# COATING CRACKS IN MATERIALS UNDER AN ANTIPLANE CONCENTRATED LOAD

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ABSTRACT

The antiplane elasticity problem for a thin cracked layer coated to an elastic half-space under a concentrated shear force is considered. The fundamental solution is obtained as a rapidly convergent series in terms of the complex potentials via iterations of Möbius transformation. The singular integral equation with a logarithmic singular kernel is derived to model a crack problem that can be solved numerically in a straightforward manner. The dimensionless mode-III stress intensity factors obtained for various crack inclinations and crack lengths are discussed in detail and provided in graphic form.

KEYWORDS

Antiplane elasticity, thin crack layer, stress intensity factor.

## INTRODUCTION

The problem of thin film layers coated to an elastic half-space has arisen considerable interest in the integrated circuit's market and composite armor protection systems. In many cases, failure of composite layer systems may occur as a result of the presence of preexisting imperfections in the thin film layer due to improper treating of manufacturing. Studies of failure mechanism of thin films deposited on a substrate have been extensively investigated by many researchers (Rice, 1983; Evans *et al.*, 1988; Hutchinson and Suo, 1991). They applied different failure criteria to predict the behavior of interfacial crack propagation and concluded that the pattern of failure mechanism is mainly dependent on the sign and magnitude of residual stresses and the relative strength of the film, the interface and the substrate. In the present study, the thin cracked layer, which is perfectly coated to an elastic half-space, subjected to an antiplane concentrated load is considered. The proposed method is based upon the complex potential theory and iterations of Möbius transformation that allows us to express the solutions as a rapidly convergent series. The above mentioned methodology has a clear advantage in deriving the solution to the heterogeneous problem in terms of the solution to the corresponding homogeneous problem that was termed *"heterogenization"* by Honein *et al.* (1992a, 1992b). In order to model a crack in the thin layer, we introduce a continuous distribution of dislocations which leads to a system of singular integral equations that can be solved numerically in a straightforward manner (Chen and Cheung, 1990; Chao and Shen, 1995) . The mode-III stress intensity factor can be directly obtained from the resulting dislocation densities. The effect of crack dimensions; geometrical configurations and material properties on the stress intensity factor is discussed in detail and displayed in graphic form.

## SOLUTION OF THE ANTIPLANE PROBLEM FOR THREE-MATERIAL MEDIA

Consider the plane-layer media with two interface boundaries $L_1$ and $L_2$ (Fig.1) where the domains $S_1$ and $S_2$ occupy the upper half-plane and lower half-plane, respectively and the middle layer $S$ is an infinite strip of thickness $2h$. Assume that all singularities are in the middle layer $S$, the solutions in the region $S_1$ and $S$, respectively are now expressed as

$$\phi_1(z) = \phi_0(z) + \alpha_1\phi_0(z) \tag{1}$$

$$\phi(z) = \phi_0(z) + \alpha_1\overline{\phi_0(A_1 z)} \tag{2}$$

with

$$\alpha_1 = \frac{(c-c_1)}{(c+c_1)} \tag{3}$$

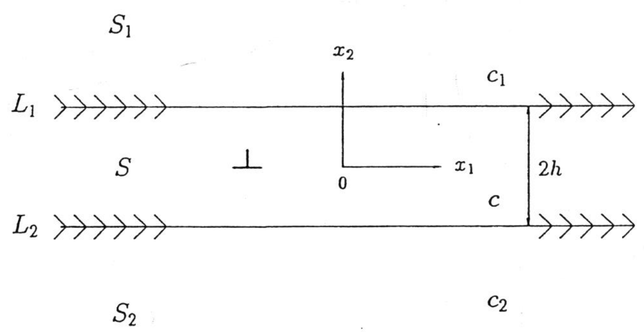


Fig.1 Three-layer media with all singularities located in the middle layer.

where the transformation function defined as $A_1 z = \bar{z} + 2ih$ carries the points in $S_1$ ( or $S$ ) into the inverse points in $S$ ( or $S_1$ ) with respect to the interface $L_1$. From the expressions given in (1) and (2), the continuity conditions are satisfied at the interface $L_1$ but not at the interface $L_2$. In order to satisfy the continuity conditions along the interface $L_2$, we construct the solution of three-material media as follows

$$\phi_1(z) = \phi_0(z) + \alpha_1\phi_0(z) + \alpha_2\overline{\phi_0(A_2 z)} + \alpha_2\alpha_1\phi_0(A_1 A_2 z) \qquad , z \in S_1 \tag{4}$$

$$\phi_2(z) = \phi_0(z) + \alpha_1\overline{\phi_0(A_1 z)} + \alpha_2\phi_0(z) + \alpha_2\alpha_1\overline{\phi_0(A_1 z)} \qquad , z \in S_2 \tag{5}$$



$$\phi(z) = \phi_0(z) + \alpha_1\overline{\phi_0(A_1 z)} + \alpha_2\overline{\phi_0(A_2 z)} + \alpha_2\alpha_1\phi_0(A_1 A_2 z) \quad , z \in S \tag{6}$$

with

$$\alpha_2 = \frac{(c-c_2)}{(c+c_2)} \tag{7}$$

where $A_2 z$ is the transformation function defined as $A_2 z = \bar{z} - 2ih$ which carries the points in $S_2$ ( or $S$) into the inverse points in $S$ ( or $S_2$) with respect to the interface $L_2$. Now, from the expressions in (4)-(6), the continuity conditions are satisfied at the interface $L_2$ but not at the interface $L_1$. In order to satisfy the continuity conditions at both the interfaces $L_1$ and $L_2$, we repeat the previous processes by obtaining the two additional terms each step and the series solution for each material medium is finally obtained as

$$\phi_1(z) = (1+\alpha_1)\{\phi_0(z) + \alpha_2\sum_{n=0}^{\infty}(\alpha_1\alpha_2)^n\overline{\phi_0(A_2 M^n z)} + \sum_{n=1}^{\infty}(\alpha_1\alpha_2)^n\phi_0(M^n z)\}, z \in S_1 \tag{8}$$

$$\phi_2(z) = (1+\alpha_2)\{\phi_0(z) + \alpha_1\sum_{n=0}^{\infty}(\alpha_1\alpha_2)^n\overline{\phi_0(A_1 N^n z)} + \sum_{n=1}^{\infty}(\alpha_1\alpha_2)^n\phi_0(N^n z)\} \quad , z \in S_2 \tag{9}$$

$$\begin{aligned}\phi(z) = {} & \phi_0(z) + \sum_{n=1}^{\infty}(\alpha_1\alpha_2)^n\phi_0(M^n z) + \sum_{n=1}^{\infty}(\alpha_1\alpha_2)^n\phi_0(N^n z) \\ & + \alpha_1\sum_{n=0}^{\infty}(\alpha_1\alpha_2)^n\overline{\phi_0(A_1 N^n z)} + \alpha_2\sum_{n=0}^{\infty}(\alpha_1\alpha_2)^n\overline{\phi_0(A_2 M^n z)} \qquad , z \in S\end{aligned} \tag{10}$$

with $M^n z = (A_1 A_2)^n z = z + 4nhi$, and $N^n z = (A_2 A_1)^n z = z - 4nhi$.
Equations (8)-(10) give the general series solutions to the antiplane three-material media problem provided that the complex potential $\phi_0(z)$ is appropriately solved. The above formal series is uniformly convergent on compact sets provided $|\alpha_1\alpha_2| < 1$. The case $\alpha_i = 1$ corresponds to the problem with the absence of the upper medium $S_1$ or the lower medium $S_2$ while $\alpha_i = -1$ corresponds to the problem with the rigid upper medium $S_1$ or the rigid lower medium. In all other cases $|\alpha_i| < 1$. Even for the case of $\alpha_i = 1$ or $\alpha_i = -1$, the series solution is still found to be uniformly convergent which will be discussed in the following sections. Note that the expressions given in (8)-(10) also hold for the corresponding problem of two circular inclusions embedded into an infinite matrix except that $A_1 z$ and $A_2 z$ are replaced by

$$A_i z = \frac{a_i^2}{\bar{z} - \bar{z}_i} + z_i \qquad , i = 1,2 \tag{11}$$

with $z_1$, $z_2$ and $a_1$, $a_2$ being the centers and the radii, respectively of the inclusions ( Honein *et al.* 1992a, 1992b).

## A THIN CRACKED LAYER BONDED TO AN ELASTIC HALF-SAPCE

In this section, we consider a thin cracked layer, which is bonded to an elastic half-space, under an antiplane concentrated force (see Fig.2). The current problem can be treated as a sum of the corresponding thin layer problem without cracks and a corrective problem. The solution associated with the former problem has been obtained from the previous section by substituting $\alpha_1 = 1$ (or $c_1 = 0$) into (10) with $\phi_0(z)$ being

$$\phi_0(z) = P\log(z_s - \hat{z}_s) \tag{12}$$

where $P$ and $\hat{z}_s$ are the magnitude and the location, respectively of the applied point load. The solution of the thin layer medium becomes

$$\begin{aligned}\phi^{(1)}(z_s) \;=\; & P[\log(z_s-\hat{z}_s)+\sum_{n=1}^{\infty}(\frac{c-c_2}{c+c_2})^n\log(z_s+4nhi-\hat{z}_s)\\ & +\sum_{n=1}^{\infty}(\frac{c-c_2}{c+c_2})^n\log(z_s-4nhi-\hat{z}_s)+\sum_{n=0}^{\infty}(\frac{c-c_2}{c+c_2})^n\log(z_s-4nhi-2hi-\bar{\hat{z}}_s)\\ & +\sum_{n=0}^{\infty}(\frac{c-c_2}{c+c_2})^{n+1}\log(z_s+4nhi+2hi-\bar{\hat{z}}_s)] \qquad (13)\end{aligned}$$

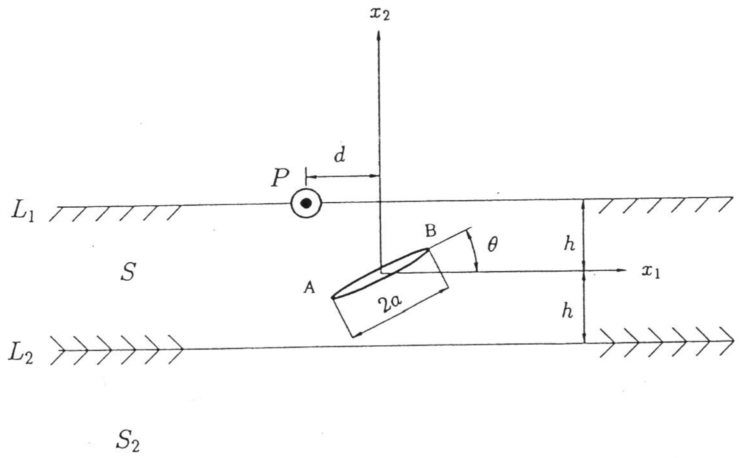


Fig. 2 A thin cracked layer bonded to an elastic half-space.

On the other hand, a corrective solution of the thin layer medium with a crack $L$ can be obtained in terms of a continuous distribution of dislocations as

$$\begin{aligned}\phi^{(2)}(z_s) \;=\; & -\frac{i}{2\pi}[\int_L b_0(s)\log(z_s-t)ds+\sum_{n=1}^{\infty}(\frac{c-c_2}{c+c_2})^n\int_L b_0(s)\log(z_s+4nhi-t)ds\\ & +\sum_{n=1}^{\infty}(\frac{c-c_2}{c+c_2})^n\int_L b_0(s)\log(z_s-4nhi-t)ds\\ & -\sum_{n=0}^{\infty}(\frac{c-c_2}{c+c_2})^n\int_L b_0(s)\log(z_s-4nhi-2ih-\bar{t})ds\\ & -\sum_{n=0}^{\infty}(\frac{c-c_2}{c+c_2})^{n+1}\int_L b_0(s)\log(z_s+4nhi+2ih-\bar{t})ds] \qquad ,t\in L \qquad (14)\end{aligned}$$

Now, a single integral equation with the unknown function $b_0(s)$ is thus established from the traction-free condition along the crack surface as

$$p^{(1)}(z_s)+p^{(2)}(z_s)=0 \qquad ,z_s\in L \qquad (15)$$



where

$$p^{(1)}(z_s)=-2Re[ic\phi^{(1)}(z_s)] \qquad (16)$$

$$p^{(2)}(z_s)=-2Re[ic\phi^{(2)}(z_s)] \qquad (17)$$

In addition, the single-valued condition of the displacement when enclosing the crack must be satisfied, i.e.

$$\int_L b_0(s)ds=0 \qquad (18)$$

Equations (15) and (18) yield to solve the constants $b_0(s)$ numerically by using the following interpolation formulae in local coordinates $s_j$ ( $1\le j\le N$) as follows

$$b_0(s_1)=b_{0,0}(\sqrt{\frac{2d_1}{s_1}}-1)+b_{0,1} \qquad (19)$$

$$b_0(s_N)=b_{0,N}(\sqrt{\frac{2d_N}{2d_N-s_N}}-1)+b_{0,N-1} \qquad (20)$$

and

$$b_0(s_j)=b_{0,j-1}\frac{2d_j-s_j}{2d_j}+b_{0,j}\frac{s_j}{2d_j} \qquad ,(2\le j\le N-1) \qquad (21)$$

where $d_j$ ($1\le j\le N$) are the half length of each line segment and $b_{0,j}$ ($0\le j\le N$) are the unknown coefficients which can be determined numerically. Once the coefficients $b_{0,j}$ are solved, the mode-III stress intensity factors can be obtained accordingly as

$$K_{III}(tip-A)=\sqrt{\pi d_1}b_{0,0} \qquad (22)$$

$$K_{III}(tip-B)=\sqrt{\pi d_N}b_{0,N} \qquad (23)$$

## RESULTS AND DISCUSSION

We first consider the problem of a crack in an anisotropic thin layer subjected to a concentrated antiplane force $P$ at its surface (see Fig.2). The anisotropic elastic constants used for the present calculations are $c_{44}=11.4Gpa, c_{45}=0, c_{55}=10.2Gpa$ for Di-potassium tartrate in the thin layer, and $c_{44}=22.3Gpa, c_{45}=0, c_{55}=32.7Gpa$ for Sodium thiosulfate in the half-space medium. In the following discussions, the center of a crack is fixed at the origin $x_1=x_2=0$. The calculated stress intensity factors for various crack lengths $a/h$, crack orientations $\theta$, are shown in Figs 3-4. When a concentrated load is applied symmetrically with respect to a straight crack parallel to the free surface, i.e., $d=0, \theta=0°$, the dimensionless stress intensity factors at tip-A and tip-B are equal in magnitude but opposite in sign. By rotating the crack angle $\theta$ in a counterclockwise sense from 0° to 90°, the absolute magnitude of $K_{III}$ at tip-B begins to increase and reaches its maxium value, and then decreases to zero at $\theta=90°$. The effect of an applied load on the value $K_{III}$ can be illustrated by the relative angle $\varphi$ and the distance between the crack tip of interest and the point of an applied load. This relative angle is defined as the angle between a line crack and a line connecting the crack tip of interest and the point of an applied load. In general, the factor $K_{III}$ increases as the crack tip approaches the point of an applied load while the value $K_{III}$ will decrease with decreasing the relative angle $\varphi$. Increasing and decreasing of the factor $K_{III}$ at tip-B would compete depending on the relative angle $\varphi$ and the distance between tip-B and the point of an applied load. When the point of an applied load colinears with the line crack, i.e., $\varphi=0°$ or $\theta=90°$ for the given case

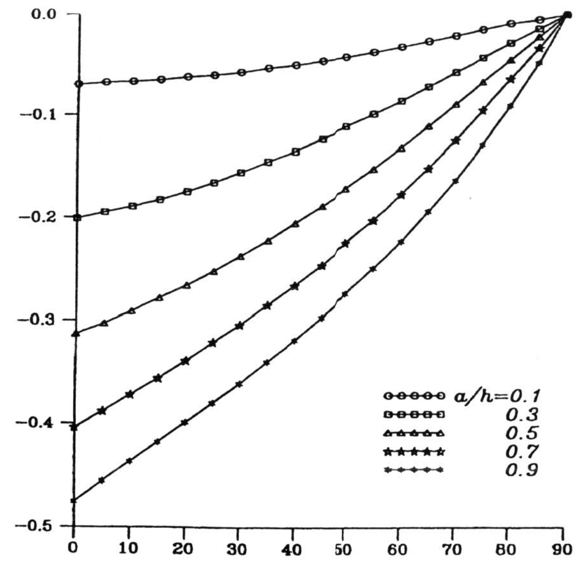


Fig.3 The mode-III stress intensity factors at tip-A versus crack inclination

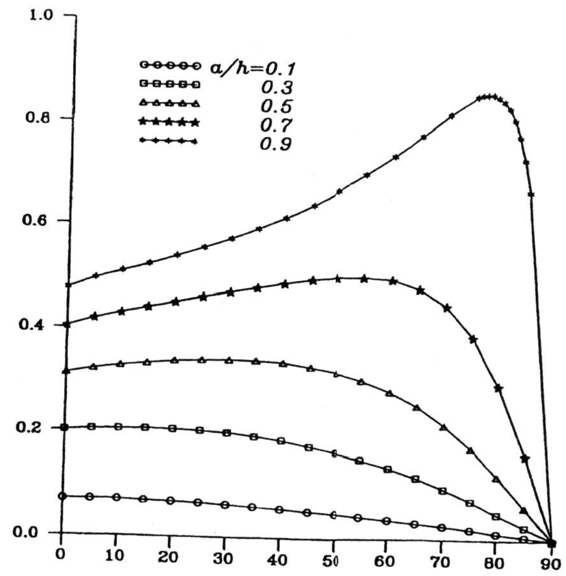


Fig.4 The mode-III stress intensity factors at tip-B versus crack inclination



$d = 0$, the factor $K_{III}$ would vanish as indicated in Fig.3 and Fig.4. On the other hand, the factor at tip-A decreases monotonously since the distance between tip-A and the point of an applied load increases and the relative angle $\varphi$ decreases when increasing the crack angle $\theta$ from 0° to 90°.

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