Standards Institution, 1977) respectively. In this connection, however, an apparent lack of consensus on the definitions of the shear and mixed mode fracture toughnesses has developed, which may be attributed to a fundamental flaw in the existing theory. The anomaly may be broadly stated as the assumption that the traditional stress intensity factors, K_I and K_{II}, may be considered to be absolute constants of a given boundary value problem, whereas it has been shown recently (Lo et al., 1996a) that they would be more consistently represented by the corresponding unified modes I and II parameters, $K_{I\theta}$ and $K_{II\theta}$, which vary from one θ plane to another, although remaining constant for a given plane. Moreover, as a consequence of the traditional assumption, Irwin's (1957) fracture criteria for the θ_{0C} plane have been inappropriately generalised as being applicable to fracture in an arbitrary θ_C plane, and this has further led to the unjustifiable tendency to associate a particular mode of loading with the same mode of fracture unconditionally. In the course of identifying and hence redressing the anomaly, Lo et al. postulated their *unified model*, whereby the mixed mode fracture of materials is generally governed by the fracture toughness K_C, which is in turn determined by the modes I and II fracture toughnesses K_{IC} and K_{IIC}, and it would therefore be necessary, in principle, to augment the standard K_{IC} test with corresponding tests for K_{IIC} and hence K_C, respectively. Accordingly, proposals for the latter tests will be presented herein on the premise of the unified model. It will also be shown that the results obtained for two widely-differing test materials, that is aluminium alloy and brittle clay, conform with the original fracture envelope derived by Lo et al., which is based on the simple conversion of the pure modes I and II loading energies to mixed mode loading energy, in direct proportion to their corresponding fracture energies.

KEYWORDS

True shear fracture, true mixed mode fracture, unified model, fracture toughness testing, aluminium alloy, brittle clay.

INTRODUCTION

A fundamental flaw has recently been reported (Lo et al., 1996a) on the traditional approach to fracture mechanics, in that, for crack propagation in the generalised θ_C plane, the assumption whereby the traditional stress intensity factors, K_I and K_{II}, may be considered to be absolute constants of a given boundary value problem would not be consistent with the basic physical relationships between Griffith's (1920) pure modes I and II critical rates of energy release, G_{IC} and G_{IIC}, and Irwin's (1957) corresponding fracture toughnesses, K_{IC} and K_{IIC}, respectively. On the other hand, it has been found that for these relationships to be so applicable, it would be necessary to generalise K_I and K_{II} in terms of the modes I and II stress intensity factors of the unified model, that is $K_{I\theta}$ and $K_{II\theta}$ respectively. Moreover, as a consequence of the fundamental flaw, a lack of consensus has developed with regard to the mechanics of pure shear and mixed mode fracture, in connection with which there has been an unjustifiable tendency to focus solely on the mode I fracture toughness, K_{IC}. The latter tendency has led, in turn, to the establishment of standards on compact tension testing only. On the other hand, the unified model does not suffer the drawbacks of the conventional approach, and furthermore indicates how the true pure mode II and mixed mode fracture toughnesses (that is, K_{IIC} and K_C respectively) may be determined consistently. Accordingly, test procedures will be specified in the subsequent discussion to determine these fracture parameters, the results of which will be shown to conform with the unified fracture envelope based on the simple conversion of pure modes I and II loading energies to mixed mode loading energy, in direct proportion to their corresponding fracture energies. In addition, the respective fracture mechanisms will be shown to be borne out by the visual and fractographical evidence obtained.

SELECTED FEATURES OF THE UNIFIED MODEL

A condensed version of the unified model originally proposed by Lo et al. (1996a) has been presented in a companion paper (Lo et al., 1996b), from which salient features will be drawn to provide the rationale for the test proposals herein. In this connection, the starting point would be the closure equation for the crack extension of Fig. 1, which is orientated in the generalised θ direction and subjected to mixed modes I and II loading, viz.

$$G_\theta = \frac{1}{\delta a}\int_0^{\delta a}[\sigma_{\theta\theta}(r,\theta)u'_{\theta\theta}(\delta a-r,-\pi) + \tau_{r\theta}(r,\theta)u'_{rr}(\delta a-r,-\pi)]\,dr\ , \tag{1}$$

where G_θ is the mixed mode rate of energy release in the θ plane, "a" the existing crack length, θ the direction of the plane of interest with respect to existing crack plane θ_0=0, "r" the radial distance from the crack tip, and $\sigma_{\theta\theta}$, $\tau_{r\theta}$, $u'_{\theta\theta}$ and u'_{rr} the circumferential and shear stresses and circumferential and radial displacements in polar coordinates, respectively. Next, by adopting the unified modes I and II stress intensity factors $K_{I\theta}$ and $K_{II\theta}$, which correspond to the generalised θ plane, in lieu of Irwin's stress intensity factors K_I and K_{II}, which pertain solely to the θ_0 plane, where

$$K_{I\theta} = \lim_{r\to 0}\sigma_{\theta\theta}\sqrt{2\pi r} = K_I\cos^3\frac{\theta}{2} - 3K_{II}\sin\frac{\theta}{2}\cos^2\frac{\theta}{2} \tag{2}$$

and

$$K_{II\theta} = \lim_{r\to 0}\tau_{r\theta}\sqrt{2\pi r} = K_I\sin\frac{\theta}{2}\cos^2\frac{\theta}{2} + K_{II}\cos\frac{\theta}{2}\left(1-3\sin^2\frac{\theta}{2}\right)\ , \tag{3}$$

it can be shown that when $\delta a \to 0$, we would have

$$G_\theta = \frac{(1+\nu)(1+\kappa)}{4E}(K_{I\theta}^2 + K_{II\theta}^2)\ , \tag{4}$$

where "E" is Young's modulus, ν Poisson's ratio and κ a known function of ν. Furthermore, since, for pure mode I crack propagation in the generalised θ_C plane, $K_{II\theta} = 0$, and $K_{I\theta} \to K_{IC}$ as $G_\theta \to G_{IC}$, we would obtain

$$G_{IC} = \frac{(1+\nu)(1+\kappa)}{4E}K_{IC}^2\ , \tag{5}$$

as in the case of Irwin's (1957) analysis of mode I crack propagation along the θ_{0C} plane. Likewise, for pure mode II crack propagation along the generalised θ_C plane, $K_{I\theta} = 0$, and $K_{II\theta} \to K_{IIC}$ as $G_\theta \to G_{IIC}$, so that

$$G_{IIC} = \frac{(1+\nu)(1+\kappa)}{4E}K_{IIC}^2\ , \tag{6}$$

as in the case of Irwin's analysis of mode II crack propagation along the θ_{0C} plane. Hence, the fundamental, physical requirement, that the relationships specified in equations (5) and (6) should be applicable to any θ_C plane in an isotropic, homogeneous medium, may be satisfied by the adoption of the unified stress intensity factors $K_{I\theta}$ and $K_{II\theta}$, as defined by equations (2) and (3) respectively (this would not be the case for the traditional stress intensity factors K_I and K_{II}).

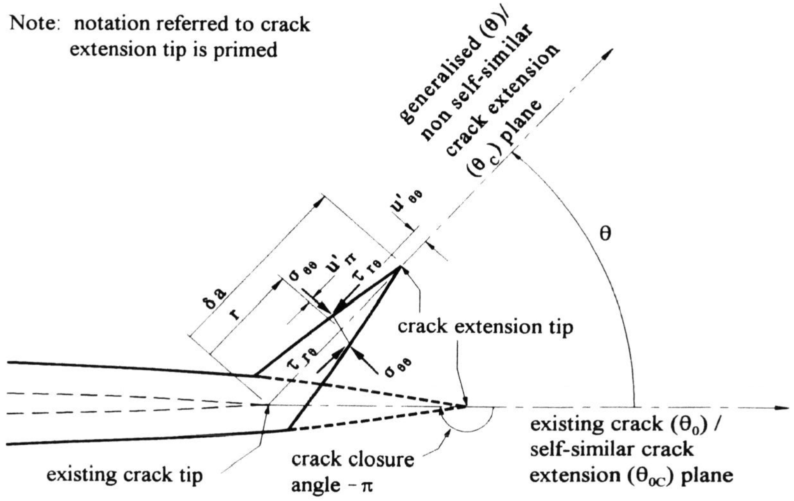

Fig. 1 Closure parameters of crack tip.

In general, however, a mixed mode fracture could occur under either pure or mixed mode applied loading, in which case, referring to equation (4), we could write

$$G_C = \frac{1}{4E}(1+\nu)(1+\kappa)K_C^2\ , \tag{7}$$

where G_C would therefore represent the mixed mode critical rate of energy release and K_C the mixed mode fracture toughness, as defined by the unified model. *Thus, by the same token as for the relationships of equations (5) and (6), K_C may be construed as a valid alternative parameter to G_C for determining crack propagation.* Consequently, the traditional fracture parameters K_{IC}, K_{IIC}, G_{IC} and G_{IIC}, which have hitherto been strictly applicable to pure mode fracture in the θ_{0C} plane only, would be generalised in terms of the unified parameters K_C and G_C, for mixed mode fracture along an arbitrary θ_C plane. Furthermore, in view of equations (4) and (7), we would have

$$K_{I\theta}^2 + K_{II\theta}^2 = K_C^2 \tag{8}$$

at fracture, where the values of $K_{I\theta}^2$ and $K_{II\theta}^2$ in the left member of equation (8), which is indicative of the loading energy, may be obtained via equations (2) and (3) respectively. On the other hand, the right member parameter reflects the fracture energy, which is a material property (for simplicity of reference, the indicative energy terms on the left and right members of equation (8) will hereinafter be abbreviated as "loading energy" and "fracture energy", respectively). In determining the mixed mode fracture toughness, K_C, it is evident that the limiting condition of pure mode I fracture along the generalised θ_C plane - for which $K_{II\theta}=0$ and therefore

$$K_{I\theta} = K_{IC} \tag{9}$$

- and that of pure mode II fracture along the same plane - for which $K_{I\theta}=0$ and hence

$$K_{II\theta} = K_{IIC} \tag{10}$$

- would have to be satisfied initially. Thereafter, an appropriate variation of K_C would have to be prescribed between the two limiting conditions. One possible approach would be to convert the component pure mode "loading energy" terms of equation (8) into equivalent mixed mode "loading energy", in direct proportion to their respective "fracture energies", that is

$$K_{I\theta}^2 + K_{II\theta}^2 = K_{I\theta}^2\frac{K_C^2}{K_{IC}^2} + K_{II\theta}^2\frac{K_C^2}{K_{IIC}^2} \tag{11}$$

Hence, in view of equation (11), the mixed mode fracture criterion would be defined as

$$\left(\frac{K_{I\theta}}{K_{IC}}\right)^2 + \left(\frac{K_{II\theta}}{K_{IIC}}\right)^2 = 1 \tag{12}$$

(vide Fig. 2). Furthermore, since K_{I0} and K_{II0} would be known quantities, it would only be necessary to determine the fracture angle, θ_C, in order to establish the point of fracture on the corresponding envelope. This may be achieved by by maximising the loading energy, a detailed account of which is provided elsewhere (Lo et al., 1996a). However, notwithstanding the foregoing derivation, Lo et al. also proposed a more generalised form of the fracture envelope which would, in principle, cater for practically any form of brittle material behaviour. Nevertheless, it is noteworthy that the unified fracture envelope of Fig. 2 will be shown, in the following discussion, to satisfy as widely differing materials as brittle clay and aluminum alloy.

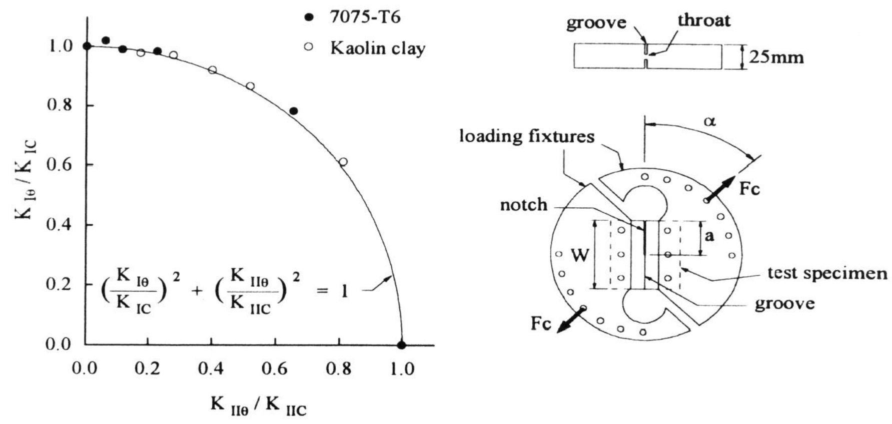

Fig. 2 Unified envelope for 7075-T6 aluminium and brittle kaolin clay.

As a final point, equation (8) indicates that for generalised mixed mode fracture to occur along an arbitrary plane, θ_C, from the crack tip,

$$K_\theta = K_C\ , \tag{13}$$

where

$$K_\theta = \pm\sqrt{K_{I\theta}^2 + K_{II\theta}^2} \tag{14}$$

Thus, K_θ has the connotation of a mixed mode stress intensity factor from the point of view of its attaining the fracture toughness, K_C, during mixed mode fracture. However, K_θ does not comply with the other characteristic requirement of a stress intensity factor, that is as a means by which the magnitudes of near field stresses may be determined by simple factoring of the terms in r and θ of their respective standard expressions (to do this, it would be necessary to employ $K_{I\theta}$ and $K_{II\theta}$ instead). Nevertheless, since, as indicated above, K_θ does have the attributes of a stress intensity factor from the standpoint of its corresponding fracture criterion, it would be appropriate to refer to it as the *unified, equivalent stress intensity factor*.

LABORATORY TESTING

In order to provide the necessary evidence for the unified notion of true shear and mixed mode fracture - as opposed to reports of conventional tests on shear and mixed mode "fracture", which were considered to have been opening modes of fracture generally (Lo et al., 1996a), the set-up proposed by Richard et al. (1986) for shear testing of aluminium alloy was adapted to study the propagation of true shear and mixed mode fractures along the θ_{0C} plane of compact test specimens of brittle kaolin clay and 7075-T6 aluminium of differing configuration, respectively. Accordingly, each specimen was subjected to a pair of co-linear, equal and opposite forces whose line of action was orientated at angle α (vide Fig. 2) to the crack plane, via the two adjacent arms of a loading fixture, such that the pure shear loading of Richard et al. - as referred to the θ_0 plane -

was effectively recovered when $\alpha=0$. However, it was surmised that there would be a general tendency for a K_{IC} fracture to propagate along the $\theta_C=\theta_{Imax}$ plane where $K_{I\theta}$ reached a maximum value of K_{Imax}, in test specimens of uniform thickness which were subjected to either pure K_{II0} or mixed K_{I0} and K_{II0} loading, in preference to either a true K_{IIC} or K_C fracture (Lo et al., 1996b). Thus, it was decided to form a pair of side-grooves to span from the notch to the opposite side of the specimen, on both faces, while keeping to the direction of the notch, in order to provide a throat segment of sufficient narrowness to activate and guide the propagation of the designated true fracture - of the latter two - along the θ_{0C} plane.

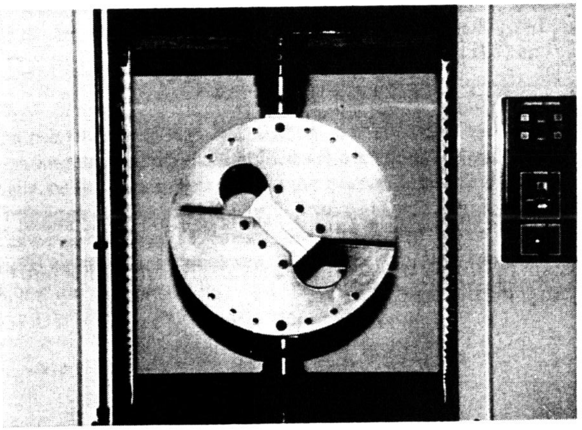

Fig. 3 Test set-up for mixed mode loading.

The test set-up (vide Fig. 3) and procedure for the aluminium alloy specimens are summarised hereafter, while the adaptations needed to deal with the much less tough kaolin clay specimens have been reported elsewhere (Lo et al., 1995). Accordingly, each aluminium alloy specimen was first machined to dimensions of 120mm by 132mm, and then two parallel lines of holes - each three in number, into which steel loading pins were to be slotted via matching holes in the loading fixture - were drilled into the test specimen on either side of a central crack starter notch - which was formed subsequently by electro-discharge machining using a wire cutter of 0·25mm diameter. The specimen was next pre-cracked by a sinusoidal load of 4 kN amplitude superimposed on a static load of 12 kN, which were applied via the Instron 1603 electro-magnetic resonance machine, while the test set-up was operating at an average resonant frequency of about 110 Hz. After 240K cycles, the length of pre-crack obtained was approximately 10 mm. Thereafter, the specimen side grooves, which were so aligned as to ensure that the pre-crack would be contained within the throat segment in the vicinity of the notch, were formed by the same wire cutter as above. The test specimen was then bolted to the two arms of the loading fixture within which it was seated (and which was similar to that proposed by Richard et al., except that it was fabricated in steel) and then subjected to mixed K_{I0} and K_{II0} loading in general. This was done by connecting the lower arm of the loading fixture to the fixed bottom frame of the Shimadzu Autograph Model AG-A computerised universal testing machine - which had an overall loading range of 1-10000 kgf and accuracy of ±1% of the maximum value of each selected range, while the opposite end of the

upper arm was connected to the movable cross-head of the machine. During a test, the load was applied monotonically to failure at the relatively slow rate of 0·5 mm/min in order to minimise inertial effects, in the course of which the cross-head displacement of the test machine versus its applied load - including the peak load at failure - were recorded automatically. As a result, either a true shear or mixed mode fracture could be propagated right across the specimen at mid-section, starting from the fatigue pre-crack and guided by the pair of side-grooves. After the test, a layer of acrylic was applied to the fracture surface of the test specimen as a protective coating, prior to examination under the JEOL JSM-T330A scanning electron microscope, so as to determine the mechanism of fracture.

VERIFICATION OF UNIFIED MODEL

The depth of grooves required to develop a high enough stress concentration in the test specimen, so as to ensure that a true shear or mixed mode fracture would indeed be guided along the intervening throat segment, was estimated via K-calibration (Barsoum, 1976), in conjunction with a three-dimensional finite element analysis of the overall test set-up. Accordingly, the modes I and II stress intensity factors of the θ_0 plane were initially obtained for a variety of unit load orientation, notch length and ungrooved specimen thickness/groove throat segment width configurations, and corresponding K-calibration curves constructed for each configuration. Then, knowing the modes I and II fracture toughnesses from tests carried out in accordance with ASTM E399-83 and as indicated herein, respectively, the requisite throat width was determined as that for which equation (13) would be satisfied before $K_{Imax}=K_{IC}$ (noting that approximately pure mode I loading conditions were found to occur at the θ_{Imax} plane, that is $K_{II\theta}\approx 0$ there). For instance, for the value of K_{CR} (= K_{IC}/K_{IIC}) of 0·52 obtained for the aluminium alloy, the ratio of throat width "T_t" to ungrooved specimen thickness "t" of 0·12 was judged to be adequate, in that it was estimated that a high enough theoretical safety margin for either true shear or mixed mode fracture to occur along the θ_{0C} plane - in preference to an opening fracture at $\theta_C=\theta_{Imax}$ - would be ensured, even in the most intransigent case of loading - that is when $\alpha=0$.

In order to test the accuracy of the unified model, a series of experiments such as described in the foregoing discussion were carried out on the aluminium alloy as well as brittle clay specimens (the latter being tested in their equivalent set-up), in which, by appropriate side-grooving to ensure that the ratio of the maximum, unified mode I stress intensity factor K_{Imax} on the θ_{Imax} plane, to unified, equivalent stress intensity factor K_θ on the θ_0 plane, would be less than the K_{IC}/K_C value of the test material, it was found that either a true shear or mixed mode fracture would indeed be propagated along the θ_{0C} plane in close agreement with unified model prediction. The example of Fig. 3 is one of five mixed mode fracture tests carried out on the aluminium alloy, where the line of action of the applied forces was at $\alpha=45°$, a/W=0·58 and T_t/t = 0·12. For that case, a true mixed mode fracture was propagated along the θ_{0C} plane in preference to an opening fracture at $\theta_{Imax}=-34°$, as predicted by the model. On the other hand, when the throat segment was not sufficiently narrow, an opening fracture developed preferentially at $\theta_C \approx -34°$, as also confirmed by the dull finish of the fracture surface as well as the "equiaxed dimple" morphology of its corresponding fractograph obtained by scanning electron microscopy. Thus, the above experimental results may be taken to be a clear vindication of the unified model, in that they satisfy the particularly stringent test of pin-pointing the exact demarcation between an opening and true mixed mode fracture. Moreover, as shown in Fig. 2, where all the results of both pure and mixed mode fracture tests on brittle kaolin clay as well as aluminium alloy specimens have been superimposed on the fracture envelope originally proposed by Lo et al. (1996a), close agreement is apparent between the predicted and experimental results, which would appear to confirm the veracity of the corresponding simple conversion of pure modes I and II loading energies to mixed mode loading energy, in direct proportion to their respective fracture energies. It is also noteworthy that all the mixed mode test results plot within the zone of mixed mode fracture, while those of the corresponding pure mode fracture tests fit exactly with the fracture envelope, as required. Indeed, the experimental plots may be taken to constitute yet another rigorous test of the unified model, in that they imply a wide distribution over the fracture surface, while still in close proximity to it.

As further confirmation of the model, the true shear and mixed mode fractures obtained were verified by visual as well as fractographic examination. Accordingly, a true shear fracture was typified by a shiny appearance as well as the "elongated dimple" morphology of its corresponding fractograph. On the other hand, for a mixed mode fracture, a combination of the dull finish of an opening fracture and shiny appearance of a shear fracture was apparent, as to be expected. This was further reflected by a mixture of the "equiaxed" and "elongated dimple" morphologies of shear and opening fractures, respectively. Finally, as supporting evidence for the unified model, it was found that the systematic propagation of a true shear fracture in a brittle clay was closely predicted in terms of its fracture load versus the notch length-to-specimen width ratio (Lo et al 1995).

REFERENCES

American Society for Testing and Materials (1986). Standard test method for plane-strain fracture toughness of metallic materials. *ASTM E399-83,* Philadelphia.

Barsoum, R. S. (1976). On the use of isoparametric joint/interface element for finite element analysis. *Int. J. Num. Meth. Engrg,* 10, 25-37.

British Standards Institution. (1977). Methods of test for plane strain fracture toughness K_{IC} of metallic materials, *BS 5447:1977.*

Griffith, A. A. (1920). The phenomena of rupture and flow in solids. *Phil. Trans. R. Soc., Lond.*, A, 221, 163-198.

Irwin, G. R. (1957). Analysis of stresses and strains near the end of a crack traversing a plate. *ASME, J. Appl. Mech.*, 24, 361-364.

Lo., K. W., Tamilselvan, T., Zhao, M. M. and Chua, K. H. (1995). True K_{IIC} and mixed mode fracture tests on brittle soil. Submitted for publication in *ASTM, J. Test. Eval.*

Lo, K. W., Tamilselvan, T., Chua, K. H. and Zhao, M. M. (1996a). A unified model for fracture mechanics. *Engrg Fracture Mech.*, 54, 189-210.

Lo, K. W. and Tamilselvan, T. (1996b). The unified equivalent stress intensity factor K_θ. Submitted for publication in the *9th Int. Conf. Fracture.*

Richard, H. A., Tenhaeff, D. and Hahn, H. G. (1986). Critical survey of mode II fracture specimens. In: *The Mechanism of Fracture* (V. S. Goel, ed.), 89-95, ASM, USA.