A METHOD TO DETERMINE CONSTITUTIVE PROPERTIES OF THIN INTERFACE LAYERS LOADED IN SHEAR

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ABSTRACT

A method to determine constitutive properties of thin interface layers loaded in shear is presented. To avoid instability a single-sided double-lap shear specimen is designed so that the stress distribution in the thin layer is non-uniform. The method is based on an exact inverse solution and is intended for determination of both hardening and softening behaviour of adhesives. The method is confined to monotonic loading.

KEYWORDS

Thin layer, adhesive, test method, constitutive behaviour, strain softening, inverse solution

INTRODUCTION

Adhesive bonding of structural components is a joining method which is attractive as an alternative or as a complement to more conventional methods like spot welding, riveting or bolting. The advantage of adhesive joints is that they introduce less stress concentration in the components to be joined than the conventional methods. In most applications the adhesive is much more compliant and has a much lower strength than the parts joined by the adhesive. Thus failure of adhesive joints often takes place by cohesive fracture in the adhesive or by adhesive fracture at the interface between the adhesive and the adherends. The strength of an adhesive joint depends, however, not only on the strength of the adhesive. The stress distribution in the adhesive will depend highly on the geometry of the joint. A lot of work, both theoretical and experimental, has been done regarding the strength of a number of specific joints, see the review in (Adams and Wake, 1984). The maximum strength is achieved if the joint is designed so that the stress distribution in the adhesive is uniform. A common experience is that the adhesive should be loaded in shear rather than in tension (peel) for the design to be successful. Results from tests performed for a specific joint geometry are, however, difficult to translate to a joint having a different geometry. The lack of reliable methods to dimension adhesive joints of arbitrary geometry is probably an important reason why adhesives have not been used up to their full potential. Kinloch (1987), for example, identifies a need to model more accurately the complete stress distributions in complex joint geometries. This requires a detailed knowledge
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This relation is to be determined from measurements. It is assumed that the adhesive layer is homogeneous, that is the constitutive relation (1) is assumed to be valid along the entire adhesive layer. It is also assumed that $q(u) = ku$ for $|u| < M$, where $k$ and $M$ are positive numbers. Examples of typical adhesive constitutive relations are shown in Fig. 2a. For the system to be in equilibrium the following differential equation must be satisfied along the system (assuming the tensile stiffness $EA$ to be independent of $x$),

$$EA \frac{d^2u}{dx^2} - q(u) = 0,$$

$$\frac{1}{EA} = \frac{2}{EA_1} + \frac{1}{EA_2}$$

(2a,b)

The boundary conditions are

$$\frac{du}{dx} \bigg|_{x=0} = -\frac{F}{EA}, \quad \frac{du}{dx} \bigg|_{x=L} = 0$$

(3a,b)

It is evident from the equations above that the original system is equivalent to a system consisting of a single bar with tensile stiffness $EA$ loaded by the force $F$ and connected to a rigid foundation via one of the adhesive layers of the original system. This equivalent system is shown in Fig. 1b. For a general constitutive relation $q(u)$ equations (1) and (2) together with the boundary conditions (3a,b) constitute a non-linear boundary value problem. In the following section an inverse formula for the experimental determination of $q(u)$ is given. The development of the stress distribution including the issue of monotonic loading is also discussed in the following.

INVERSE FORMULA FOR THE CONSTITUTIVE RELATION

Since, in general, the stress distribution in the adhesive layer of the test specimen will be non-uniform, it may not seem obvious how the constitutive relation $q(u)$ can be determined by performing measurements on the present test specimen. An inverse solution for the constitutive relation can, however, be derived by multiplying equation (2a) with $du/dx$ and integrating along the system,

$$\int_0^L EA \frac{d^2u}{dx^2} \frac{du}{dx} dx = \int_0^L q(u) \frac{du}{dx} dx$$

(4)

The integral on the left hand side may be evaluated directly which leads to

$$\left[ \frac{EA}{2} \left( \frac{du}{dx} \right)^2 \right]_0^L = \int_0^L q(u) \frac{du}{dx} dx$$

(5)

By introducing $\delta = u(0)$ and $\delta = u(L)$ as the relative displacement at the left and right adhesive boundaries, respectively, and using the boundary conditions (3a,b), equation (5) may be written

$$\frac{F^2}{2EA} = \int_0^\delta q(u) \frac{du}{dx}$$

(6)

The assumption of a long specimen ($L \to \infty$) implies that $\delta = 0$. Thus, for a long specimen equation (6) reads

$$\frac{F^2}{2EA} = \int_0^\delta q(u) \frac{du}{dx}$$

(7)
If the constitutive law is known this equation provides a relation between the force $F$ and the relative displacement $\delta$ at the point of load application. In general numerical models of $q(u)$ explicit relations may be obtained. It displacement $\delta$ will increase monotonically with $F$ (and vice versa for a prescribed displacement and $\delta$ of course obtained.

$$F = \sqrt{k E A \delta}$$

(8)

Figure 2b shows the force displacement relation for the adhesive types shown in Fig. 2a. If formula for the constitutive relation is obtained,

$$q(\delta) = \frac{F}{E A} \frac{dF}{d\delta}$$

(9)

This is the key equation of the present paper. The constitutive relation may be used of this equation. An inverse solution, of the same form as the one given here, has previously been presented.

The physical meaning of equation (9) is obvious, but equations (6) and (7) may be explained in terms of energy release rates and use of the $J$-integral (Eschley, 1953 and Rice, 1968),

$$J = \int_{\Gamma} \left( W \, dy - T \cdot \frac{\partial u}{\partial x} \, ds \right)$$

(10)

where $\Gamma$ is a curve surrounding the crack tip and taken in a counter-clockwise sense, $W$ is the strain energy density, $T$ is the traction vector defined according to the outward normal along the crack and $u$ is the displacement vector. It should be noted that the $J$-integral in equation (10) is evaluated as the total energy release rate. For this purpose half of the symmetric shear specimen

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x}$$

(12a,b)

containing one adhesive layer is studied, see Fig. 3. Evaluation of the $J$-integral along the two curves $\Gamma_1$ and $\Gamma_2$ yields the following values,

$$J_1 = \frac{F^2}{2EA} + \int_0^L q(u) du$$

$$J_2 = \int_0^L q(u) du$$

(11a)

Path independence of $J$ implies that $J_1 = J_2$, which is equivalent to equation (6). Here $J_a$ may be interpreted as the elastic energy released by the system for a unit increase of the "crack length" $a$ in Fig. 3. Further, $J_b$ is the energy consumed by the adhesive layer for a unit increase of $a$. This is the physical explanation of equation (6) from which equation (7) and subsequently the inverse formula (9) are deduced.

In the derivations performed above it has been assumed that the constitutive relation is valid both at loading and unloading. Thus the adhesive is assumed to be non-linearly elastic in the general case. It is not plausible that an adhesive will behave the same in unloading as in loading. Due to damage and plasticity unloading follows a different path. The present equations will, however, be valid also for a material that as long as the entire adhesive layer is loaded monotonically. In the next section, where the stress distribution in the layer is studied in detail, it is shown that for a monotonically increasing load $F$ (or displacement $\delta$) the entire adhesive layer will be loaded monotonically.

STRESS DISTRIBUTION IN THE ADHESIVE LAYER

Even though the stress distribution in the adhesive layer is governed by a non-linear boundary value problem the relation between the applied load $F$ and the relative displacement $\delta$ can be determined quite easily, see equation (7). In fact, the stress distribution in the layer can be determined in a similar manner if the constitutive relation is known. The procedure of multiplying the governing equation (2a) with $du/dz$ and integrating along the system, used in the previous section, is used here as well. Instead of integrating along the entire system, however, the integration is performed from an arbitrary coordinate $x$ to the right end of the specimen. By use of the boundary condition (3b) the resulting equation for a long specimen ($x = 0$)

$$\frac{du}{dx} = \frac{2H(u)}{EA}$$

$$H(u) = \int_0^\infty q(\tilde{u}) d\tilde{u}$$

(12a,b)

Figure 3: Paths for evaluation of $J$-integrals. Upper half of shear specimen is studied.
where \( H \geq 0 \) represents the strain energy per unit length of the adhesive at the position \( x \). Taking the square root of equation (12a) leads to

\[
\frac{du}{dx} = -\sqrt{\frac{2H(u)}{EA}}
\]

where the sign is given by physical reasons explained later in this text. This equation expresses a local relation between the gradient of the relative displacement and the strain energy density of the adhesive. The equation may be separated and integrated from the point of load application to an arbitrary point \( x \). This leads to the following implicit relation between the relative displacements \( u(x) \) and \( \delta \).

\[
x = \sqrt{\frac{EA}{2}} \int_{u(0)}^{u(x)} \frac{d\delta}{\sqrt{H(\delta)}}
\]

(14)

For a linear elastic adhesive as in equation (8) this equation gives an exponentially decaying displacement field,

\[
u(x) = \delta e^{-\sqrt{\frac{EA}{2x}}}
\]

(15)

as is to be expected. This shows that the negative sign in equation (13) is correct for small loads and that both \( u \) and \( q \) are positive along the entire system for a linear elastic adhesive. In order to show that the negative sign in equation (13) is correct for a non-linear adhesive it is first assumed that \( u > 0 \) for all \( x \). This is obviously true at the onset of non-linear behaviour. From \( u > 0 \) it follows that \( q \geq 0 \). Furthermore from equation (2a) \( d^2u/dx^2 \geq 0 \), that is \( du/dx \) is increasing with \( x \). From the boundary conditions \( (3a,b) \) it follows that \( du/dx \) is non-positive and increasing with \( x \). Thus, for \( u > 0 \) the negative sign in equation (13) is correct for a non-linear constitutive relation. Evaluation of equation (14) for two different values of \( \delta (\delta < \delta_0) \) yields two different values of the relative displacement \( (u, u_a) \) for a given position \( x \). After some manipulations this gives

\[
\int_{u_a}^{u} \frac{d\delta}{\sqrt{H(\delta)}} = \int_{u_a}^{u_a} \frac{d\delta}{\sqrt{H(\delta)}}
\]

(16)

This equation shows that an increase of the prescribed displacement \( \delta \) leads to an increase of the displacement \( u(x) \) at every point of the system \( (u_a, u_a) \). This implies further that, when the linear relation is no longer valid, the relative displacement \( u(x) \) is still positive everywhere. According to the previous explanation this means that the sign chosen in equation (13) is correct also for a non-linear constitutive relation. Accordingly the entire adhesive layer is loaded monotonically.

The development of the stress distribution is remarkably simple. By use of equation (14) it may be shown that the distance \( \Delta x \) between two points where the relative displacement is \( u_a \) and \( u_a \), respectively, is independent of the prescribed displacement \( \delta \) and given by

\[
\Delta x = x_a - x_a = \sqrt{\frac{EA}{2}} \int_{u_a}^{u_a} \frac{d\delta}{\sqrt{H(\delta)}}
\]

(17)

This means that the displacement field and the stress distribution will translate undistorted in the positive \( x \)-direction as \( \delta \) is increased. The translational velocity of the distributions is found from equation (14) as

\[
x = \sqrt{\frac{EA}{2H(\delta)}}
\]

(18)

where a dot denotes time-differentiation. Figure 4 shows the development of the stress distribution for one of the adhesive types shown in Fig. 2a. The figure shows the stress distribution for situations at constant increments of the prescribed displacement \( \delta \). This corresponds to a decreasing translational velocity \( x \), since \( H(\delta) \) is an increasing function. The rightmost curve in Fig. 4 depicts the situation when the adhesive at \( x = 0 \) is unable to carry any further load. For a further increase of \( \delta \) the "crack length" \( a \) (see Fig. 3) will increase with a speed proportional to \( \delta \). The stress distribution will remain the same as in Fig. 4 but it will translate together with the "crack tip".

DISCUSSION

In a physical realization of the method proposed in this paper it is essential to assure that the assumptions made in the analysis are fulfilled. Beside the requirements that the bars behave elastically and that the specimen is sufficiently long it is also important that the design ensures that symmetry prevails and that bending of the bars is not introduced at the points of load application. The procedure will require that displacements of small magnitude are determined with good enough accuracy so that the measured relation between the applied force \( F \) and the displacement \( \delta \) can be differentiated with sufficient accuracy. It should also be noted that in its present form the method is not suited for determining unloading characteristics of adhesives. The main advantage of the method is that instability is avoided because of the non-uniform stress distribution. This is believed to be essential when the softening behaviour of an adhesive is determined.
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REFERENCES


