CONSTRAINT EFFECTS IN MIXED MODE LOADING

J.W. HANCOCK, A. NEKKAL AND A.D. KARSTENSEN

Department of Mechanical Engineering
University of Glasgow
Glasgow G12 8QQ
Scotland

Abstract. The structure of mode I crack tip fields has been examined for non-hardening plane strain plasticity under contained yielding. The analysis is based on modified boundary layer formulations in which the displacements corresponding to K and T fields are applied as boundary conditions remote from the tip. The same technique has been used to establish the nature of mixed mode fields. In both cases slip line fields have been constructed. In mode I, constraint loss has been correlated with T which leads to a family of fields which are deviatorically similar, but differ largely hydrostatically. At the angle of maximum hoop stress, mixed mode fields are also deviatorically similar, but hydrostatically different. The constraint of mixed mode fields has then been related to the constraint of mode I J-Q/T fields, and used to infer mixed mode failure for stress controlled brittle failure.

Key words: Mixed mode, Prandtl field, Constraint, T-stress, Q fields, Slip line fields

1. Introduction

The structure of elastic-plastic crack tip fields in contained yielding can be studied using boundary layer formulations, first introduced by Rice and Tracey (1974). In this numerical technique, the displacements corresponding to an outer elastic field, characterised by the stress intensity factor, K are applied to an arbitrary remote boundary surrounding the crack tip. A more refined version of the technique is to use the displacements corresponding to the first two terms of the Williams (1957) expansion, K and T to characterise the outer field, and thus develop modified boundary layer formulations. This technique has been used by Bilby et al (1986), Betegón and Hancock (1991) and Du and Hancock (1991), O'Dowd and Shih (1991a,b) to study the loss of constraint of elastic-plastic crack tip fields of strain hardening materials under mode I plane strain conditions. The same principle has been use in mixed mode loading by Du *et al.* (1992) to examine constraint loss due to T. In the present work interest is focused on non-hardening plasticity, with the object of constructing plane strain slip line fields for crack tip plasticity. The objective is to compare the nature of the fields and the constraint loss which arises in mode I loading due to compressive values of the T stress, and constraint loss which arises from combination of mode I and II loading.

2. Numerical Methods

Plane strain mode I crack tip deformation was modelled by a boundary layer formulation using half of the focused mesh shown in Fig. 1.

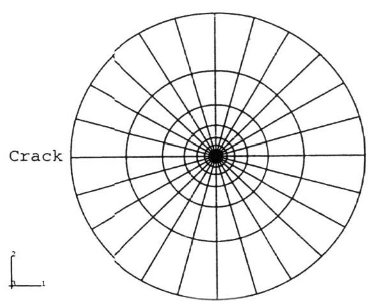

Fig. 1. The mesh for the mixed mode problems.

The mesh consisted of twelve rings of twelve isoparametric second order hybrid elements concentric with the crack tip. The crack tip thus consisted of twenty five initially coincident, but independent nodes. The boundary conditions applied to the outer circumference of the mesh corresponded to the displacements associated with an elastic mode I K field, plus those due to a T stress related to K by a biaxiality parameter β, such that both terms increased in a proportional manner. In the mixed mode problems the outer field comprised the displacements calculated from the Westergaard equations, for a plane strain mixed mode problem characterised by the mode I stress intensity factor K_I and the mode II stress intensity factor K_{II}. In all the mixed mode problems T was zero and the full mesh shown in Fig 1 was used. The crack tip in the full mesh consisted of forty nine initially coincident, but independent nodes. The ratio of the K_I to the K_{II} component was defined by an elastic mixity, M^e, following Shih (1974):

$$M^e = \frac{2}{\pi} tan^{-1} \left(\frac{K_I}{K_{II}} \right) \tag{1}$$

Six values of mixity, given in Table 1, were analysed between pure mode I for which M^e is 1 and pure mode II for which M^e is 0.

Material response was based on small strain J_2 flow theory in which the material was plastically non-hardening, with a tensile yield stress σ_0 which can be related to the yield stress in shear by, $k = \sigma_0/\sqrt{3}$. The elastic response used a finite Young's Modulus with Poisson's ratio of 0.3. Slip line field theory strictly applies to rigid elastic incompressible solids, however a compressible elastic response was been adopted to avoid numerical problems associated with incompressible behaviour. A small number of solutions were obtained with Poisson's ratio 0.49 to investigate the effect of compressibility, and were shown to have little effect. Solutions were obtained for plastic zones very much smaller than the radius in accord with the procedure described by Rice and

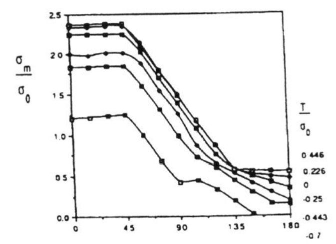

Fig. 2. The angular distribution of the mean stress about the crack tip (r=0) at various levels of T/σ_0.

Tracey (1974). Under these conditions solutions without a T stress were self similar, which is a necessary condition in small scale yielding. For such small levels of crack tip plasticity, the reaction forces on the nodes on the outer boundary were linearly related to the applied displacements demonstrating a good representation of small scale yielding. In both mode I and mixed mode problems J was determined by the virtual crack extension method of Parks(1977) as modified Shih and co-workers (Li, Shih and Needleman(1985), and implemented in ABAQUS(1989)

	M^e
K_I	1.00
$K_I = 4K_{II}$	0.84
$K_I = 2K_{II}$	0.71
$K_I = K_{II}$	0.50
$K_I = 0.5K_{II}$	0.30
K_{II}	0.00

Table 1. Elastic mixity for range of plane strain mixed mode fields.

In plane strain incompressible deformation the crack tip field can be described in terms of elastic and plastic sectors. The angular span of the elastic sector can simply be determined be extrapolating the Mises stress along radial lines to the tip (r=0). Within the plastic sectors, there are two possibilities. The first is a constant stress region in which the slip lines are straight, and in consequence the hydrostatic stress is constant. The alternative is a centred fan, comprising radial lines and arcs, such that the shear stress, $\sigma_{r\theta}$ is independent of angle. This implies that the mean stress varies linearly with angle. In the present work these features have been used to construct crack tip slip line fields. The span of the elastic sectors has been identified from the Mises stress at the crack tip, while the span of the centred fan and constant stress regions has been inferred from the angular distribution of the mean stress.

3. Results

The distribution of mean stress around the crack tip (r=0) is given in Fig. 2 for mode I deformation, with levels of T, ranging from $0.45\sigma_0$ to $-0.7\sigma_0$. Directly ahead of the crack there is a region of constant mean stress with an angular span of $\pm\pi/4$. This corresponds to a constant stress region with straight slip lines directly ahead of the crack. At angles greater than $\pi/4$ there is a region where the mean stress varies linearly with angle, corresponding to a centred fan. Finally in the region adjoining the crack flanks the yield criterion was only satisfied for tensile values of T. Thus the Prandtl field (and J-dominance) is recovered when T is positive, while negative values of T lead to fields with an elastic sector on the crack flanks and a loss of constraint directly ahead of the crack. The mode I slip line fields are finally assembled in Fig. 3, following Du and Hancock (1991).

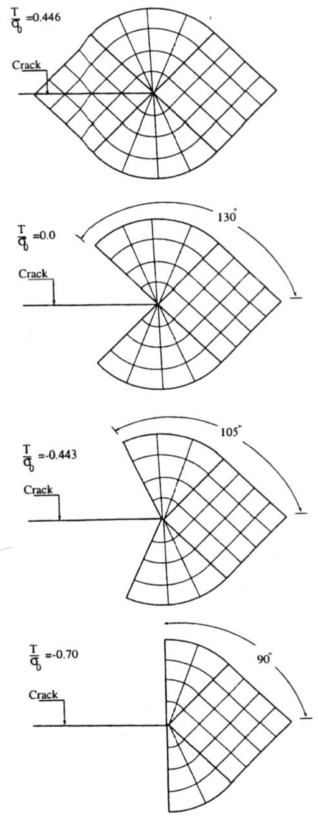

Fig. 3. Mode I slip line fields in contained yielding at various levels of T/σ_0.

In the mixed mode fields the same technique has been used to assemble the elastic and plastic sectors surrounding the tip. The angular distribution of the Mises stress is shown in Fig. 4. For fields with mixities greater than 0 there is an elastic sector on one of the crack flanks, in contrast to the fields discussed by Shih (1974), while in mode I ($T = 0$) there is an elastic sector on both flanks.

The plastic sectors consist of a constant stress region with an angular span of $\pi/2$, and centred fans. The effect of mixity is to cause the constant stress region ahead of the crack to rotate, so that in mixed mode loading the maximum principal stress is not symmetrically distributed directly

ahead of the crack. As well as causing the direction of maximum principal stress to rotate, the mode II component also cause a loss in crack tip constraint. This is clearly shown in Fig. 5 in which the mean stress in the constant stress sector decreases with mixity. The assembled slipline fields are shown in Fig. 6 and are more extensively discussed by Hancock, Karstensen and Nekkal (1996).

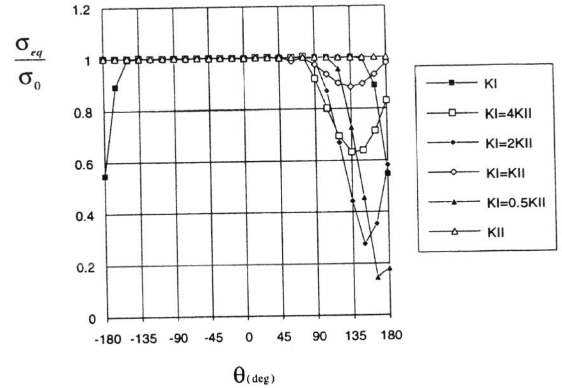

Fig. 4. The angular distribution of the Mises stress around the crack tip at different levels of elastic mixity.

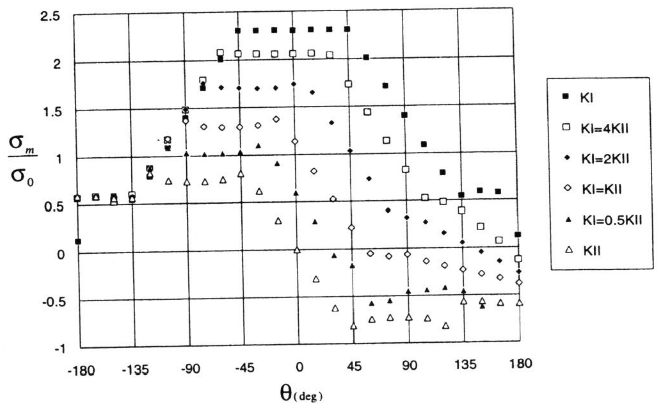

Fig. 5. The angular distribution of the mean stress around the crack tip at different levels of elastic mixity.

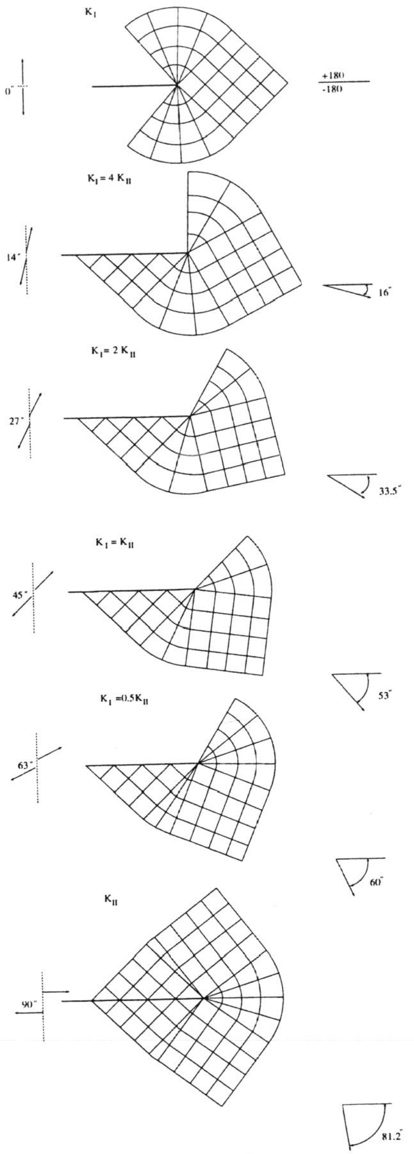

Fig. 6. Slip Line field for the family of mixed mode problems.

Hancock, Karstensen and Nekkal (1996) have also examined the structure of strain hardening field and it is relevant to note that the salient features of the non-hardening fields observed in both mode I and mixed mode loading extend to weak and moderately hardening materials. In these cases attention has been focused on the plane on which the maximum principal stress and minimum shear stress occur. This is weakly dependent on the hardening rate over the range of interest (Shih 1974). The important point is that the stress profiles for all the mixities are parallel. At this orientation, they can therefore be regarded as a family of fields which differ by a second order term which is independent of distance, and this term is largely hydrostatic in nature, as it necessarily is in the non-hardening solutions. This implies that constraint and the stress levels associated with a mixed mode fields can be correlated with the constraint of unconstrained mode I fields. This relationship between mixity and T and Q is shown in Figs. 7 and 8 respectively. O'Dowd and Shih (1991a,b) have argued that mode I fields are deviatorically similar but differ hydrostatically through a parameter Q, and it is now clear that elastic-plastic mixed mode fields are also deviatorically similar and differ largely hydrostatically in the sectors around the plane of maximum stress. This allows unconstrained mode I and mixed mode toughness data to be correlated for stress controlled brittle fracture.

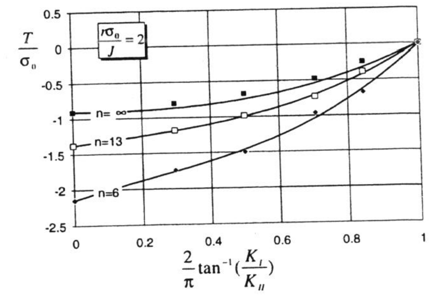

Fig. 7. The relation between mixity and T for different strain hardening rates.

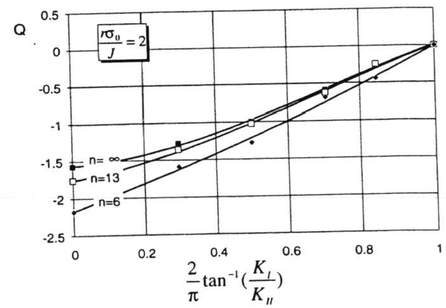

Fig. 8. The relation between mixity and Q for different strain hardening rates.

4. Conclusions

In mode I, constraint loss gives rise to a family of elastic-plastic crack tip fields which can be described by J and a second parameter which determines the level of crack tip constraint (Q/T). In the region of maximum principal stress, this family of fields differs in a largely hydrostatic manner. Mixed mode fields can be interpreted as belonging to the same family such that constraint loss by mixed mode loading results in a family of fields which differ largely hydrostatically on the plane of maximum hoop stress. This establishes a relation between Q and mixity, which enables the constraint enhanced toughness observed in unconstrained mode I fields to be correlated with the constraint enhanced toughness in mixed mode loading for stress controlled brittle fracture.

5. Acknowledgements

J.W. Hancock and A. Nekkal acknowledge the support of the EPSRC and Rolls-Royce and Associates, and discussions with Prof. J. Sumpter and Dr. Du. A Karstensen acknowledges the support of Professor J.D.G Sumpter and the D.R.A. Access to ABAQUS was under academic license from H.K.S.

Reference

ABAQUS V.5.3 (1992). ABAQUS Manual. *Hibbitt, Karlsson and Sorenson Inc.* Providence, Rhode Island.

Betegón, C. and Hancock, J. W. (1991). 'Two-parameter characterization of elastic-plastic crack-tip fields'. *Journal of Applied Mechanics* **58**, 104–110.

Bilby, B. A., Cardew, B. A., Goldthorpe, M. R. and Howard, I. C. (1986). A finite element investigation of the effect of specimen geometry on the field of stress and strain at the tip of stationary cracks. In 'Size Effect in Fracture'. Inst. Mech. Eng.. London, UK. pp. 37–46.

Du, Z. Z. and Hancock, J. W. (1991). 'The effect of non-singular stresses on crack tip constraint'. *Journal of Mechanics and Physics of Solids* 39, 555–567.

Du, Z. Z., Betegón, C. and Hancock, J. W. (1992). 'J dominance in mixed mode loading'. *International Journal of Fracture* **52**, 191–206.

Hancock, J., Karstensen, A. and Nekkal, A. (1996). 'Mixed mode and mode I crack tip fields unified by constraint'. *Journal of Mechanics and Physics of Solids*. Submitted for publication.

Hill, R. (1950). *The Mathematical Theory of Plasticity*. Clarendon Press. Oxford.

Hutchinson, J. W. (1968a). 'Singular behavior at the end of a tensile crack in a hardening material'. *Journal of Mechanics and Physics of Solids* 16, 13–31.

Karstensen, A. D., Nekkal, A. and Hancock, J. W. (1995). 'Constraint estimation scheme for edge crack bars in tension and bending'. Submitted for publication in Journal of Mechanics and Physics of Solids.

Li, F. Z., Shih, C. F. and Needleman, A. (1985). 'A comparison of methods for calculating energy release rate'. *Engineering Fracture Mechanics* **21**, 405–421.

O'Dowd, N. P. and Shih, C. F. (1991a). 'Family of crack-tip fields characterized by a triaxiality parameter: Part I - structure of fields'. *Journal of Mechanics and Physics of Solids* **39**, 939–963.

O'Dowd, N. P. and Shih, C. F. (1991b). 'Family of crack-tip fields characterized by a triaxiality parameter: Part II - fracture applications'. *Journal of Mechanics and Physics of Solids* **40**, 989–1015.

Parks, D. M. (1977). 'The virtual crack extension method for non-linear material behavior'. *Computer Methods in Applied Mechanics and Engineering* **12.**, 353–364.

Prandtl, L. (1920). 'Über die härte plastischer körper'. *Nachr. Ges. Wiss.* pp. 74–85.

Rice, J. R. and Rosengren, G. F. (1968). 'Plane strain deformation near a crack tip in a power law hardening material'. *Journal of Mechanics and Physics of Solids* **16**, 1–12.

Rice, J. R. and Tracey, D. M. (1974). 'Computational fracture mechanics'. *Journal of Mechanics and Physics of Solids* **22**, 17–26.

Shih, C. F. (1974). Small-scale yielding analysis of mixed mode plane-strain crack problems. In 'Fracture Analysis, ASTM STP 560'. American Society for Testing and Materials. Philadelphia, PA. pp. 187–210.

Williams, M. L. (1957). 'On the stress distribution at the base of a stationary crack'. *ASME Journal of Applied Mechanics* **24**, 111–114.