A HIGHER ORDER FATIGUE CRACK PATHS SIMULATION BY THE MVCCI-METHOD

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ABSTRACT

The Modified Virtual Crack Closure Integral (MVCCI)-method has proved to be a highly effective and versatile numerical procedure for the fracture analysis of various crack problems in linear elasticity. In this paper it is shown that under proportional loading conditions the computer aided prediction of curved fatigue crack paths can further be improved in accuracy in combination with the MVCCI-method. The numerical crack growth simulation is still based on a step-by-step technique but using a piecewise curved approximation of the crack path. By this method both, the new locus of the crack tip and the direction of the next step of crack extension can be computed simultaneously by a virtual tangential crack extension. For a non-symmetrical specimen under combined bending and shear loading the comparison of computationally predicted and experimentally obtained curved crack trajectories show an excellent agreement in all cases considered.

KEYWORDS

Curved fatigue cracks, crack path simulation, virtual crack closure integral method

INTRODUCTION

From failed structures and components it is known in engineering practice that cracks frequently originate and extend in regions characterized by complicated geometrical shapes and asymmetrical loading conditions. Hence the developing crack paths are found to be curved and standard solutions for coplanar cracks do not apply. Therefore the prediction of such curved crack paths is essential for the accurate evaluation of the final fracture modes of cracked structures and components. In the present paper the crack growth simulation for proportional loading conditions is also based on a step-by-step finite element analysis, like in investigations of several authors (Bergquist and Gnex, 1978; Theilig, 1979; Sumi, 1985a, 1990b; Linnig, 1993; Theilig and Buchholz, 1994). The objective of this paper is to improve the conventional simulation technique in accuracy by using a piecewise curved approximation of the crack path on the basis of quantities which the straightforward Modified Virtual Crack Closure Integral Method (Buchholz, 1984; Krishnamurty *et. al.*, 1985; Raju, 1987) can provide. In order to show the significance of the proposed simulations technique the computational results are compared with findings from a detailed experimental investigation by the aid of a non-symmetrical specimen, especially designed for combined bending and shear loading (Theilig, 1979).

CRACK GROWTH SIMULATION

Consider a crack in a two-dimensional linear elastic body under proportional mixed-mode loading conditions. The state of stress ahead of the crack tip is given by

$$\sigma_{11}(x_1,0) = \frac{k_I}{\sqrt{2\pi x_1}} + T + b_I\sqrt{\frac{x_1}{2\pi}} + O(x_1), \tag{1}$$

$$\sigma_{22}(x_1,0) = \frac{k_I}{\sqrt{2\pi x_1}} + b_I\sqrt{\frac{x_1}{2\pi}} + O(x_1), \tag{2}$$

$$\sigma_{12}(x_1,0) = \frac{k_{II}}{\sqrt{2\pi x_1}} + b_{II}\sqrt{\frac{x_1}{2\pi}} + O(x_1), \tag{3}$$

where k_I and k_{II} are the stress intensity factors (SIFs) and the coefficients T, b_I and b_{II} are also determined from the boundary conditions. It is known that in such a situation the crack will propagate in a curved manner after kinking out of the original plane, (Fig. 1).

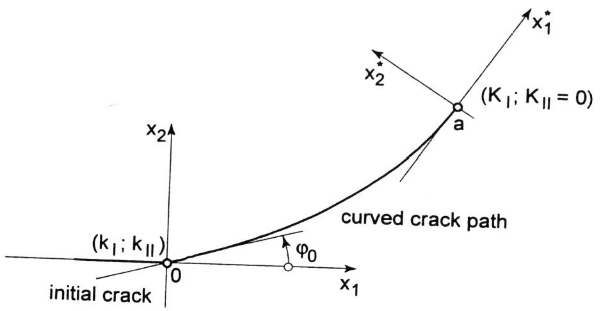

Fig. 1. A kinked and curved crack and the coordinate systems

For several mixed-mode criteria the direction φ_0 of crack extension depends only on the ratio k_{II}/k_I of the SIFs of the original straight crack, whereas for others a further dependence on Poisson's ratio ν is found. But for small ratios $0 < |k_{II}/k_I| < 0.1$ practically the same values $\varphi_0 = -2\,k_{II}/k_I$ are predicted by all criteria. This direction will result in the state of local symmetry at the branched crack tip ($K_{II} = 0$). After crack initiation under monotonic loading, the generalization of the local symmetry criterion can be regarded as the natural basis for the evolution of the crack path in a homogeneous isotropic material, (Goldstein and Salganik, 1974). Thus, it can be stated that a continuously growing crack will form a curved path that experiences pure mode I at any crack tip position, Fig. 1. Therefore the state of stress ahead of the crack tip is given by

$$\sigma_{11}(x_1^*,0) = \frac{K_I}{\sqrt{2\pi x_1^*}} + T^* + b_I^*\sqrt{\frac{x_1^*}{2\pi}} + O(x_1^*) \tag{4}$$

$$\sigma_{22}(x_1^*,0) = \frac{K_I}{\sqrt{2\pi x_1^*}} + b_I^*\sqrt{\frac{x_1^*}{2\pi}} + O(x_1^*) \tag{5}$$

$$\sigma_{12}(x_1^*,0) = b_{II}^*\sqrt{\frac{x_1^*}{2\pi}} + O(x_1^*) \tag{6}$$

The Cartesian coordinate system (x_1^*, x_2^*) is defined with the origin at the actual crack tip with the x_1^*-axis along the tangential direction of the crack path at the tip. It can be stated that continuous crack deflections can only be caused by the non-singular stresses. According to Sumi (1985a, 1990b) the crack path prediction can be performed by using the first order perturbation solution of a slightly kinked and curved crack. A virtual crack profile (Fig. 2) is assumed in the form

$$l(x_1) = \alpha x_1 + \beta x_1^{3/2} + \gamma x_1^2 + O(x_1^{5/2}) \tag{7}$$

where α, β and γ are the shape parameters.

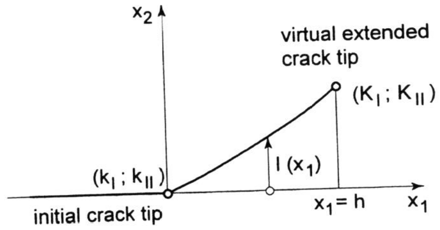

Fig. 2. A slightly kinked and curved crack

Then the SIFs at the virtually extended crack tip are given by

$$K_I = \left(k_I - \frac{3}{2}\alpha k_{II}\right) - \frac{9}{4}\beta k_{II} h^{1/2} + \left[\frac{b_I}{2} - \frac{5}{4}\alpha b_{II} - 3\gamma k_{II} + k_I \bar{k}_{11} - \alpha k_I\left(\bar{k}_{12} + \frac{3}{2}\bar{k}_{21}\right) + k_{II}\bar{k}_{12} - \alpha k_{II}\left(\bar{k}_{11} + \frac{3}{2}\bar{k}_{22}\right)\right] h + O(h^{3/2}), \tag{8}$$

$$K_{II} = \left(k_{II} - \frac{1}{2}\alpha k_I\right) - \left(\frac{3}{4}\beta k_I - 2\sqrt{\frac{2}{\pi}}\alpha T\right) h^{1/2} + \left[\frac{b_{II}}{2} - \frac{1}{4}\alpha b_I - \frac{3\sqrt{2\pi}}{4}\beta T + \gamma k_I + k_I \bar{k}_{21} + \alpha k_I\left(\frac{1}{2}\bar{k}_{11} - \bar{k}_{22}\right) + k_{II}\bar{k}_{22} + \alpha k_{II}\left(\frac{1}{2}\bar{k}_{12} - \bar{k}_{21}\right)\right] h + O(h^{3/2}), \tag{9}$$

where the quantities $k_{11}, k_{12}, k_{21}, k_{22}$ represent the effects of the far field boundary conditions. The crack growth criterion of local symmetry requires that the SIF K_{II} vanishes at any crack tip position. Therefore the shape parameters of the natural crack path are obtained as

$$\alpha = -2\,k_{II} / k_I\,, \tag{10}$$

$$\beta = \frac{8}{3}\sqrt{\frac{2}{\pi}}\,\frac{T}{k_I}\,\alpha\,, \tag{11}$$

$$\gamma = -\left(k_{II}\,\bar{k}_{22} + k_I\,\bar{k}_{21} + \frac{b_{II}}{2}\right)\frac{1}{k_I} + \left\{\left[k_I\left(2\bar{k}_{22} - \bar{k}_{11}\right) + \frac{b_I}{2}\right]\frac{1}{2k_I} + 4\left(\frac{T}{k_I}\right)^2\right\}\alpha\,. \tag{12}$$

If we consider a straight crack under local symmetry at the initial crack tip, i.e. $k_{II} = 0$, we find $\alpha = \beta = 0$. Therefore

$$l\left(x_1\right) = \gamma\, x_1^2 \tag{13}$$

is holding, with

$$\gamma = -\left(\frac{b_{II}}{2} + k_I\,\bar{k}_{21}\right)\frac{1}{k_I} \tag{14}$$

and with eq. (8)

$$K_I = k_I + \left(\frac{b_I}{2} + k_I\,\bar{k}_{11}\right)h \tag{15}$$

is found. In this case the crack will propagate with a continuous deflection. But in the case of a self-similar virtual crack extension of the straight crack under consideration

$$\bar{K}_I = k_I + \left(\frac{b_I}{2} + k_I\,\bar{k}_{11}\right)h\,, \quad \bar{K}_{II} = \left(\frac{b_{II}}{2} + k_I\,\bar{k}_{21}\right)h \tag{16}$$

are obtained. From eqs. (15) and (16) $K_I(h) = \bar{K}_I(h)$ is found in consequence of the considered slightly curved crack extension. According to eqs. (13 - 16) one finally gets for a selected Δh

$$\Delta\varphi = -2\,\frac{\Delta\bar{K}_{II}}{k_I}\,, \quad \Delta l = -\frac{\Delta\bar{K}_{II}}{k_I}\,\Delta h\,, \quad \Delta a \approx \Delta h\left[1 + \frac{2}{3}\left(\frac{\Delta\bar{K}_{II}}{k_I}\right)^2 - \frac{2}{5}\left(\frac{\Delta\bar{K}_{II}}{k_I}\right)^4\right]. \tag{17}$$

It is seen that under the condition $K_I = 0$ the change of the slope and the locus of the crack tip can be interpreted as the consequence of $\Delta\bar{K}_{II} \neq 0$ for a virtual tangential crack extension Δh (Fig. 3).

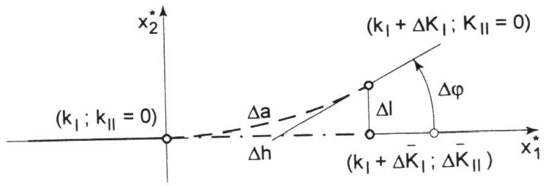

Fig. 3. Curved crack propagation

Therefore, in the case of proportional loading conditions the analysis of a crack path can be carried out by a small virtual tangential crack extension in combination with a finite change of the crack path. Due to the finite crack extension the calculation of $\bar{K}_I$ and $\Delta\bar{K}_{II}$ is necessary after each step Δh. This can effectively be done by using the finite element method. According to Fig. 4 it holds

$$\varphi_{j+1} = \varphi_j - 2\,\frac{\Delta\bar{K}_{II_j}}{K_{I_{j-1}}}\,, \quad \Delta l_j = -\frac{\Delta\bar{K}_{II_j}}{K_{I_{j-1}}}\,\Delta h_j\,, \quad a_j = a_{j-1} + \left[1 + \frac{2}{3}\left(\frac{\Delta\bar{K}_{II_j}}{K_{I_{j-1}}}\right)^2 - \frac{2}{5}\left(\frac{\Delta\bar{K}_{II_j}}{K_{I_{j-1}}}\right)^4\right]\Delta h_j\,. \tag{18}$$

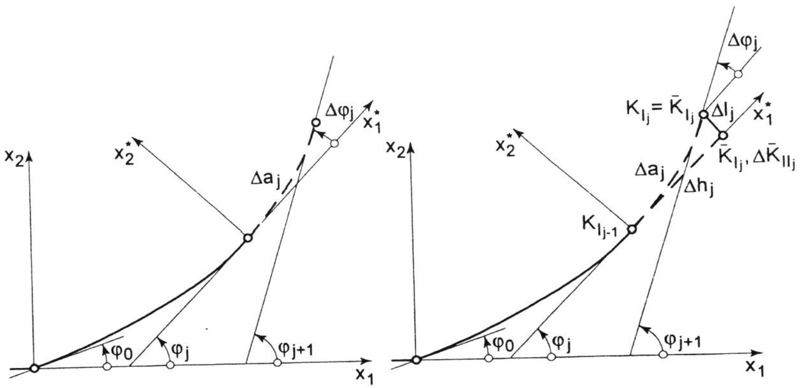

Fig. 4. Step by step approach with curved increments for crack path simulation

From eqs.(18) the need for an efficient numerical mode separation technique in conjunction with the step-by-step finite element analysis can be seen. Particularly with respect to this requirement the MVCCI-method has proved to be highly advantageous, because it delivers the separated strain energy release rates of two modes simultaneously with good accuracy and without any additional effort. For 8-noded quadrilaterals, which form the actual crack tip in Fig. 5, the following finite element representation of Irwin's crack closure integral relations can be given (Buchholz, 1984; Krishnamurthy *et al.*, 1985; Raju, 1987)

$$\bar{G}_I = \frac{1}{2\Delta h t}\left(F_{2,i}\,\Delta u_{2,i-1} + F_{2,i+½}\,\Delta u_{2,i-½}\right), \quad \Delta\bar{G}_{II} = \frac{1}{2\Delta h t}\left(F_{1,i}\,\Delta u_{1,i-1} + F_{1,i+½}\,\Delta u_{1,i-½}\right). \tag{19}$$

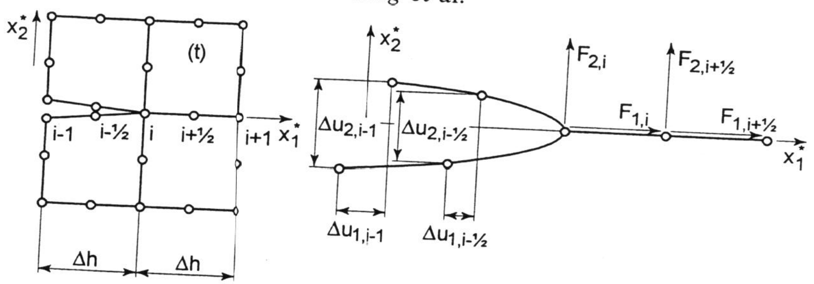

Fig. 5. Modified virtual crack closure integral method

NUMERICAL RESULTS

In order to evaluate the significance of the improved crack path simulations a special non-symmetrical specimen for lateral force bending has been designed (Theilig, 1979). Along the circular shaped transition region of the specimen notches have been attached at positions α_N= 0, 20, 40 deg. from which cracks initiated and extended with characteristically curved crack paths during the fatigue tests, Fig. 6. For each notch position five specimens have been investigated under proportional loading conditions. These crack paths were measured at the center line of the broken specimens. It was found that the experimental scatterband is rather narrow. Furthermore, it can be recognized that the local positions of the pre-cracks in the roots of the notches essentially determine the scattering.

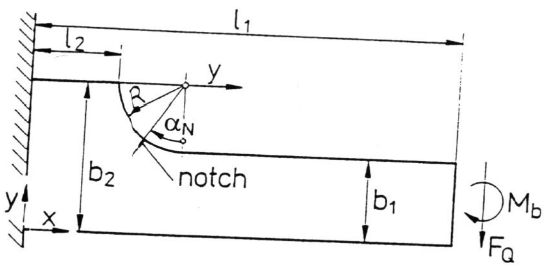

Fig. 6. Geometry and applied boundary conditions of the lateral force bending specimen

For the numerical calculations the length l_1 of the finite element model was chosen to 125mm. In accordance with the design of the lateral bend specimen the further dimensions are given with l_2 = 25mm, b_1= 25mm, b_2 = 45mm and t = 15mm (thickness). The lateral force was chosen to F_Q = 1kN with the consequence that M_b = 200Nm. Considering linear elastic material behaviour Young's modules and Poisson's ratio were chosen to E = 2.1 10^5 N/mm^2 and ν = 0.3. Fig. 7 shows the two basic finite element discretisations, which were found to be suitable for the selected virtual tangential crack increments Δh.

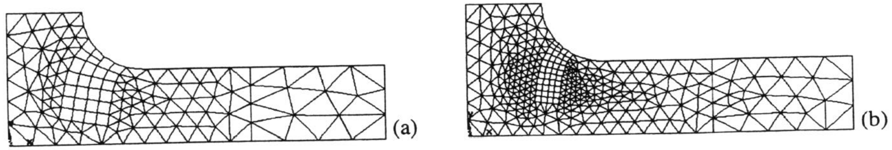

Fig. 7. FE-meshes with simulated crack extension steps α_N = 40deg., (a: Δh=4mm,b: Δh=2mm)

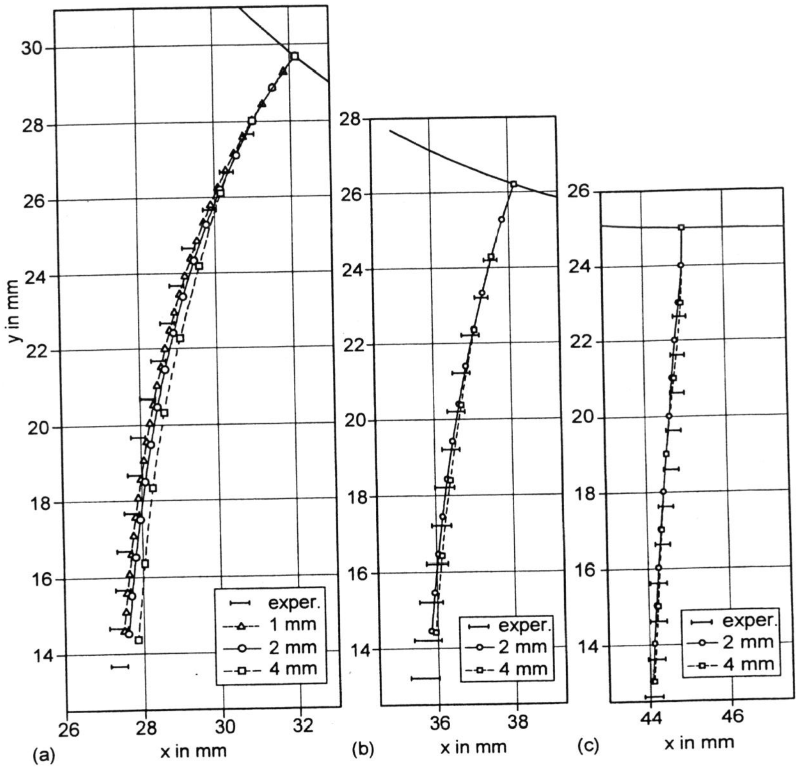

Fig. 8. Simulated and experimentally obtained crack paths
(a: α_N = 0 deg., b: α_N = 20 deg., c: α_N = 40 deg.)

After each simulation step of virtual tangential crack extension by Δh a remeshing is necessary in such a manner, that new crack tip elements are generated providing additional nodes in order to model the new curved crack surfaces. The calculations were carried out for Δh = 4mm, Δh = 2mm and Δh = 1mm. In Fig. 8 the computationally simulated crack trajectories for all notch positions are shown together with the experimentally obtained scatterbands from the related fatigue crack growth experiments. For all extension increments Δh an excellent agreement is found.

SUMMARY

This investigation has shown that the numerical tool of the MVCCI-method is delivering very good crack path simulation results with linear strain elements and only moderately refined finite-element-nets around the crack tip. The step-by-step simulation process with a piecewise curved approximation of the crack path offers an efficient way for the numerical analysis of fatigue crack growth in complex two dimensional structures under proportional loading conditions. From the excellent agreement of the numerical and experimental results one can conclude that the applied criterion of local symmetry works well. The step-by-step simulation in conjunction with the evaluation by the MVCCI-method provides the basis for a general computational approach to the fracture analysis of complex crack configurations and loading conditions that may be found in engineering applications within acceptable accuracy.

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