ON CREEP DAMAGE EVOLUTION IN THIN PLATES

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ABSTRACT

Thin–walled structures (plates, shells, tubes etc.) made from metals or alloys are showing typical creep behaviour under moderate loads at a high temperatur (below 0.3 - 0.4 of the melting temperature of the material). For estimation of the operating time or repair cycles it is necessary to take into account the creep behaviour and the damaging as first step of the formation of cracks (Continuum Damage Mechanics approach). Using the flow theory of creep and a damage evolution equation based on the Lecky-Hayhurst-criterion the influence of the maximum principal stress on the damage development in rectangular plates is investigated. The limit state of the plates is defined by the critical damage parameter value. This value is connected with the coalescence of microdeffects in the material. So the limit state can be the starting point of the fracture mechanics analysis of the failure propagation.

KEYWORDS

Continuum damage mechanics, creep-damage, limit state, metals, alloys

INTRODUCTION

The creep in a metal or an alloy can be described by three models, connected with the existence of three different stages: primary, secondary and tertiary creep (Smith, 1990). This is an idealization of uniaxial creep curves (strain versus time), which slope can be used as a measure for the classification of the creep stages. After an instantaneous rapid elongation (elastic strain) we can obtain that the strain rate decreases with time (primary creep). Then we get a part of the creep curve in which the creep rate is approximatly constant (secondary or steady state creep). The third part of the creep curve is characterized by material softening and an acceleration of the creep process.

The acceleration of creep during the tertiary stage is often caused by coalescence of micro-cavities on grain boundaries. The creep fracture is therefore generally intergranular. The nucleation and growth of the micro-cavities is connected with the full creep life. But their effect on the creep rate during the primary and secondary creep is negligible (Riedel, 1987).

For engineering applications of creep models it is necessary to find a suitable phenomenological description. In creep mechanics are proposed several approaches (Skrzypek, 1993). One of the simplest is the flow theory based on the von Mises creep potential and Norton's creep law (Rabotnov, 1969). This theory ignore the primary and the tertiary creep stages and can be used only in the case of secondary creep. The creep law parameters can be identified by uniaxial tests.

The simplest extension of the flow theory based on the von Mises creep potential and Norton's creep law based on the introduction of a continuity (Kachanov, 1958) or damage (Rabotnov, 1959) parameter. With help of the principle of strain equivalence (Lemaitre, 1990) we can transform the constitutive equations for secondary creep into equations for the third stage. Till now the physical interpretation of this procedure is not clear, but the proposed theory gives adequate results with respect to experimental data.

The analysis of the time dependent behaviour of thin-walled structures is presented by different authors. Most of them investigate the creep behaviour of plates, shells etc. under stationary loads using constitutive equations for primary and secondary creep and geometricaly linear shell theory, sometimes extended to stability problems. The solution of plate creeping problems with respect to damage evolution processes was discussed, e.g., by Bodnar and Chrzanowski (1991, 1994). Here we evaluate the damage evolution on the upper and the lower side of the plate by an initial-boundary value problem, formulated for geometrically nonlinear plate bending (Naumenko, 1996). It was pressume that the damage accumulation is influenced by the maximum principal stress and the hydrostatic pressure (Hayhurst and Leckie, 1977). In the numerical examples the hydrostatic pressure dependence was neglected. The numerical results show significant difference of the damage evolution on the upper and lower side of the plate, which is important for the prediction of the rupture time.

GOVERNING MECHANICAL EQUATIONS

Constitutive equations for secondary creep and their extension to damage processes

In the case of small strains the total strain tensor can be additive splitted into an elastic part ε^{el} and an irreversible creep part ε^{cr}

$$\varepsilon = \varepsilon^{el} + \varepsilon^{cr} \tag{1}$$

Assuming the existence of a creep potential Ω, the constitutive equation for the creep strain rate tensor can be expressed in form of the normality rule (Lemaitre and Chaboche, 1990)

$$\dot{\varepsilon}^{cr} = \frac{\partial \Omega}{\partial \sigma}, \tag{2}$$

where σ is the stress tensor. In the case of isotropic material behaviour the potential Ω depends on three invariants of the stress tensor, temperature and internal state variables (Rabotnov, 1969). Assuming incompressibility and independence of secondary creep on the

kind of loading (tension, compression etc.), we can introduce the potential Ω as a function of the von Mises equivalent stress

$$\sigma^{vM} = \sqrt{\frac{3}{2} s \cdot\cdot s}, \qquad s = \sigma - \frac{1}{3} J_1(\sigma) I, \tag{3}$$

where s is the deviatoric part and $J_1(\sigma)$ is the first invariant of the stress tensor. Then the constitutive equation for the creep strain rate tensor can be written as

$$\dot{\varepsilon}^{cr} = \frac{3}{2} \frac{\partial \Omega(\sigma^{vM})}{\partial \sigma^{vM}} \frac{s}{\sigma^{vM}} \tag{4}$$

In the case of Norton's creep law the potential Ω and the Eq. (4) can be expressed as

$$\Omega = \frac{a}{n+1} (\sigma^{vM})^{n+1}, \qquad \dot{\varepsilon}^{cr} = \frac{3}{2} a (\sigma^{vM})^{n-1} s \tag{5}$$

where a, n are the material parameters, defined by uniaxial tests at a constant temperature.

The damage state of the material can be described by the introduction of a continuous mechanical internal variable. A simple definition of a continuity or damage parameter was given by Kachanov, 1958 and Rabotnov, 1969. It is based on the concept of the reduction of the cross-section area of a specimen. The constitutive equation for creep strain rate has been formulated for one dimensional case as the modified Norton's creep law and the damage rate was postulated as a function of stress and of actual damage state. The extension to multiaxial stress states was considered by Hayhurst and Leckie (1977). The damage evolution is expressed in terms of invariants of the stress tensor. The following extended constitutive equations are used here

$$\dot{\varepsilon}^{cr} = \frac{3}{2} F(\sigma^{vM}) K(d) \frac{s}{\sigma^{vM}}, \qquad \dot{d} = R(d) H\left[\langle \chi(\sigma) \rangle\right] \tag{6}$$

$$\varepsilon^{cr}|_{t=0} = 0, \quad d|_{t=0} = 0, \quad 0 \le d \le d_* \tag{7}$$

$$\langle \chi(\sigma) \rangle = \begin{cases} \chi(\sigma), & \chi(\sigma) > 0 \\ 0, & \chi(\sigma) \le 0 \end{cases} \tag{8}$$

$$\chi(\sigma) = \alpha \sigma_1 + \beta J_1(\sigma) + (1 - \alpha - \beta) \sigma^{vM}, \tag{9}$$

where d is a scalar damage variable, d_* is a critical damage value, $\chi(\sigma)$ is a multiaxial damage criterion with σ_1 as the maximum principal stress. The material parameters α, β denote the material sensitivity to the failure mode. The functions F, K, R, H must be identified from uniaxial creep experiments. A suitable approximation for metalls and alloys is (Moratschkowski and Naumenko, 1995)

$$F(\sigma) = a\sigma^n, \qquad K(d) = (1-d)^{-m}, \tag{10}$$

$$H(\sigma) = b\sigma^k, \qquad R(d) = (1-d)^{-l}, \tag{11}$$

with a, b, n, k, m, l as material parameters. The given Eqs. (6) represent mathematically a set of first order coupled differential equations with corresponding initial conditions (7). Their application to the analysis of structures leads to the formulation of an initial–boundary value problem. The damage parameter d is defined to be an unknown continuous function of coordinates with respect to the location of the material points of the analyzed structure. The time to failure can be estimated by the solution of the initial–boundary value problem as the time from the initial state (instantaneous elastic state) to the critical state characterized by the critical damage value d_*, which should be determined experimentally.

Initial-boundary value problem for rectangular plates

We consider a thin plate with reference to an orthogonal Cartesian coordinate system x_1, x_2, $x_3 \equiv z$, where x_1 and x_2 are coordinates of the plate middle surface and z denotes the transverse direction ($-h/2 \le z \le h/2$ with h as the plate thickness). It is assumed that the plate is uniformly heatet without thermal stresses and loaded with a stationary applied force q_z.

Using the Kirchhoff's hypotheses and assuming that the displacements of the middle surface u_1, u_2 are small in comparison with the deflections w, the strain–displacement relations can be expressed by (cp. Reismann, 1988)

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + w_{,i}w_{,j}) + z\mu_{ij}, \qquad \mu_{ij} = -w_{,ij}, \qquad i,j = 1,2, \tag{12}$$

where are ε_{ij} are the components of the strain tensor, μ_{ij} are the curvature changes and twist of the middle plane. The geometrically nonlinear terms in Eq. (12) denote the influence of finite deflections. Their consideration is necessary for an adequate prediction of creep deformation and failure time of thin-walled structures (Altenbach and Naumenko 1995).

The constitutive equations are written in the form of the Hooke's law

$$\sigma_{ij} = C_{ijkl}\left(\varepsilon_{kl} - \varepsilon_{kl}^{cr}\right), \tag{13}$$

where σ_{ij} are the components of the stress tensor. It will be assumed that the material is homogeneous and isotropic. Then the elastic material parameter tensor is determined as

$$C_{ijkl} = \frac{E}{2(1-\nu^2)}\left[(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})(1-\nu) + 2\nu\delta_{ij}\delta_{kl}\right], \tag{14}$$

where E is the Young's modulus, ν is the Poisson's ratio and δ_{ij} is the Kronecker's symbol. By integration of the Eq. (13) over the plate thickness, the constitutive relations for the membrane forces N_{ij} and the plate moments M_{ij} with the additional creep components N_{ij}^{cr}, M_{ij}^{cr} are obtained

$$N_{ij} = \frac{1}{2}g_{ijkl}^{I}(u_{k,l} + u_{l,k} + w_{,k}w_{,l}) - N_{ij}^{cr}, \qquad M_{ij} = g_{ijkl}^{II}\mu_{kl} - M_{ij}^{cr} \tag{15}$$

$$N_{ij}^{cr} = \int_h C_{ijkl}\varepsilon_{kl}^{cr}\,dz, \qquad M_{ij}^{cr} = \int_h C_{ijkl}\varepsilon_{kl}^{cr} z\,dz, \tag{16}$$

with $g_{ijkl}^{I} = C_{ijkl}h$, $g_{ijkl}^{II} = C_{ijkl}h^3/12$, $i,j,k,l = 1,2$. The quasistatic equilibrium equations of a differential element of the deformed middle surface can be written as

$$N_{ij,j} = 0, \quad Q_i = M_{ij,j}, \quad Q_{i,i} + w_{,ij}N_{ij} + q_z = 0 \tag{17}$$

where Q_i are the transverse shear forces.

On the plate boundaries (edges) the kinematic and (or) static boundary conditions must be satisfied

$$\begin{gathered} u_1|_\Gamma = u_1^*, \quad u_2|_\Gamma = u_2^*, \quad w|_\Gamma = w^*, \quad w_{,n}|_\Gamma = w_{,i}n_i = \vartheta^*, \\ N_{ni}|_\Gamma = N_{ij}n_j = N_i^*, \; M_n|_\Gamma = M_{ij}n_i n_j = M^*, \; Q_n|_\Gamma = Q_i n_i + (M_{ij}n_j\tau_i)_{,k}\tau_k = Q^* \end{gathered} \tag{18}$$

where n_i and τ_i are the components of the normal and tangent unit vectors to the plate boundary Γ and u_1^*, u_2^*, w^*, N_i^*, M^*, Q^* are given kinematic and static parameters. The initial–boundary value problem is completely defined by the set of Eqs. (12) – (18) with respect to the constitutive law (6) and the initial conditions. Setting $\varepsilon_{ij}^{cr} = 0$ the solution of the elastic problem follows from these equations.

NUMERICAL SOLUTION PROCEDURE

The solution of the creep problem is performed by time–step discretization. The coupled creep–damage equations (6) are sensitive to the time–integration procedure. In order to obtain the stable solution (especially for the tertiary creep state) the implicit Euler method is used. The numerical procedure is realized in the form of the fixed–point iteration scheme (cp. Lemaitre and Chaboche, 1990). The time step sizes are choosen in dependence on the convegence of the iterations. At each time (iteration) step the following variational problem with fixed creep load components must be solved

$$\begin{aligned} I(\varphi, w) \;=\; & \frac{1}{2}\int_A \left[-(g_{ijkl}^{I})^{-1}N_{ij}(\varphi)N_{kl}(\varphi) + g_{ijkl}^{II}\mu_{ij}(w)\mu_{kl}(w)\right] dA \\ & - \int_A \frac{1}{2}N_{ij}(\varphi)w_{,ij}w\;dA - \int_A \left[q_z w + (g_{ijkl}^{I})^{-1}N_{ij}^{cr}N_{kl}(\varphi) + M_{ij}^{cr}\mu_{ij}(w)\right] dA, \end{aligned} \tag{19}$$

with $N_{11}(\phi) = \phi_{,22}$, $N_{22}(\phi) = \phi_{,11}$, $N_{12}(\phi) = -\phi_{,12}$, where ϕ is the Airy's function and A is the plate area. The creep loads N_{ij}^{cr}, M_{ij}^{cr} are defined from creep strain and damage fields, which are assumed to be known at the time step t_i and must be determined from the previous time step t_{i-1}. Details of discretization procedures and results of the numerical testing as well as some examples of verification are discussed by Altenbach and Naumenko (1995), Moratschkowski and Naumenko (1995) and Naumenko (1996).

NUMERICAL EXAMPLE

We consider a clamped square plate under uniformly distributed load. The calculations are performed for the case $q_z = 0.3$ MPa, $h = 3\cdot10^{-3}$m, $l = 8\cdot10^{-2}$m with l as plate length. The material parameters are given for a nickel–based alloy (cp. Moratschkowski and Naumenko 1995): $a = 9.89\cdot10^{-10}\text{MPa}^{-n}$/h, $n = k = 2.4$, $m = l = 3.4$, $b = 5.45\cdot10^{-8}\text{MPa}^{-k}$/h. It is difficult to carry out the multiaxial creep experiments for the identification of the material constants α and β in all application cases. In order to show the sensitivity of the material behaviour and of the damage evolution in plates to the mode of failure the numerical analysis has been performed. It is assumed that the damage evolution of material is caused only by the maximum principal stress. Therefore the Eq. (9) is used by $\beta = 0$. By the integration of the damage evolution equation (6) the isochronous rupture surfaces can be obtained. Fig. 1 shows these surfaces by different values of $0 \leq \alpha \leq 1$ in the dimensionless coordinates σ_1/σ_0, σ_2/σ_0 with σ_0 as the uniaxial failure stress.

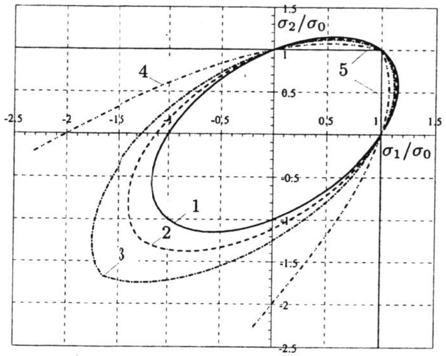

Fig. 1 Isochronous rupture surfaces: 1 – $\alpha = 0$; 2 – $\alpha = 0.1$; 3 – $\alpha = 0.2$; 4 – $\alpha = 0.5$; 5 – $\alpha = 1$

The simulation of time–dependent behaviour of plates was performed by different values of α. The results for the evolution of deflection, stress and damage fields are given by Naumenko, 1996. Here we show the distribution of the field damage parameter on the upper and lower side of the plate at the critical time t_*, Fig. 2. By $\alpha = 0$ the damage criterion (8) denotes the von Mises equivalent stress. The tensile and compressive stress states have the same influence on the damage evolution (Fig. 1). For the plate problem considered the bending stress state dominate and the small membrane stresses occur due to geometrical nonlinearity of the plate bending. Therefore the difference between the damage distributions on the upper and lower side of the plate is small, Fig. 2. By $\alpha > 0$ the damage evolution is influenced by the maximum principal stress that damage zones in the plate corresponds more to the tensile stress state. By $\alpha = 0.5$ the distribution of damage is completely nonsymmetrical across the plate thickness.

CONCLUSIONS

In the paper a model for numerical analysis of thin rectangular plates under stationary bending load is proposed. The phenomenological constitutive equations are formulated with the introduction of the scalar damage parameter to characterize tertiary creep. The corresponding initial–boundary value problem is formulated using the governing equations of the geometrically nonlinear plate theory. The numerical example shows the sensitivity of damage states in a square plate to the maximum principal stress. The maximum principal stress influences the significant difference of damage evolution by tensile and compressive loading.

The used Continuum Damage Mechanics approach allows a description of deterioration processes in the material such as micro–crack initiation by multiaxial stress states. The

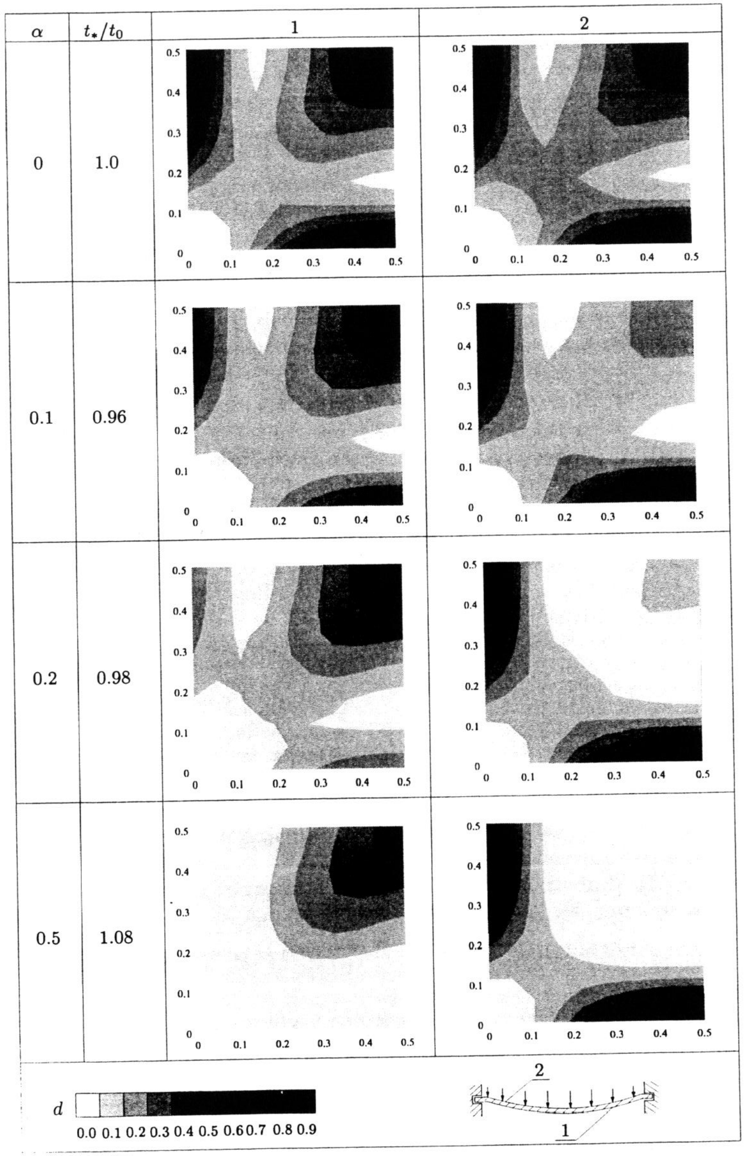

Fig. 2 Distribution of the damage variable on two plate sides, t_0 is the failure time in the case $\alpha = 0$

time-dependent behaviour of the thin-walled structure such as deflection growth, stress redistribution and stress relaxation due to material damage, known as a latent failure of the structure (cp. Scrzypek, 1993) can be predicted in terms of the classical continuum mechanics. The failure state is defined by the critical value of the damage parameter. The stress, strain and damage fields obtained is the starting point for the analysis of the failure propagation in the structure by use of the Fracture Mechanics approach (cp. Chaboche 1987).

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