THE THEORY OF LONGTERM FATIGUE FRACTURE PROVIDING FOR CRACKS INITIATION AND PROPAGATION

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ABSTRACT

The paper presents the general theory of longterm fatigue fracture and some applications to the fatigue behaviour of elastic solids with initial cracks or stress concentrators. The regime of high cyclic loading is considered when the maximum stress in a cycle does not exceed the yield point and the main solid mass is deformed linear-elastically. The high cyclic fatigue problem is first formulated within the framework of Deformable Solid Mechanics. The fracture model is based on the joint consideration of the boundary-value problem of the elasticity theory and of the fatigue damage kinetics problem of the continuum damage theory. Fatigue fracture is considered as the crack propagation stage. Some problems of longterm fatigue fracture of thin isotropic plates with different stress concentrators are solved. Theory predictions are in good agreement with test results.

KEYWORDS

Fatigue fracture, high cycle fatigue, crack initiation, crack propagation, damage kinetics, thin plates, stress concentrators.

INTRODUCTION

The problem of the modelling of high cycle fatigue fracture is of important theoretical and practical significance and it is extensively discussed in scientific and engineering literature. The study of the fatigue process is one of several early research efforts in fracture mechanics. Over the years, an enormous body of experimental data has been accumulated in this area of investigations. Tests are highly numerous and cover various classes of materials, different structural elements, a wide temperature and loading duration range.

A large number of empirical relations to represent fatigue crack growth and fatigue fracture diagrams has been formulated on the basis of experimental data obtained. Various aspects of this problem are covered in an exhaustive literature (Frost and Dugdale, 1957; Hoeppner and Krupp, 1974; Panasyuk et al., 1977; and others). The well-known Paris and Erdogan's equation (1963) is most often used among them in practical applications. However, all empirical relations satisfy only approximated experimental data and cannot be extended to different loading conditions and structural elements.

Only single results have been obtained in the endeavour of the theoretical modelling of fatigue fracture processes. In fact only solutions proposed by Bolotin (1990), Golub and Panteleyev (1993), Golub and Plashchinskaya (1994) can be considered as such models. However, the solutions have been obtained in the case of infinite plates if the analytical expression for the stress distribution in the neighborhood of the crack tip is known. In case of finite dimension solids, the difficulties are connected with the necessity of the determination of the current stress-strain state taking time dependent boundary conditions into account.

The objective of this paper is the further development of the theoretical approach in high cycle fatigue problems and the modelling of fatigue fracture for finite dimension solids allowing for crack initiation and propagation stages.

TWO-STAGE FATIGUE FRACTURE THEORY

The fatigue fracture process is considered from the phenomenological point of view and it is solved as the coupled boundary-value problem and damage kinetics problem.

Assumed Relations. As an example of the plane stress state, consider the problem of the fatigue fracture processes modelling under high cyclic loading. Let a thin isotropic plate (see Fig.I) of 2a in length and 2b in width is subjected to a uniaxial symmetrically varying cyclical stress

$$\tilde{\sigma} = \sigma_a \sin(2\pi n), \qquad (I)$$

which is applied along the 2b length. Here σ_a is the alternating stress and n is the number of cycles change in load that will be treated as continuous time from now on. The plate may be weakened with open stress concentrator or pointed crack. The crack can be an incipient defect and it can originate during cyclic loading. It will be also assumed that all stresses cycle at same frequency and that their amplitudes do not change during the cycling.

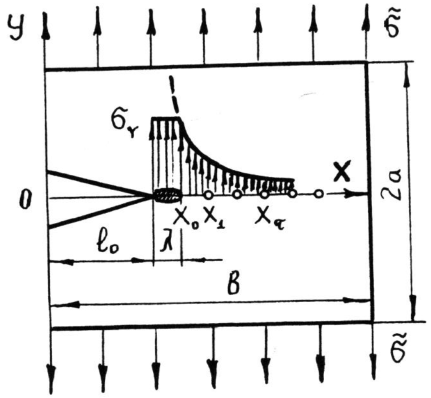

Fig.I. Scheme of a thin plate cyclic loading with crack.

We examine the high cyclic loading regime when the maximum stresses do not exceed the material yield point, so that the main body of the plate is deformed linear-elastically. In this case, fatigue fracture is first determined by the damage accumulation process given with an evolutionary equation

$$\frac{d\omega_f}{dn} = B\left(\frac{\tilde{\sigma}}{1-\omega_f}\right)^m, \qquad (2)$$

where ω_f is the scalar damage parameter, and B, n are material constants.

The alternating stress σ_a does not exceed the material yield point σ_Y under high cyclic loading, so that the main body of the plate is deformed linear-elastically. All nonlinear effects are concentrated into a thin plastic zone having a space scale λ_f which is formed in the vicinity of the fatigue crack tip (see Fig.I). The law of changing in the plastic zone length λ can be given by Rice's (1967) relation

$$\lambda_0 = \left(\frac{\sigma_a}{\sigma_Y}\right)\cdot\frac{\ell_0}{3} \Longrightarrow \lambda_f = \left(\frac{\sigma_a}{\sigma_Y}\right)^2\frac{\ell_f}{3}, \qquad (3)$$

where σ_{max} is the maximum stress, ℓ_0 is the initial, and ℓ_f is the current length of a fatigue crack.

Main Solving Equations. During loading conditions (I), in the smooth part of a plate the uniaxial stress state and in the neighbourhood of a stress concentrator the plane stress state are realized. With σ_{ij}^{max} and σ_{ij}^{min} to denote the maximum and minimum amplitudes of a cyclic stress component, the alternative stress σ_{ij}^a is defined by

$$\sigma_{ij}^a = \frac{1}{2}\left(\sigma_{ij}^{max} - \sigma_{ij}^{min}\right) \qquad i,j = I,2. \qquad (4)$$

The problem of the long-term fatigue fracture modelling consists in the finding of the function

$$F_\ell\left\{\sigma_{ij}^a(x,n), \dot{\ell}_f, \ell_f, n\right\} = 0 \qquad \forall n > n_*, \qquad (5)$$

which establishes the relationship between stress tensor components σ_{ij}^a $(\cdot)$, the current fatigue crack length ℓ_f, the fatigue crack growth rate $\dot{\ell}_f$ and the number of loading cycles n. The initial plate geometry L_0 is known (see Fig.I) and can include the initial crack of ℓ_0 length.

The fatigue fracture phenomenon is interpreted as a two-stage process including the incubation stage and the stage of the crack propagation. The function F_ℓ is determined for each stage from the joint solution of the boundary-value problem of the elasticity theory and of the fatigue fracture kinetics problem of the continuum damage theory. The conditions of the fatigue crack initiation and of the fatigue fracture front movement taking the continuum damage mechanics as a basis are written as follows

$$\max\left\{\omega_f(x_*, n_*)\right\} = 1 \quad \text{and} \quad \max\left\{\omega_f(\Sigma_f, n)\right\} = 1. \qquad (6)$$

Here X_* are coordinates of a maximal loaded point, n_* is the duration of the incubation stage, and Σ_f is the fatigue fracture front, which moves.

We define the incubation stage as a period while the material is damaged without failing in the sence of the fracture front formation and propagation. The initial crack opens only but not moves during the incubation stage. In this case the stress state of a plate varies only by means of change in external load. So, the fatigue fracture model for the incubation stage is given by the set of governing equations

$$
\left.
\begin{array}{lll}
(a) & \begin{cases} \sigma_{ij,j}(x_i)=0 \\ \nabla^2\sigma_{ij}(x_i)=0 \end{cases} & i,j=1,2 \\
(b) & \begin{cases} \sigma_{11}(x_i)=\tilde{\sigma}(n);\ \sigma_{12}(x_i)=0 \\ \sigma_{22}(x_i)=\sigma_{21}(x_i)=0 \\ \sigma_{ij}(x_i)\nu_i=0 \end{cases} & \begin{array}{l} x_i\in L_{01}(x;\pm b) \\ x_i\in L_{02}(\pm a;y) \\ x_i\in L_{0c} \end{array} \\
(c) & \sigma_{eqv}=\gamma\sigma_1+(1-\gamma)\tau_{oct} & \\
(d) & \begin{cases} d\omega_f=B(\sigma_{eqv})^m(1-\omega_f)^{-m}dn \\ \omega_f(x_i,0)=0 \end{cases} & \\
(e) & \max\{|\omega_f(x_*,n_*)|\}=1 &
\end{array}
\right\} \qquad (7)
$$

consisting of the elasticity theory equations (a), the boundary conditions (b), the equivalent stress (c) chosen in the form of a mixed invariant between the main principle stress σ_1 and the octahedral shear stress τ_{oct}, the fatigue damage evolutionary equation with the initial condition (d), and the condition of the fracture front formation (e).

The solution of the set of Eqs.(7) gives the value of n_*. At the incubation stage the plate boundary L_0 does not depend on the number of loading cycles n bacause the damage state does not influence the state of stress, the set of Eqs.(7) is not coupled, and its solution may be obtained in an analytical form.

The stage of the fatigue crack propagation is connected with the formation of new free surfaces. As a result the external contour of a plate changes, the boundary conditions vary over time, and the stress-strain state varies not only in view of the variation of loads, but also by means of the crack movement. The change in the stress state leads to the change in the damage kinetics. On the other hand, the damage accumulation process gives rise to the crack growth, to the change in the boundary conditions and to the change in the stress state respectively. So, the fatigue fracture model for the crack propagation stage is written in the form

$$
\left.
\begin{array}{lll}
(a) & \begin{cases} \sigma_{ij,j}(x_i)=0 \\ \nabla^2\sigma_{ii}(x_i)=0 \end{cases} & i,j=1,2 \\
(b) & \begin{cases} \sigma_{11}(x_i)=\tilde{\sigma}(n);\ \sigma_{12}(x_i)=0 \\ \sigma_{22}(x_i)=\sigma_{21}(x_i)=0 \\ \sigma_{ij}(x_i)\nu_j=0 \end{cases} & \begin{array}{l} x_i\in L_{01}(x;\pm b) \\ x_i\in L_{02}(\pm a;y) \\ x_i\in L_f(n) \end{array} \\
(c) & \sigma_{eqv}=\gamma\sigma_1+(1-\gamma)\tau_{oct} & \\
(d) & \begin{cases} d\omega_f=B(\sigma_{eqv})^m(1-\omega_f)^{-m}dn \\ \omega_f(x_i,n_*)=\omega_* \end{cases} & \\
(e) & \max\{|\omega_f(x_i,n)|\}=1 &
\end{array}
\right\} \qquad (8)
$$

where $L_f(n)$ is the current crack surface contour depending on the number of loading cycles.

The law of fatigue crack growth (5) is the solution of the set of Eqs.(7). This set is a coupled and essentially nonlinear one and can be solved numerically only.

Basic Mechanical Effects. The formulation of the initial conditions for the starting of the crack stage propagation from the solution obtained at the incubation stage is the fundamental moment of the approach suggested. The conditions are given by the value of n_* which connects the set of Eqs.(7) to the set of Eqs.(8).

The salient feature of the approach is that it also takes into account the influence of damage accumulated along the crack propagation front Σ on the crack growth rate. Indeed, differentiating Eq.(6) as a complex function, we have

$$\frac{d\Sigma}{dn}=-\frac{\partial\omega_f}{\partial n}\cdot\left(\frac{\partial\omega_f}{\partial x}\right)^{-1} \qquad (9)$$

where x is the coordinate of an arbitrary point ahead of the propagating crack front. So, if ω_f did not depend on x and were a constant, the crack growth rate would obey $\Sigma\Rightarrow 0$, because $\partial\omega_f/\partial x\Rightarrow 0$. In this event the crack opening time would coincide with the time of complete fracture, and the crack growth stage would not even exist.

Another distinctive attribute of the above-formulated models is that the fatigue crack growth rate also strongly depends on the law of variation of the length of the plastic zone during crack propagation in a center-cracked infinite plate. So, using the approximate asymptotic stress distribution in the neighbourhood of the crack tip (x, $y=0$), proposed by Williams (1957), and taking the plastic zone of length λ into account, we have the relation for σ_{yy}

$$\sigma_{yy}=\frac{\sigma_a}{2}\sqrt{\frac{\ell_f(n)}{x+\lambda(\ell_f)-\ell_f(n)}} \qquad (10)$$

which is interpreted graphically by the solid curve in Fig.1 and in fact is the solution of Eqs.(7a) and (7b). Substituting Eqn.(10) into (7d) and taking into account the condition of fracture front movement (6), and applying simple transformations, we obtainethe following for the fatigue crack growth rate

$$\frac{d\ell_f}{dn}=\left(1+\frac{1}{m}\right)B\frac{(\sigma_a)^m(\ell_f)^{m/2}}{(2\lambda_f)^{m/2-1}}\Rightarrow\frac{d\ell_f}{dn}=\left(1+\frac{1}{m}\right)\frac{B(\Delta K)^m}{(2\sqrt{\pi})^m(2\lambda_f)^{m/2-1}} \qquad (11)$$

Here $\Delta K = 2\sigma_a\sqrt{\pi \ell_f}$ is the peak-to-peak amplitude of the stress intensity factor, and it is also assumed, that $\ell_f = \ell_f(n)$ and $\lambda(\ell_f) = \lambda_f$.

So, using the first relation from Eqn.(3) for the plastic zone length λ and substituting it into Eqn.(II), the fatigue crack growth rate is written as follows

$$\frac{d\ell_f}{dn} = \left(1+\frac{1}{m}\right)B\left(\frac{\Delta K}{2\sqrt{\pi}}\right)^m \cdot \left(\frac{\sigma_Y}{\sigma_a}\right)^{m-2} \cdot \left(\frac{3}{2\ell_0}\right)^{m/2-1} \qquad (I2)$$

and using the second relation from Eqn.(3), the fatigue crack growth rate is written in the form

$$\frac{d\ell_f}{dn} = \left(1+\frac{1}{m}\right)B\left(\frac{\Delta K}{2\sqrt{\pi}}\right)^2 \cdot \left(\frac{3\sigma_Y}{2}\right)^{m/2-1} . \qquad (I3)$$

It can be seen from Eqn.(I2) that, in the case of a constant-length plastic zone the crack growth rate essentially depends on the coefficient m, which can varies from 2 to I0-I2 when the material becomes more brittle.

In the case of a variable-length plastic zone, the crack growth rate (Eqn.(I3)) is given by a square-law dependence on the stress intensity factor ΔK. Such dependence is typical for more plastic materials. The crack growth rate decreases as the plastic zone length increases during the crack propagation.

CALCULATION RESULTS AND DISCUSSION

Within the framework of the theory suggested some problems of longterm fatigue fracture of thin isotropic plates containing different stress concentrators and subjecting to uniaxial high cycle loading are solved.

Procedure of Coefficients Determination. To solve problems of fatigue fracture based on Eqs.(7) and (8), it is necessary to have two groups of characteristics. The first group includes the coefficients m and B and specifies the resistance of a material to fatigue damage accumulation. The second group includes the limits of short-term strength σ_Y and ε_Y. All the necessary material characteristics are determined from standard experiments involving the axial loading of smooth cylindrical specimens.

The coefficients m and B are determined from tests of smooth specimens in fatigue under symmetrical loading. All points of a smooth solid are equal in strength, the incubation stage essentially coincides with complete fracture, and from (7) we obtain the following expression for the number of cycles to fracture

$$n_R = \frac{1}{(1+m)B(\sigma_a)^m} \qquad (I4)$$

Here it is assumed that $n_R = n_*$, and $\sigma_a = \sigma_{eqv}$. Eqn.(I4) approximates experimental data on the fatigue fracture by a straight

line in logarithmic coordinates, and the values of m and B determined by the position and slope of this line.

The value of the yield point σ_Y and the value of the corresponding strain ε_Y are determined from a stress-strain diagram.

Methods of Solution. The main idea of the solution method proposed consists in the development of a computational scheme which allow to solve the problem about the stress state and the Cauchy problem for the damage function separately.

During the incubation stage, as it was mentioned above, the set of corresponding governing Eqs.(7) is not a coupled one and the solution for the incubation stage duration n_* can be obtained in the analytical form as

$$n_* = \frac{1}{(1+m)B(\sigma_{eqv})^m} \Longrightarrow n_* = \frac{1}{(1+m)B(\sigma_Y)^m} . \qquad (I5)$$

Here the first relation is used in the case of smooth initial stress concentrators and the second one in the case of pointed initial cracks.

During the crack propagation stage, the set of corresponding governing Eqs.(8) is coupled one and the law of fatigue crack growth can be written only in the form

$$n(\ell_f) = n_* + \sum \left[\Phi^{(\tau-1,\tau)}\right]^{-1}\left\{1-\omega_{\tau,0}\sum_{k=1}^{\tau-1}\left[\Phi^{(k-1,\tau)}(n_k) - \Phi^{(k-1,\tau)}(n_{k-1})\right]\right\} \qquad (I6)$$

where τ is the number of partion points along the crack movent trajectory (see Fig.I); $\Phi(\cdot)$ is the operator of the solution of the elasticity theory equations (8a,b); and $\omega_{\tau,0}$ is the damage value accumulated in an arbitrary point X_τ at the moment $n_0 = n_*$ of the incubation stage termination.

Eqn.(I6) is solved numerically. The calculation process can be organized in the following manner.

- First the initial stress state σ^0_{ij} is calculated using one of the known numerical methods, for example by means of the Finite Element Method or Boundary Element Method.
- The most loaded point is determined, the value of the equivalent stress σ_{eqv} is computated and the incubation stage duration n_* is calculated.
- The path of a crack movement is divided into the finite number of intervals and starting from the initial stress state σ^0_{ij} the value of fatigue damage $\omega_{\tau,0}$ is computated in each knot of partition X_τ at the moment $n_0 = n_*$.
- A small increment $\Delta\ell_f = X_\tau - \ell_{\tau-1}$ is given to the current crack length ℓ_τ, the stress state $\sigma^{L_{\tau-1}}_{ij}$ is computated for the plate geometry $L_{\tau-1}$, and the corresponding value of the equivalent stress $\sigma^{L_{\tau-1}}_{eqv}$ is calculated.
- The value of fatigue damage $\omega_{\tau-1}$ is computated in each node of partition $X_K (K>\tau)$ at the moment of time $n_{\tau-1}$, using the condition of fracture front movement the moment of time n_τ

of the crack tip transfer in the node X_τ is determined, and therefore the value of the number of cycles increment $\Delta n_\tau = n_\tau - n_{\tau-1}$ corresponding to the fatigue crack length increment $\Delta \ell_\tau$ is calculated.

This algorithem leads to the jump-like law of the fatigue crack growth that corresponds to the physical nature of the fatigue fracture process. In case when the number of partition of the crack movement trajectory will tend to infinity, the crack law propagation will be continuous.

<u>Comparison with Experiments</u>. The thin isotropic plates with central slots, with central smooth circular holes, with central circular hole and small radial cracks growing from it, and plates with side notches are considered. Fatigue fracture of thin plates is solved for two classes of materials including brittle (EP7I8 alloy at 20 °C) and plastic (HN55MVC steel at 750 °C) materials.

The boundary element method is used to solve numerically the set of governing Eqs.(8) in case of crack growth.

The calculation results are yielded as the fatigue crack length ℓ_f plotted against the number of cycles n with the stress amplitude σ_a as a parameter. On the whole, it is shown that the fatigue crack growth more intensively with the increasing of stresses and of the initial crack length, and with the taking the dimensions finiteness of plates into account. These results are supported by known results and experiments.

As an example, Fig.2 showed also the calculation results (lines) obtained for infinite (solid line) and finite (dashed lines) size plates with central slot (a) and with smooth central hole (b). The plates is symmetrical in geometry and loading as

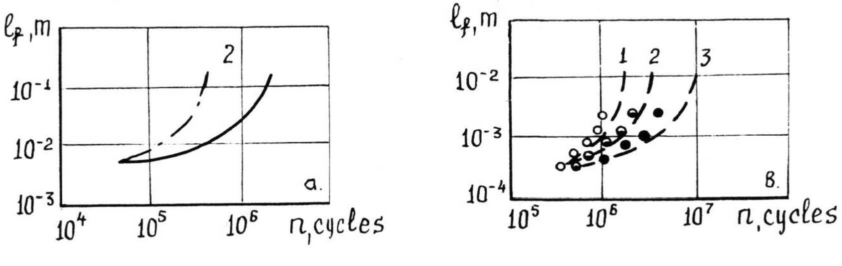

Fig.2. Kinetics of fatigue crack growth considering dimentions finiteness of plates (a) and stress variation (b).

shown in Fig.I. It is made of EP7I8 alloy (σ_Y = 700 MPa; B = I,7.I0^{-12} MPa^m .cyc.; m = 2,6) with a thickness of I mm. The experimental data have been presented by circles.

On the whole, as can be seen from Fig.2b, the calculations agree quite satisfactorily with the experiment. The maximum error of fatigue crack length is 20 %.

SUMMARY AND CONCLUSION

The problem of high cyclic fatigue fracture modelling has been considered in terms of Continuum Damage Mechanics. It is assumed that the material undergoes damage and small elastic strains. The hypothesis of effective stress is used to produce the coupling between elasticity and damage. The concept of a thin plastic zone in the neighbourhood of the fatigue crack tip which moves together with the crack is employed.

An constitutive model is formulated to analyze problem of fatigue fracture with allowance for two-stages process including the incubation stage and the crack propagation stage. The model is based on the joint consideration of the elasticity theory and of the continuum damage theory. A numerical example is presented to show the effectiveness of the coupled model used. A thin plate with a center hole that is subjected to uniaxial tension-compression is analyzed using the boundary element method. The results obtained from the fatigue fracture model are compared with experiments.

Let us emphasize that, all calculations have been made using material coefficients which have been determined by testing of smooth cylindrical specimens only.

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