THE EFFECT OF T STRESS ON THE NEAR TIP STRESS FIELD OF AN ELASTIC-PLASTIC INTERFACE CRACK

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ABSTRACT

Recently the authors have shown that the near tip stress field of a plastically dissimilar bi-material interface crack can be characterized by two parameters, J and M, where J is the J integral, M is a material mismatch constraint parameter. In the J-M formulation, small scale yielding conditions with zero T stress are enforced. In this study, the effect of T stress on the near tip stress field of the interface crack has been investigated. It is found that for a given material mismatch, the T stress shifts the near tip stress level of the interface crack up and down without significantly affecting the M. This observation suggests that the effect of T stress and material mismatch on the crack tip can be separated. Based on this observation, a J-Q-M formulation is presented.

KEYWORDS

Interface crack, geometry constraint, material mismatch constraint, J-Q-M formulation

INTRODUCTION

A number of studies have revealed that a crack between weld metal and heat-affected-zone in a weldment is in many cases the most critical one (Thaulow et al., 1994; Minami et al., 1993). These cracks can be usually represented by an interface crack with identical elastic constants. By using modified boundary layer (MBL) model with zero T stress, the authors have investigated the effect of material mismatch on the near tip stress field of the interface crack. It is shown that the near tip stress field can be characterized by two parameters, the J and a material mismatch constraint parameter M (Zhang et al., 1995a, 1996a). Applications of the J-M framework have been made (Zhang et al., 1995b, 1996b).

For an interface crack specimen, the near tip stress field is also influenced by the specimen geometry, crack size and loading mode, here referred to as geometry constraint. Therefore, it is necessary to study the effect of geometry constraint on the near tip field of the interface crack. Using the same numerical procedure as the previous study (Zhang et al., 1995a, 1996a) and the same MBL model but with varying T stress, it is found that the presence of T stress shifts up an down the stress level at the near tip of the interface crack and does not significantly disturb the mismatch effect. It is therefore suggested that the effect of the geometry constraint and material mismatch constraint can be approximately separated. Based on this observation, a J-Q-M formulation is presented.

NUMERICAL PROCEDURES

The same numerical procedures used in the previous study (Zhang et al., 1995) have been used. Fig. 1 shows the bi-material MBL model loaded by J and T,

$$
\begin{aligned}
u(r,\theta) &= K_I \frac{1+\nu}{E}\sqrt{\frac{r}{2\pi}}\cos(\frac{\theta}{2})(3-4\nu-\cos\theta) + T\frac{1-\nu^2}{E} r\cos\theta \\
v(r,\theta) &= K_I \frac{1+\nu}{E}\sqrt{\frac{r}{2\pi}}\sin(\frac{\theta}{2})(3-4\nu-\cos\theta) - T\frac{\nu(1+\nu)}{E} r\sin\theta
\end{aligned} \qquad (1)
$$

where $K_I = \sqrt{EJ/(1-\nu^2)}$ under plane strain condition, r and θ are the co-ordinates centred at crack tip with θ =0 corresponding to the interface. In Fig. 1 the material below the interface is called reference material and the material above is defined as mismatch material.

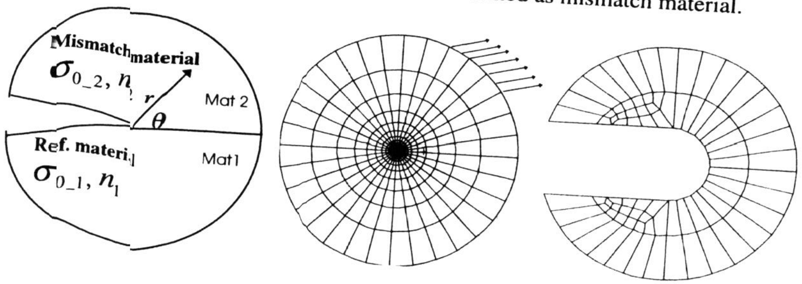

Fig. 1 Bi-material MBL model

The yield strength of the reference material is kept constant while the yield strength of the mismatch material is changing. A mismatch ratio of yield stress of the mismatch material to the reference material is introduced for convenience,

$$
m = \frac{\sigma_{0\_2}}{\sigma_{0\_1}}. \qquad (2)
$$

Following plastic hardening rule has been considered for both of the materials,

$$
\sigma_i = \sigma_{0\_i}(1+\frac{\bar{\varepsilon}_i^p}{\varepsilon_{0\_i}})^{n_i}, \qquad (3)
$$

where σ_i is the flow stress corresponding to the equivalent plastic strain $\bar{\varepsilon}_i^p$, σ_{0_i} is the yield stress, ε_{0_i} is the yield strain $\varepsilon_{0\_i} = \sigma_{0\_i}/E$, and n_i is the strain hardening exponent, of

material i. In this study only the results of the cases where both materials have the same hardening exponent $n_1 = n_2 = 0.1$ are reported. The elastic constants of the materials are $E = 500\sigma_{0\_1}$ and $\nu = 0.3$.

The finite element meshes in a global scale and at the crack tip are also shown in Fig. 1. There are 6383 nodes and 2066 8-node elements. Reduced integration scheme was used. The radius of the initial notch at the crack tip is 1×10^{-5} of the model radius. Although small scale yielding (SSY) conditions were enforced in the analyses, large deformation formulation has been used.

The virtual crack extension method in ABAQUS was used for the calculation of the J-integral. The computed J-integral is path-independent outside the finitely deformed zone at the crack tip. Similar observation has been reported by Aoki. The error between the computed J and the applied J at the boundary is less than 5%. Nevertheless, in the presentation of the results, the applied J at the boundary is used.

The T stress applied to the model varied from -0.6, -0.4, -0.2 to 0.5. Four mismatch ratios, m=0.75, 0.85, 1.25 and 1.5 have been analyzed.

THE EFFECT OF T STRESS ON THE HOMOGENOUS MATERIAL CRACK

O'Dowd and Shih (1991 and 1992) have investigated the T stress effect on the crack tip stress field. A MBL model with large deformation formulation was used. By evaluating the difference field $\sigma_{ij}^{Diff\_Q}$, between the full range SSY plane strain solutions, σ_{ij} and the solution with zero T, $\sigma_{ij}^{T=0}$,

$$
\sigma_{ij}^{Diff\_Q} = \sigma_{ij} - \sigma_{ij}^{T=0}, \qquad (4)
$$

they found that the difference field in the forward sector $|\theta| \le \pi/4$ of the annulus $J/\sigma_{0\_1} \le r \le 5J/\sigma_{0\_1}$ can be approximated by a hydrostatic field,

$$
\sigma_{ij}^{Diff\_Q} \approx Q\sigma_{0\_1}\delta_{ij} \qquad (5)
$$

where δ_{ij} is the Kroneker delta and the dimensionless parameter Q is a measure of the constraint level at the crack tip.

To a more general approximation, the difference field can be expressed,

$$
\sigma_{ij}^{Diff\_Q} \approx Q_{ij}(R)\sigma_{0\_1} f_{ij}^Q(\theta) \qquad (6)
$$

where $R = r/(J/\sigma_{0\_1})$, $Q_{\theta\theta}(R)$ and $Q_{rr}(R)$ are the values of the corresponding difference stress components at θ =0, $Q_{r\theta}(R)$ is the absolute maximum of the shear component of the difference field in the range -60° < θ < 0° (because the value at θ =0 is zero) and $f_{ij}^Q(\theta)$ are the normalized angular functions of the difference field with respect to $Q_{ij}(R)$.

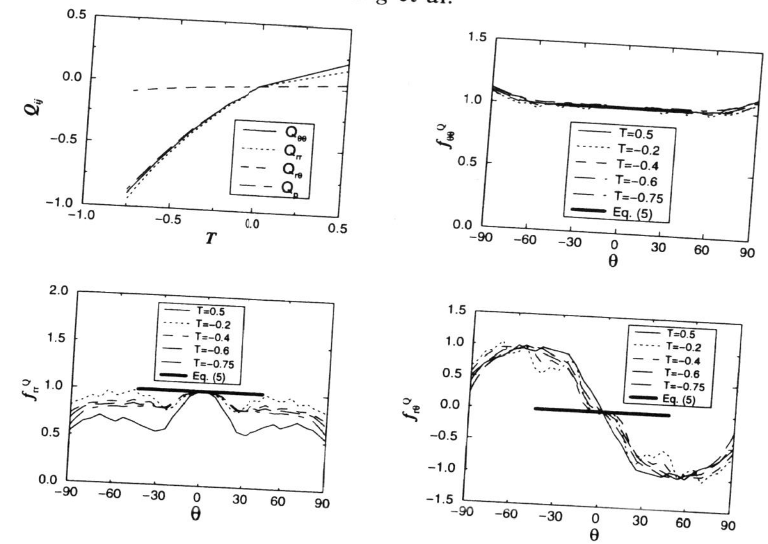

Fig. 2 a) Q_{ij} as a function of T stress, b), c) and d) $f_{ij}^{Q}(\theta)$ for different T stress

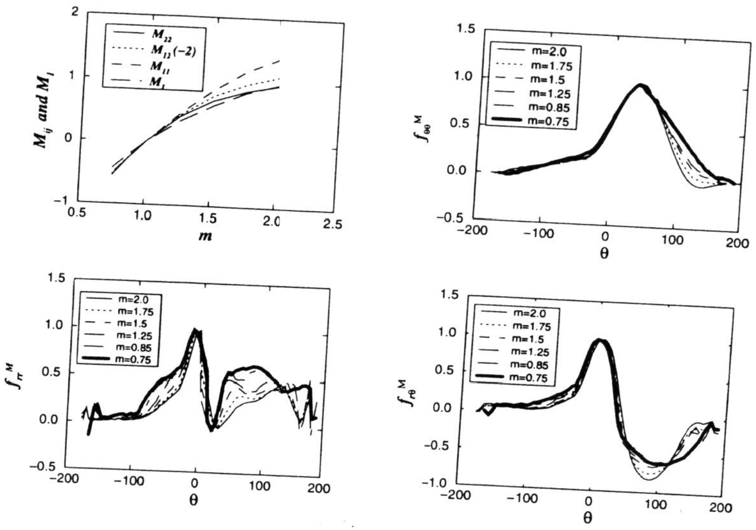

Fig. 3 a) M_{ij} as a function of mismatch ratio m, b), c) and d) $f_{ij}^{M}(\theta)$ for different mismatch ratios m.

$Q_{ij}(R=2)$ as a function of the T stress and the corresponding $f_{ij}^{Q}(\theta)$ are shown in Fig. 2. In order to compare, Eq. (5) is also shown in Fig. 2. From Fig. 2 we observe that the absolute value of Q_{rr} is larger than that of $Q_{\theta\theta}$ for a negative T. The ratio of $Q_{rr}/Q_{\theta\theta}$ is about 1.1 at R=4 and 1.06 at R=2 for the material with moderate hardening exponent n=0.1 and negative T. It can be seen from Fig. 2 that $\sigma_{\theta\theta}^{Diff\_Q}$ can be better approximated by Eq. (5) in the limited range than $\sigma_{rr}^{Diff\_Q}$ for the positive T and large negative T cases. By neglecting the shear component, we may rewrite the Eq. (6) such as

$$\sigma_{ij}^{Diff\_Q} \approx Q\sigma_0 \tilde{f}_{ij}^{Q}(\theta), \tag{7}$$

where $\tilde{f}_{\theta\theta}^{Q}(\theta) = f_{\theta\theta}^{Q}(\theta)$, $\tilde{f}_{rr}^{Q}(\theta) = \alpha f_{rr}^{Q}(\theta)$, $\tilde{f}_{r\theta}^{Q}(\theta) \approx 0$, α is a constant. Eq. (7) is used in the J-Q-M formulation as follows.

INTERFACE CRACK TIP FIELD WITH ZERO T STRESS

Considering the bi-material MBL model shown in Fig. 1 with zero T stress, the authors have studied the difference field, $\sigma_{ij}^{Diff\_M}$, between the interface crack tip field, σ_{ij}, and the SSY solution of the reference material, $\sigma_{ij}^{M=0}$ (Zhang et al. 1995a, 1996a),

$$\sigma_{ij}^{Diff\_M} = \sigma_{ij} - \sigma_{ij}^{M=0}, \tag{8}$$

where $\sigma_{ij}^{M=0}$ is equal to $\sigma_{ij}^{T=0}$ in Eq. (4).

It is found that the difference field is slightly dependent on the normalized radial distance to the crack tip (see next section) and can be approximated by the following expression,

$$\sigma_{ij}^{Diff\_M} \approx M_{ij}(R)\sigma_{0\_1} f_{ij}^{M}(\theta + 12\beta) \tag{9}$$

where β=0 for $m \geq 1$ (overmatch) and β=1 for m<1 (undermatch). In Eq. (9), M_{ij} is the maximum amplitude of a difference field measured in the reference material side except $\sigma_{\theta\theta}$ component, and $f_{ij}^{M}(\theta)$ are the normalized angular functions of the difference field by M_{ij}. M_{ij} as a function of m is shown in Fig. 3a and $f_{ij}^{M}(\theta)$ in the rest of Fig. 3. Fig. 3a shows that the absolute value of M_{rr} is larger than that of $M_{\theta\theta}$, and the absolute value of $M_{r\theta}$ is nearly half of that of the normal component, $M_{\theta\theta}$. Based on this observation and by neglecting the radial dependence, we can rewrite Eq. (9)

$$\sigma_{ij}^{Diff\_M} \approx M\sigma_{0\_1} \tilde{f}_{ij}^{M}(\theta + 12\beta) \tag{10}$$

where $\tilde{f}_{rr}^{M}(\theta) \approx 1.15 f_{rr}^{M}(\theta)$, $\tilde{f}_{\theta\theta}^{M}(\theta) = f_{\theta\theta}^{M}(\theta)$ and $\tilde{f}_{r\theta}^{M}(\theta) = -0.5 f_{\theta\theta}^{M}(\theta)$ for the present materials.

THE EFFECT OF T STRESS ON THE INTERFACE CRACK TIP FIELD

Radial Dependence

The T stress effect on the near tip stress field of the interface crack is studied. In studying the radial dependence of the difference field, the following difference field is evaluated,

$$\sigma_{ij}^{Diff} = \sigma_{ij}^{M,T} - \sigma_{ij}^{M=0,T=0}. \tag{11}$$

The difference field by Eq. (11) includes both the effect of T stress and material mismatch. The radial dependence of the difference field $\sigma_{\theta\theta}^{Diff}$ for the cases with m=1.5 at θ=-2.38° and -45° (both in the reference material) are displayed in Fig. 4. It can be seen that the difference field, $\sigma_{\theta\theta}^{Diff}$, is very weakly dependent on the normalized radial distance to the crack tip in the range of T stress analyzed. Similar behaviour can be seen for other stress components at other angles and mismatch conditions.

Angular Dependence

In examining the angular dependence, the difference field due to material mismatch,

$$\sigma_{ij}^{Diff} = \sigma_{ij}^{M,T} - \sigma_{ij}^{M=0,T}. \tag{12}$$

has been used. In the following, we shall examine whether the mismatch effect on the crack tip stress field is disturbed by the T stress. The normalized angular functions of the difference field are plotted as functions of the T stress in Fig. 5 for the cases m=0.85. Similar behaviour can be seen for other mismatch cases. We observe that the T stress does not significantly affect the normalized angular functions. The T stress effect on the maximum amplitudes used for the normalization, M_{ij}, are displayed in Fig. 6. For the purpose of comparison, the results for T=0 are also included in figures. It can be seen that the T stress does not significantly affect the M_{ij}, especially M_{rr}.

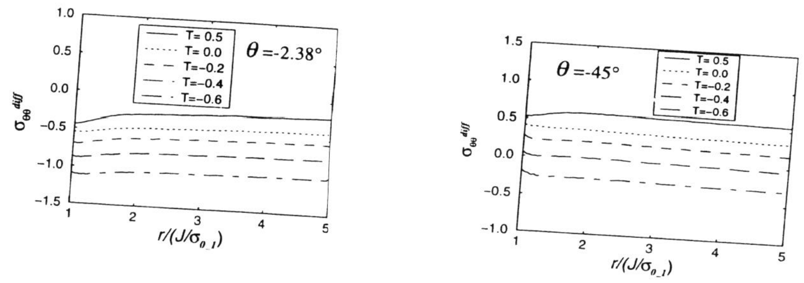

Fig. 4 Radial dependence of the difference field by Eq. (11).

J-Q-M FORMULATION

The above results have demonstrated that the difference field in the presence of both T stress and material mismatch has a minor dependence on the normalized radial distance to the crack tip. The T stress does not significantly affects the difference field caused by material mismatch. The later observation suggests that the effect of T stress and material mismatch can be separated. Combing Eq. (7) and (10), the near tip stress field of the interface crack with non-zero T stress can be approximated

$$\sigma_{ij} \approx \sigma_{ij}^{M=0,T=0} + Q\sigma_{0\_1}\tilde{f}_{ij}^{Q}(\theta) + M\sigma_{0\_1}\tilde{f}_{ij}^{M}(\theta + 12\beta). \tag{13}$$

It should be reminded that the range with 0.75 ≤ m ≤ 1.5 and -0.6 ≤ T ≤ 0.5 have been analyzed, and therefore Eq. (13) applies for that range. For the same reason as discussed by Zhang et al. (1995a), Eq. (13) is most accurate in the reference material side -45° ≤ θ ≤ 0°.

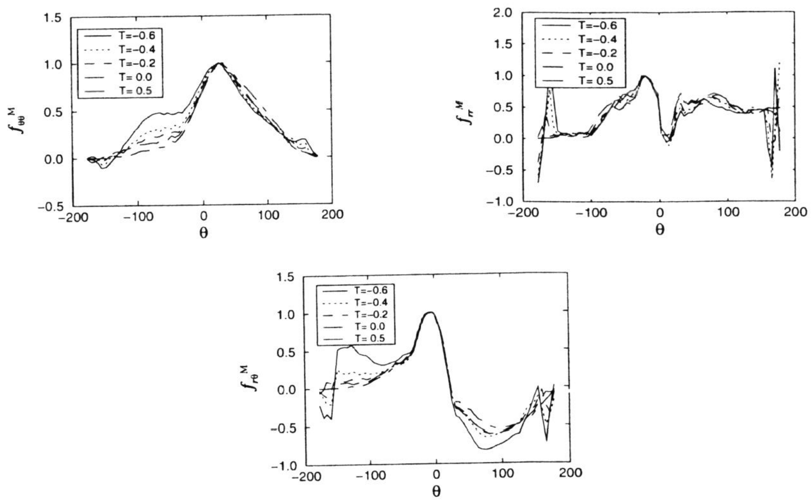

Fig. 5 T stress effect on the normalized angle functions $f_{ij}^{M}(\theta)$ for the cases with m=0.85.

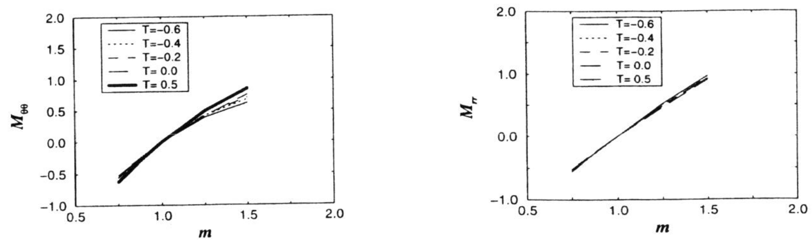

Fig. 6 T stress effect on the $M_{\theta\theta}$ and M_{rr}

CONCLUDING REMARKS

Two parallel theories have been proposed for characterizing the constraint effect on the near tip stress field, with J-Q for geometry constraint and J-M for material mismatch constraint . In this paper, it has been found that in the mixture of the two constraints, the T stress does not significantly disturb the difference field caused by material mismatch. This observation implies that the effect of material mismatch and T stress is separable. A three parameter (J-Q-M) characterization of the stress field is proposed. Further study on the J-Q-M formulation as well as its application to bi-material interface crack specimen will be carried out in the near future.

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