A HIGHER ORDER FATIGUE CRACK PATHS SIMULATION
BY THE MVCCI-METHOD

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ABSTRACT

The Modified Virtual Crack Closure Integral (MVCCI)-method has proved to be a highly effective and versatile numerical procedure for the fracture analysis of various crack problems in linear elasticity. In this paper it is shown that under proportional loading conditions the computer aided prediction of curved fatigue crack paths can further be improved in accuracy in combination with the MVCCI-method. The numerical crack growth simulation is still based on a step-by-step technique but using a piecewise curved approximation of the crack path. By this method both, the new locus of the crack tip and the direction of the next step of crack extension can be computed simultaneously by a virtual tangential crack extension. For a non-symmetrical specimen under combined bending and shear loading the comparison of computationally predicted and experimentally obtained curved crack trajectories show an excellent agreement in all cases considered.

KEYWORDS

Curved fatigue cracks, crack path simulation, virtual crack closure integral method

INTRODUCTION

From failed structures and components it is known in engineering practice that cracks frequently originate and extend in regions characterized by complicated geometrical shapes and asymmetrical loading conditions. Hence the developing crack paths are found to be curved and standard solutions for coplanar cracks do not apply. Therefore the prediction of such curved crack paths is essential for the accurate evaluation of the final fracture modes of cracked structures and components. In the present paper the crack growth simulation for proportional loading conditions is also based on a step-by-step finite element analysis, like in investigations of several authors (Bergquist and Gnex, 1978; Theilig, 1979; Sumi, 1985a, 1990b; Limnig, 1993; Theilig and Buchholz, 1994). The objective of this paper is to improve the conventional simulation technique in accuracy by using a piecewise curved approximation of the crack path on the basis of quantities which the straightforward Modified Virtual Crack Closure Integral Method (Buchholz, 1984; Krishnamurty et al., 1985; Raju, 1987) can provide. In order to
show the significance of the proposed simulations technique the computational results are compared with findings from a detailed experimental investigation by the aid of a non-symmetrical specimen, especially designed for combined bending and shear loading (Theilig, 1979).

CRACK GROWTH SIMULATION

Consider a crack in a two-dimensional linear elastic body under proportional mixed-mode loading conditions. The state of stress ahead of the crack tip is given by

$$\sigma_{11}(x_1, \theta) = \frac{k_1}{2\pi x_1} + T + b_1 \sqrt{\frac{\pi}{2\pi x_1}} + O(x_1), \quad (1)$$

$$\sigma_{22}(x_1, \theta) = \frac{k_1}{2\pi x_1} + b_2 \sqrt{\frac{\pi}{2\pi x_1}} + O(x_1), \quad (2)$$

$$\sigma_{12}(x_1, \theta) = \frac{k_2}{2\pi x_1} + b_2 \sqrt{\frac{\pi}{2\pi x_1}} + O(x_1). \quad (3)$$

where $k_1$ and $k_2$ are the stress intensity factors (SIFs) and the coefficients $T$, $b_1$, and $b_2$ are also determined from the boundary conditions. It is known that in such a situation the crack will propagate in a curved manner after kinking out of the original plane, (Fig. 1).

![Fig. 1. A kinked and curved crack and the coordinate systems](image)

For several mixed-mode criteria the direction $\varphi_0$ of crack extension depends only on the ratio $k_2/k_1$ of the SIFs of the original straight crack, whereas for others a further dependence on Poisson's ratio $\nu$ is found. But for small ratios $0 < |k_2/k_1| < 0.1$ practically the same values $\varphi_0 = -2 \alpha$ are predicted by all criteria. This direction will result in the state of local symmetry of the branched crack tip ($K_{II} = 0$). After crack initiation under monotonic loading, the generalization of the local symmetry criterion can be regarded as the natural basis for the evolution of the crack path in a homogeneous isotropic material, (Goldstein and Salganik, 1974). Thus, it can be stated that a continuously growing crack will form a curved path that experiences pure mode I at any crack tip position, Fig. 1. Therefore the state of stress ahead of the crack tip is given by

$$\sigma_{11}(x_1, \theta) = \frac{K_1}{2\pi x_1} + T + b_1 \sqrt{\frac{\pi}{2\pi x_1}} + O(x_1), \quad (4)$$

$$\sigma_{22}(x_1, \theta) = \frac{K_2}{2\pi x_1} + b_2 \sqrt{\frac{\pi}{2\pi x_1}} + O(x_1), \quad (5)$$

$$\sigma_{12}(x_1, \theta) = b_2 \sqrt{\frac{\pi}{2\pi x_1}} + O(x_1). \quad (6)$$

The Cartesian coordinate system ($x_1', x_2'$) is defined with the origin at the actual crack tip with the $x_1'$-axis along the tangential direction of the crack path at the tip. It can be stated that the $x_1'$-axis is a perturbation solution of a slightly kinked and curved crack. A virtual crack profile (Fig. 2) is assumed in the form

$$l(x_1) = \alpha x_1 + \beta x_1^{1/2} + \gamma x_1^2 + O(x_1^{3/2}) \quad (7)$$

where $\alpha$, $\beta$ and $\gamma$ are the shape parameters.

![Fig. 2. A slightly kinked and curved crack](image)

Then the SIFs at the virtually extended crack tip are given by

$$K_1 = \left( k_2 - \frac{3}{2} \alpha k_{11} \right) \left[ \frac{3}{4} \beta k_{11} h^{1/2} + \frac{b_1}{2} - \frac{3}{4} x k_{11} + k_1 k_{11} - \alpha k_1 \left( k_{12} + \frac{3}{2} k_{23} \right) \right] h + O(h^{3/2}), \quad (8)$$

$$K_{II} = \left( k_2 - \frac{1}{2} \alpha k_{11} \right) \left[ \frac{3}{4} \beta k_{11} - 2 \sqrt{\alpha} \right] h^{1/2} + \left[ \frac{b_1}{2} - \frac{1}{4} \alpha k_{11} - \frac{3}{2} \sqrt{\alpha} \beta T + \gamma k_{11} + k_1 k_{11} + \alpha k_1 \left( k_{12} + \frac{1}{2} k_{23} - k_{22} \right) \right] h + O(h^{3/2}), \quad (9)$$
where the quantities $k_{11}, k_{12}, k_{1}, k_{22}$ represent the effects of the far field boundary conditions. The crack growth criterion of local symmetry requires that the SIF $K_H$ vanishes at any crack tip position. Therefore the shape parameters of the natural crack path are obtained as

$$\alpha = -2k_H/k_1, $$

$$\beta = 3\sqrt{T/k_1} \alpha, $$

$$\gamma = -\left(k_{11} k_{22} + k_1 k_{21} + \frac{b_H}{2} k_1 \right) \frac{1}{k_1} \left[1 + \frac{1}{2} \left(2 k_{22} - k_{11} \right) + \frac{b_H}{2} k_1 \right] \frac{1}{k_1} \Delta \varphi $$

If we consider a straight crack under local symmetry at the initial crack tip, i.e. $k_1 = 0$, we find $\alpha = \beta = 0$. Therefore

$$l(x_1) = \gamma x_1^2 \tag{13}$$

is holding, with

$$\gamma = -\left(\frac{b_H}{2} + k_1, k_{11} \right) \frac{1}{k_1} \tag{14}$$

and with eq. (8)

$$K_1 = k_1 + \left(\frac{b_H}{2} + k_1, k_{11} \right) \Delta h \tag{15}$$

is found. In this case the crack will propagate with a continuous deflection. But in the case of a self-similar virtual crack extension of the straight crack under consideration

$$\bar{K}_1 = k_1 + \left(\frac{b_H}{2} + k_1, k_{11} \right) \Delta h, \quad \bar{K}_H = \left(\frac{b_H}{2} + k_1, k_{11} \right) \Delta h \tag{16}$$

are obtained. From eqs. (15) and (16) $K_1(\Delta h) = \bar{K}_1(\Delta h)$ is found in consequence of the considered slightly curved crack extension. According to eqs. (13 - 16) one finally gets for a selected $\Delta h$

$$\Delta \varphi = -2 \frac{\Delta \bar{K}_H}{k_1}, \quad \Delta l = -\frac{\Delta \bar{K}_H}{k_1} \Delta l, \quad \Delta a = \Delta h \left[1 + \frac{2}{3} \left(\frac{\Delta \bar{K}_H}{k_1} \right) - \frac{2}{5} \left(\frac{\Delta \bar{K}_H}{k_1} \right)^4 \right]. \tag{17}$$

It is seen that under the condition $K_1 = 0$ the change of the slope and the locus of the crack tip can be interpreted as the consequence of $\Delta \bar{K}_H \neq 0$ for a virtual tangential crack extension $\Delta h$ (Fig. 3).

From eqs. (18) the need for an efficient numerical mode separation technique in conjunction with the step-by-step finite element analysis can be seen. Particularly with respect to this requirement the MVCCI-method has proved to be highly advantageous, because it delivers the separated strain energy release rates of two modes simultaneously with good accuracy and without any additional effort. For 8-noded quadrilaterals, which form the actual crack tip in Fig. 5, the following finite element representation of Irwin’s crack closure integral relations can be given (Buchholz, 1984; Krishnamurthy et al., 1985; Raju, 1987)

$$G = \frac{1}{2 \Delta h \left(F_{11} \Delta u_{2,1-1} + F_{11} \Delta u_{2,1-1} \right)}, \quad \Delta G = \frac{1}{2 \Delta h \left(F_{11} \Delta u_{2,1-1} + F_{11} \Delta u_{2,1-1} \right)}. \tag{19}$$
NUMERICAL RESULTS

In order to evaluate the significance of the improved crack path simulations a special non-
symmetrical specimen for lateral force bending has been designed (Theilig, 1979). Along the
axial shaped transition region of the specimen notches have been attached at positions α =
0, 20, 40 deg. from which cracks initiated and extended with characteristicaly curved crack
paths during the fatigue tests, Fig. 6. For each notch position five specimens have been
investigated under proportional loading conditions. These crack paths were measured at the
center line of the broken specimens. It was found that the experimental scatterband is rather
narrow. Furthermore it can be recognized that the local positions of the pre-cracks in the roots
of the notches essentially determine the scattering.

For the numerical calculations the length $l$ of the finite element model was chosen to 125 mm.
In accordance with the design of the lateral bend specimen the further dimensions are given
with $l_2 = 25$ mm, $b_2 = 25$ mm, $b_1 = 45$ mm and $t = 15$ mm (thickness). The lateral force was
material behaviour Young’s modulus and Poisson’s ratio were chosen to $E = 2.1 \times 10^5$ N/mm$^2$
and $\nu = 0.3$. Fig. 7 shows the two basic finite element discretisations which were found to be
suitable for the selected virtual tangential crack increments $\Delta h$.

After each simulation step of virtual tangential crack extension by $\Delta h$ a remeshing is necessary
in such a manner, that new crack tip elements are generated providing additional nodes in order
to model the new curved crack surfaces. The calculations were carried out for $\Delta h = 4$ mm,
$\Delta h = 2$ mm and $\Delta h = 1$ mm. In Fig. 8 the computationally simulated crack trajectories for all
notch positions are shown together with the experimentally obtained scatterbands from the
related fatigue crack growth experiments. For all extension increments $\Delta h$ an excellent
agreement is found.
SUMMARY

This investigation has shown that the numerical tool of the MVCCI-method is delivering very good crack path simulation results with linear strain elements and only moderately refined curved approximation of the crack path offers an efficient way for the numerical analysis of conditions. From the excellent agreement of the numerical and experimental results one can conclude that the applied criterion of local symmetry works well. The step-by-step simulation in conjunction with the evaluation by the MVCCI-method provides the basis for a general computational approach to the fracture analysis of complex crack configurations and loading conditions that may be found in engineering applications within acceptable accuracy.

REFERENCES


