NOTCH EFFECT IN HIGH CYCLE FATIGUE

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ABSTRACT

The notch effect in fatigue can be interpreted in terms of local criterion assuming that the stress range is higher than an effective value in the process zone characterized by the effective distance. These parameters can be established on the notch tip stress distribution and are connected with the notch stress inensity factor range.

This approach allows to get by computing the fatigue stress concentration factor and the notch sensitivity index.

KEYWORDS

Notch effect, fatigue initiation

INTRODUCTION

The presence of a defect in a structure is more dangerous than a simple reduction of the net cross section. This effect is generally called " notch effect " and can be observed for fatigue and fracture.It can be easily seen on a critical global stress σ_g^c versus non dimensional defect size a/W diagram. This diagram is generally called the Feddersen's diagram. (figure 1).

Load transmission on the specimen ligament without stress concentration leads to that critical nominal stress σ_g^c which is equal to the ultimate strength Rm of the material.This gives a linear decreasing of the critical global stress with the flaw size according to :

$$\sigma_g^c = \sigma_N^c = Rm\left(1 - a/W\right) \qquad (1)$$

Where a is the flaw size, σ_N^c the critical nominal stress and W the specimen width.

In the presence of a notch, the critical global stress exhibits a non-linear relationship versus the non dimensional defect size and its value is less than that obtained according to equation (1), except for very small and very large defects according to :

$$\sigma_g^c (a)^{\alpha} = const \qquad (2)$$

where (α) is a constant less than 0,5 of the fracture process. The stress value is higher than that given by the criterion '' local stress equal to ultimate strength'' :

$$\sigma_l^c = \sigma_N^c / k_t \tag{3}$$

k_t is the elastic stress concentration factor.

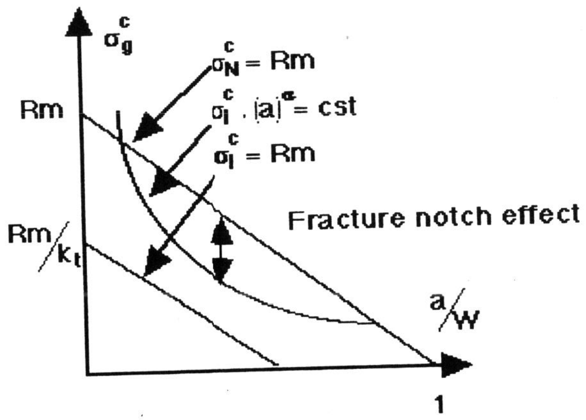


Figure 1 : representation the notch effect in fracture.

For the fatigue process, we can consider that an effective stress range $\Delta\sigma_e$ plays the major role. For a smooth specimen, this effective stress range is similar to the global stress range :

$$\Delta\sigma_e = \Delta\sigma_g \tag{4}$$

For the same life duration N_r and for a notched specimen, the effective stress range can be considered as a material constant but the global stress range is less and equal to :

$$\frac{\Delta\sigma_e}{k_f} = \Delta\sigma_g \tag{5}$$

This value is higher than the local stress range $\Delta\sigma_l$:

$$\frac{\Delta\sigma_e}{k_t} = \Delta\sigma_l \tag{6}$$

The fatigue process needs a physical volume called process zone. In a simple way, this volume can be assimilated to a cylinder with a thickness equal to those of the structure. The diameter of the process zone is called the effective distance X_e. In other words, the stress gradient cannot be too important in this area and the average stress is relatively high. The role of the stress gadient in the fatigue process was mentionned prevouisly by Brandt [1] and Buch [2]. The stress distribution in the process zone of notched specimen can be described by the concept of notch stress intensity factor [3]. The value of notch stress intensity factor range can be used as a fatigue initiation criterion for notched specimen as it can be seen in this paper.

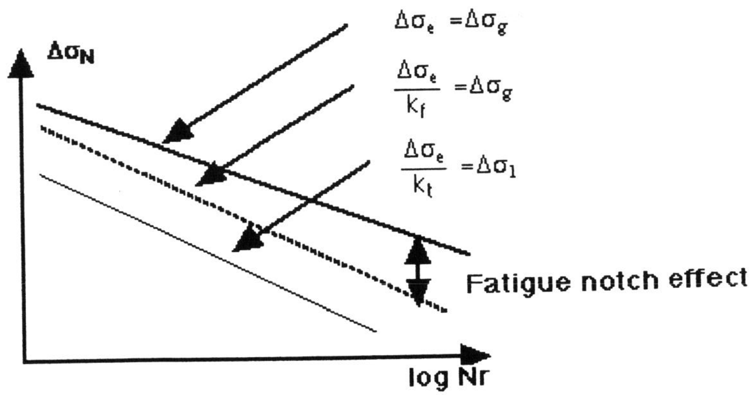


Figure 2 : Notch effect in fatigue

2) STRESS DISTRIBUTION AT NOTCH TIP.

A typical stress distribution at the notch tip of a three points bending (3PB) specimen can be seen on figure 3. More precisely, we have plotted the normal to the notch plane stress versus the distance ahead the notch tip. Two typical elements from this distribution can be considered :

- the maximum stress σ_{yy}^{max}
- the maximum stress disribution $\sigma_{yy} = f(\sigma_{yy}^{max}, \rho)$

The maximum stress at notch tip σ_{yy}^{max} is related to the nominal stress σ_N by the mean of the stress concentration factor. Two kinds of stress concentration factors have to be considered :

- the elastic stress concentration factor k_t for $\sigma_{yy}^{max} \leq Re$
- the elastoplastic stress concentration factor k_S for $\sigma_{yy}^{max} \geq Re$

Where Re is the yield stress. Several relationships between the elastic, the elasto-plastic stress concentration factor and the elastoplastic strain concentration factor can be found in the literature [4]. They have the following form :

$$\frac{k_t^2}{k_\sigma k_\varepsilon} = f(\sigma_N) \tag{7}$$

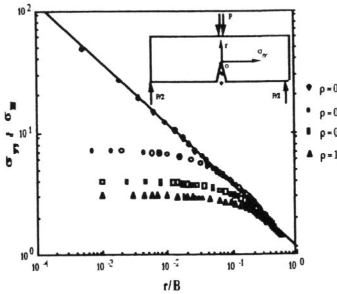


Figure 3 : Stress distribution at notch tip, 3PB specimen.

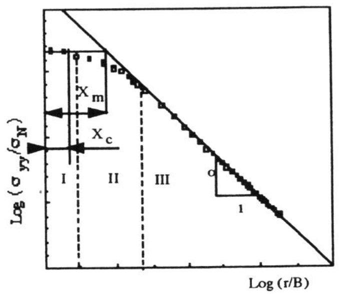


Figure 4 : Schematic notch tip stress distribution in a bilogarithmic graph.

We can notice that the well known Neuber's formula [5] gives a relationship independent the of stress level :

$$k_t^2 = k_\sigma k_\varepsilon \qquad (8)$$

The elastic or the elasto-plastic stress distribution at notch tip exhibits a decreasing dependance with the notch tip distance .It is relatively more complicated than the crack tip stress distribution exhibiting a ($r^{-1/2}$) dependance. This distribution is presented in a bilogarithmic graph (figure 4). The logarithm of the non dimensionnal normal stress is plotted versus the logarithm of the non dimensional distance r/b where b is the ligament size.The stress distribution diagram can be divided into 3 parts :

- zone I where the non dimensionnal normal stress is practically constant and equal to the stress concentration factor $\sigma_{yy}/\sigma_N = k_\sigma$;
- zone III where the non dimensional normal stress exhibits a power dependance with the non dimensional distance (C and a are constants with a ≤ 0,5) :

$$\frac{\sigma_{yy}}{\sigma_N} = C\left(\frac{r}{b}\right)^{-\alpha} \qquad (9)$$

- zone II intermediate between zone I and zone III.

Stress distribution in zone III can be assimilated to a "pseudo stress singularity". This distribution can be considered only for a distance greater than X_m defined on figure 2 with the following form

$$\sigma_{yy} = \frac{K_\rho}{(2\pi r)^\alpha} \text{ for } r \geq Xm \qquad (10)$$

where K_ρ is the so called notch stress intensity factor (NSIF).

This notch stress intensity factor gives a description of the stress gradient at a notch tip, which plays an important role on the fatigue process emanating from notches.

Each notch can be considered with the two characteristic geometrical parameters : the notch radius r, and the notch angle, ψ. We can distinguish different cases of notches and with different stress intensity factors :

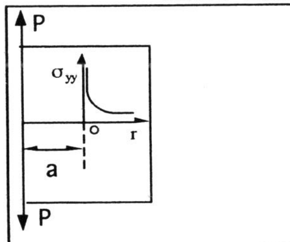
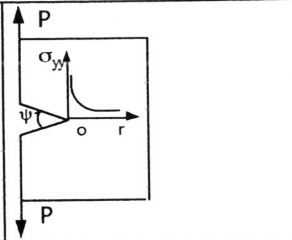
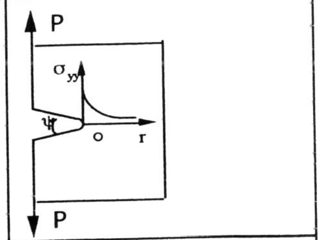
- a crack is a particular case of notch with ρ = 0 and ψ = 0 ;
- an infinite acuity notch in the William's sense [6] with ρ = 0 and ψ ≠ 0 ;
- a simple notch for which, ρ and ψ are different from zero.

We will distinguish 3 kinds of stress intensity factors namely :

- (crack) stress intensity factor K_I ;
- infinite acuity notch stress intensity factor K^*_I ;
- notch stress intensity factor K_ρ.

A general picture of these different stress intensity factors can be seen in table 1.

Table 1 Different kinds of stress intensity factors

		
stress intensity factor K_I	infinite acuity notch stress intensity factor K^*_I	notch stress intensity factor K_ρ.
$K_I = \lim_{r \to 0} \sqrt{2\pi r}\, \sigma_{yy}$	$K^*_I = \lim_{r \to 0} 2\pi^\alpha \sigma_{yy} r^\alpha$	$\sigma_{yy} = \frac{K_\rho}{\sqrt{2\pi r}^\alpha}$

3) THE USE OF THE NOTCH STRESS INTENSITY FACTOR RANGE TO DEFINE THE EFFECTIVE STRESS IN FATIGUE

The stress range distribution at notch tip can be computed in fatigue by using the cyclic stress strain curve after transformation to take into account the fact that the origin of loading is the lowest point of the hysteresis loop. The cyclic stress strain curve is given by :

$$\Delta\sigma = K'\Delta\varepsilon^{n'} \qquad (11)$$

The equation of the rising part of the hysteresis loop is :

$$\Delta\sigma = K_1 \Delta\varepsilon^{n_1} \ (12) \qquad \text{with } n' = n_1 \text{ and } K_1 = K'^{\,2n'-1} \qquad (13)$$

A typical exemple of the stress range distibution in a bilogaritmic graph can be seen on figure 4. This graph is relative to a single edge notch specimen with a notch angle Ψ = 45° and a notch radius ρ = 0,2mm. The stress distribution is typical and similar to those decribed previously.

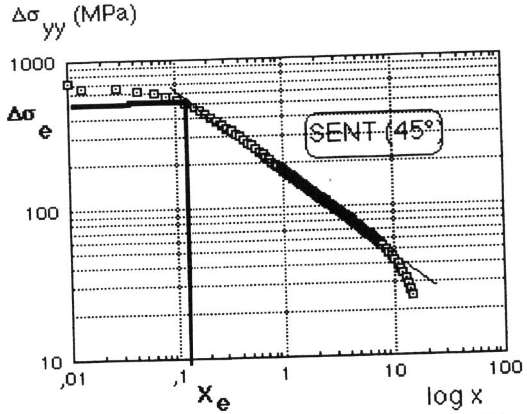


Figure 5 : Stress distribution at notch tip in a bilogaritmic graph ; SENT specimen with a 45° notch angle.

The upper limit of the linear part of the distribution has the following coordinates (X_e ; $\Delta\sigma_e$) where X_e is the effective distance and $\Delta\sigma_e$ the effective stress range. The choice of the value of the effective distance is based on the following considerations :

- X_e is greater than the grain size and less than the notch radius . This is due to the physical nature of the fatigue process ;
- the effective stress range is also defined by :

$$\frac{\Delta\sigma_e}{k_f} = \Delta\sigma_g \tag{14}$$

which leads to the values of the distance X* :

$$\Delta\sigma_e = \frac{K_\rho}{(2\pi X^*)^\alpha} \tag{15}$$

Experimenal investigations [7] showthat X_e ª X^*. A typical Wöhler curve $\Delta\sigma_e$ f(log Nr) where N_r is the number of cycles for life duration can be plotted with this criteria (figure 6). The different curves $\Delta\sigma_e$ f (log N_r) obtained from specimens with different notch radius exhibit a small difference. Considering the natural scattering of fatigue tests, this kind of Wöhler curves can be considered as intrinsic to the material. Using the Creager's solution [8] for notch tip stress distribution, Clark and Price [9] obtain the following fatigue criterion :

$$\Delta K/\sqrt{\rho} = f(N_r) \tag{16}$$

This criterion cannot be compared directly to the previous .More exactly the Creager's solution leads to the criterion

$$\Delta K/\sqrt{2\pi\rho} = f(N_r) \tag{18}$$

A comparison beetween these two criteria shows an effective stress higher than the 'Clark and Price' stress.

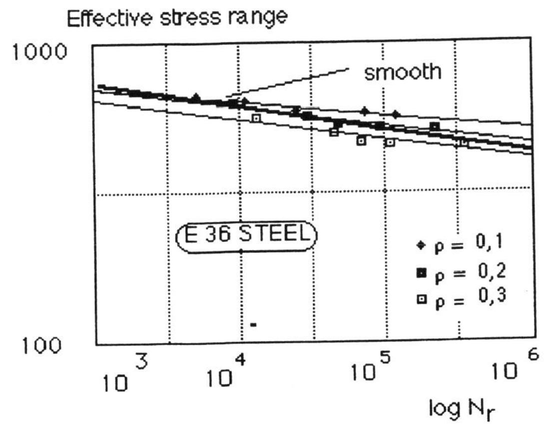


Figure 6 : Wölher's curves drawn with the effective stress from smooth and notched specimens.

This kind of Wöhler's curve can also be computed for mode II fatigue initation using slanted notch ring specimen [10] (figure 7 and 8). The local shearing stress amplitude is calculated according to the following formula where θ is the crack intiation angle (70°5 for pure mode II).

$$\tau_{xy} = \frac{\Delta K_{II}^{\rho}}{(2\pi X_e)^{\alpha}} \cdot \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin 3\frac{\theta}{2}\right) \tag{19}$$

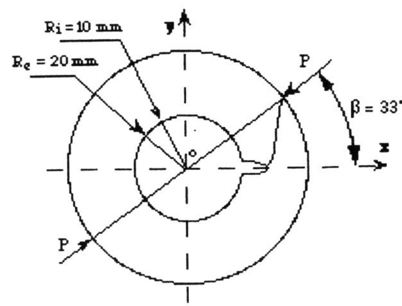


Figure 7 : Ring specimen for mode II fatigue initiation test.

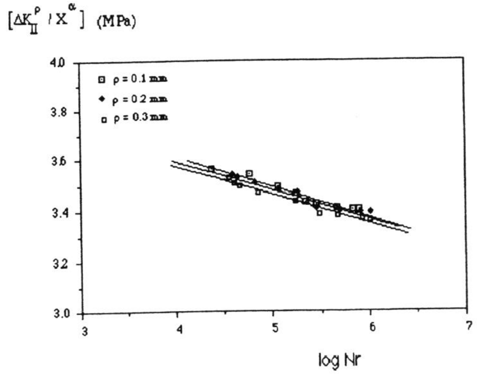


figure 8 : Mode II fatigue initiation curve.

4) THE NOTCH SENSITIVITY INDEX

The notch effect in fatigue can be characterized by the fact that the Wöhler's curve obtained with notched specimens is below the smooth specimen one, i.e. for the same stress level the life duration is reduced (figure 9). It is necessary to mention that this life duration is greater than that we can get by the unique consideration of the local stress range.

$$\Delta\sigma_l = k_t \Delta\sigma_N \qquad (20)$$

Where k_f is the stress concentration factor and $\Delta\sigma_N$ the nominal stress range. By assumption, the smooth specimen Wöhler's curve is intrinsic to the material. A life duration N_r is obtained for a nominal stress range $\Delta\sigma_N$ on a smooth specimen and for an effective stress range $\Delta\sigma_e$ on notched specimens.

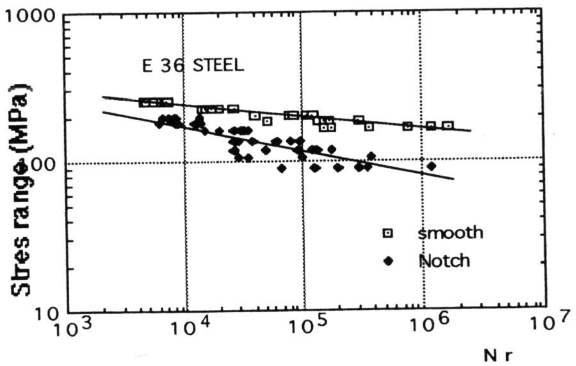


Figure 9 : Wöhler's curves for smooth and notched specimens (E36 steel).

The fatigue stress concentration factor k_f is defined as :

$$k_f = \frac{\Delta\sigma_e}{\Delta\sigma_N} \qquad (21)$$

The effective stress range is actually the mean stress which acts on the fatigue process volume. This volume can be considered as a cylinder of thickness B and diameter X_e where X_e is the effective distance. We define usually a fatigue sensitivity index to show the difference beetween the elastic stress concentration factor k_t and the fatigue stress concentration factor k_f .This difference is generally considered as the result of the plastic relaxation at notch tip.It is probably due to the fact that the fatigue process needs a physical volume and is not only sensitive to the local stress amplitude. Three definitions of the fatigue sensitivity index can be found in the litterature.

The oldest defined by :
$$q_1 = \frac{k_f - 1}{k_t - 1} \qquad (22)$$

The most commonly used :
$$q_2 = \frac{k_f}{k_t} \qquad (23)$$

The most recent :
$$q_3 = \frac{k_f}{k_\sigma} \qquad (24)$$

Where k_s is the elasto-plastic stress concentration factor. Many empirical formula have been proposed for the relationship beetween the value of the notch sensitivity index and the notch radius ρ. They are summarized in table 2. The accuracy of these empirical formula is not known and needs to be checked with numerous fatigue tests. In addition, the evolution of the q parameters with the number of cycles is not always formulated.

The relationship of the notch sensitivity index q_3 is easily interpreted by considering the stress range distribution at notch tip in a bilogarithmic graph. Figure 10 gives an exemple of such a diagram which has been computed using the cyclic stress-strain curve of the material. As a definition the maximal stress range $\Delta\sigma_{max}$ is equal to :

$$\Delta\sigma_{max} = k_t \Delta\sigma_N \qquad (25)$$

The effective stress range is :

$$\Delta\sigma_e = k_f \Delta\sigma_N \qquad (26)$$

In zone III of this diagramm, the stress distribution obeys to the following relationship :

$$\Delta\sigma = \frac{\Delta K_\rho}{(2\pi r)^\alpha} \qquad (27)$$

Where ΔK_ρ is the notch stress intensity factor range. In a bilogarithmic graph, on the line corresponding to equation (27), the X_e and X_m distance are associated to the $\Delta\sigma_e$ and $\Delta\sigma_{max}$ range. The value of q_3 is :

$$q_3 = \frac{k_f}{k_\sigma} = \frac{\Delta\sigma_e}{\Delta\sigma_{max}} = \left(\frac{X_e}{X_m}\right)^\alpha \qquad (28)$$

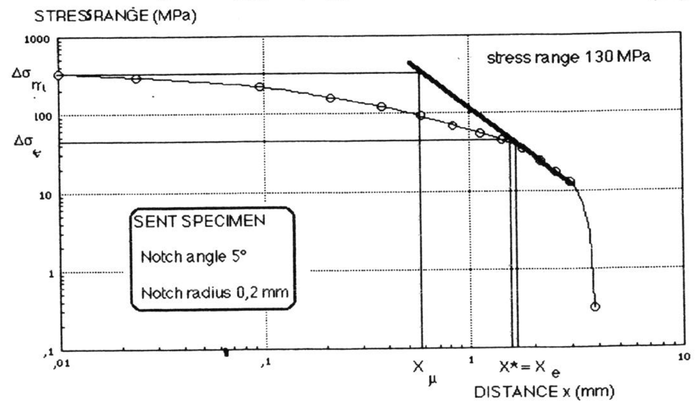


Figure 10 : Stress range distribution at the notch tip ; (SENT Ψ= 45°, ρ = 0,2mm).

Table 2 Notch sensitivity index formula

Authors	Formula	Constants
NEUBER [5]	$q_1=\left[1+\sqrt{\left(\rho'/\rho\right)}\right]^{-1}$	r' = constant
PETERSON [11]	$q_1=\left[1+\left(\rho'/\rho\right)\right]^{-1}$	r' = f(Rm) Rm ultimate strength
HARRIS[12]	$q_1=1-\exp\left(\dfrac{-\rho}{a_h}\right)$	$a_h=f(Rm)$
LUKAS ET KLESNIL [13]	$\begin{cases} q_1=\dfrac{1}{F_\sigma}\sqrt{\dfrac{\rho}{4l_0}} & \text{pour } \rho\le 4l_0 \\ q_1=1 & \text{pour } \rho\ge 4l_0\end{cases}$	F_σ : gemetrical factor l_0 material constant
TOPPER et EL HADDAD [14]	$q_1=\left[1+4{,}5\,l_c/\rho\right]^{-1/2}$	l_c : maximal length of a non propagating crack
HEYWOOD [15]	$q_2=\left[1+\left(2\cdot\rho'/\rho\right)\right]^{-1}$	r' = f(Rm)
STIELER [16]	$q_2=1+\sqrt{S_g\cdot\chi}$	$S_g=f(R_e)$ $\chi=\left(\dfrac{d\sigma}{dx}\right)/\sigma_{max}$
SWITEK[17]	$q_2=\dfrac{\left(1-\dfrac{X_c\cdot C}{\rho+X_0}\right)}{H}$	$X_0=\dfrac{6{,}3\,X_c}{3-H}$ H = constant , X_c distance
BOUKHAROUBA and PLUVINAGE [4]	$q_3=\left(\dfrac{\rho_0+B.\rho}{X_c+A.\rho}\right)^{\alpha}$	A et B constants

The definition of the effective distance is based on the following considerations:
the effective distance X_e is greater than the characteristic distance X_c and less than the notch radius.
The beginning of the domain III with abscisa X^* generally satisfy this condition and indicates the beginning of the most stressed area. The values of X^* and X_e are generally closed together with an acceptable confidence level. We can considere that $X^* = X_e$ (29)

Generally the distance X_m and X_e vary linearly with the notch radius :

$$X_m = A + B\rho \qquad (30)$$

$$X_e = X_0 + C\rho \qquad (31)$$

where A, B, C and X_0 are constants. Consequently the fatigue sensitivity index q_3 is given by :

$$q_3 = \left(\frac{X_o + C\rho}{A + B\rho}\right)^m \qquad (32)$$

This expression gives similar values to the experimental ones (figure 11). In figure 12, we can notice that the influence of notch in fatigue is less important with this formalism as with the others. This is due to the fact that the effect of plastic relaxation has been separated from the effect of fatigue relaxation introducing in the q_3 formula the elasto-plastic stress concentration factor k_σ.
The major interest of this formula is to allow an estimation of the fatigue notch sensitivity index with a numerical simulation and only needs the knowledge of the cyclic stress strain curve and the smooth specimen Wöhler's curve.

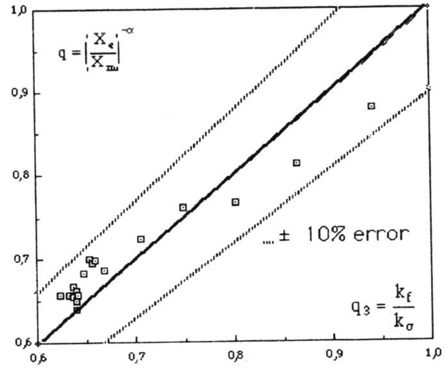


Figure 11 : Relationship beetween the experimental and theoritical notch sensitivity index.

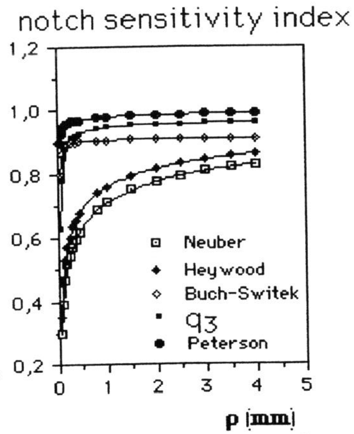


Figure 12 . Comparison of different models for the notch sensitivity index.

It is also possible to know with several computings the evolution of the fatigue stress concentration with the number of cycles. This value trends to an assymptotic value of 1 for low value of number of cycles and to k_t value for unlimited life duration.

CONCLUSION

Prediction of life duration of notched components are now made with empirical relationships beetween fatigue sensitivity index and notch radius and the material Wöhler's curve.
The notch effect in fatigue can be interpreted in terms of local criterion assuming that the stress range is higher than an effective value in the process zone characterized by the effective

distance. These parameters can be established on the notch tip distribution and are connected with the notch stress intensity factor range.
This approach allows to get by computing the fatigue stress concentration factor and the notch sensitivity index.

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