FATIGUE LIFE UNDER BIAXIAL STRESS STATE WITH DIFFERENT CROSS-CORRELATION COEFFICIENTS OF NORMAL STRESSES

T. ŁAGODA and E. MACHA
Technical University of Opole
ul.Mikołajczyka 5, 45-233 Opole, Poland

ABSTRACT

The paper contains the results of fatigue tests of 10HNAP steel under uniaxial cyclic loadings, uniaxial non-Gaussian random loadings and biaxial random loadings with various cross-correlation coefficients (-1,0,+1). Cruciform specimens were tested under plane stress state. With use of the weighed average stress amplitudes it was possible to include the results from three various fatigue tests in one graph in the scatter band with the factor of 3. The equivalent stress history was determined according to some criteria of multiaxial random fatigue. The criteria of normal stresses, normal strains and shear stresses did not give satisfactory results. The effective calculations of fatigue life times were obtained according to the modified criterion of maximum normal stresses in the critical plane and with the criterion of maximum normal and shear stresses in the critical plane.

KEYWORDS

Multiaxial random fatigue, multiaxial fatigue criteria, random loadings, non-proportional loading, cross-correlation coefficiant, fatigue life, critical planes, weighed average stress amplitude

INTRODUCTION

Recently some experimental results have been used for analysis of influence of the cross-correlation between the random stress or strain state components on fatigue life of the materials. The first attempts of theoretical approach to the problem have been presented by Łagoda and Macha (1993). There are also some results for the plane stress state under cyclic out-of-phase loadings which can be treated as a specific case of random loadings where a cross-correlation between stresses occurs. Łagoda et al. (1995) state that under biaxial cyclic tension-compression the permissible ratio of non-proportional stress amplitudes to those proportional ones decreases as the stress cross-correlation coefficient decreases (from 1 to -1). It means that at the constant amplitudes of stresses, σ_{axx} and σ_{ayy}, a change of the stress cross-correlation coefficient from $r = 1$ (in - phase stresses) to $r = -1$ (stresses displaced at π) causes drop of the fatigue life. Determination of influence of the cross-correlation coefficient (Bendat and Piersol, 1980) between normal stresses, $\sigma_{xx}(t)$ and $\sigma_{yy}(t)$, on the fatigue life under biaxial random stress state is the aim of the paper. Next, the influence should be taken into account in the algorithm for calculations. The authors try to find a parameter which could be able to show the test results obtained under uniaxial cyclic and random loadings together with those obtained under biaxial random loadings with one fatigue curve σ - N.

EXPERIMENTS

The following constants of the $\sigma_a - N$ curves for 10HNAP steel were determined under uniaxial cyclic tension - compression

$$\lg N = A - m \lg \sigma_a = 29.69 - 9.82 \lg \sigma_a \tag{1}$$

$\sigma_{af} = 252.33 \pm \delta\sigma_{af}$, $\delta\sigma_{af} = 18.75\text{MPa}$, $N_o^{'} = 1.281 \times 10^6$ cycles; $A \in (22.39; 36.98)$; $m \in (6.89; 12.75)$.

The fatigue tests under uniaxial random loadings with zero expected values were carried out for medium- and long-life time. Random histories of loadings with non-Gaussian probability distribution and wide-band frequency spectra were generated with the matrix method. It was found that under the tested loadings the experimental life time T_{exp} could be efficiently estimated with the rain flow algorithm and Palmgren-Miner hypothesis, when damages are cumulated for amplitudes greater than the fatigue limit by a half, i.e. for $\sigma_{ai} \geq 0.5\sigma_{af}$.

Let us substitute the histogram of amplitudes by the weighed average stress amplitude

$$\sigma_{aw} = \left(\frac{1}{N_b} \sum_{i=1}^{k} n_i \sigma_{ai}^m \right)^{1/m} \quad \text{for } \sigma_{ai} \geq 0.5\sigma_{af} \quad \text{when } \max_i(\sigma_{ai}) \geq \sigma_{af} - \delta\sigma_{af} \tag{2}$$

where:

N_b - block length in observation time T_o,

n_i - number of cycles at the stress amplitude σ_{ai}.

Thus we can determine constants of the fatigue curve $\sigma_{aw} - N$ under uniaxial random tension-compression

$$\lg N = A_r - m_r \lg \sigma_{aw} = 27.51 - 8.99 \lg \sigma_{aw} \tag{3}$$

where:

$$N = N_b \frac{T_{exp}}{T_o} \tag{4}$$

and $A_r \in (21.34; 33.69)$; $m_r \in (6.46; 11.52)$.

It can be seen that parameters A, m of the cyclic curve (1) and A_r, m_r of the random curve (3) are close together respectively. We can say the same about their confidence intervals, determined at probability of 0.95.

The cruciform specimens of 10HNAP steel were tested under biaxial stress state. Their shape is shown by Będkowski (1994). The central part of each specimen had a spherical shape, its radius of curvature was 150 mm. The specimen thickness in its thinnest part was 1 mm.

Fatigue tests of the cruciform specimens under biaxial stress state were done for the loading cross-correlation coefficients close to $r_{F_{xx},F_{yy}} \approx -1, 0, +1$. The probabilistic characteristics of

loadings, $F_{xx}(t)$ on X-axis and $F_{yy}(t)$ on Y-axis were similar to those ones applied during uniaxial tests. The first detailed results of the tests are given by Będkowski (1994) and by Lagoda and Macha (1996). 24 specimens were tested and 6 of them were tested under cyclic loadings. Fig.1 shows parts of the stress histories in x and y directions for loading correlations close to -1, 0 and +1 respectively. Fig.2 shows power spectral densities and moduli of cross-spectral densities for the same loading correlations -1, 0 and +1.

The test results for the specimens, (parameters of random stresses and the obtained fatigue lives, T_{exp}) are shown in Table 1. From the table it appears that if the coefficient of cross-correlation between normal stresses under biaxial stress state decreases from positive to negative values the fatigue life of 10HNAP steel decreases.

Table 1

Results of fatigue tests of cruciform specimens of 10HNAP steel for different cross-correlation coefficients of normal stresses $\sigma_{xx}(t)$ and $\sigma_{yy}(t)$

Lp	Variances $\mu_{xx,xx}$ $[MPa^2]$	Variances $\mu_{yy,yy}$ $[MPa^2]$	Covariance $\mu_{xx,yy}$ $[MPa^2]$	Mean value $\bar{\sigma}_{xx}$ [MPa]	Mean value $\bar{\sigma}_{yy}$ [MPa]	$r_{\sigma_{xx}\sigma_{yy}}$	$U_{x/y}$	T_{exp} [s]
1*	4984	5847	-4621	0,8	-4,1	-0,86	0,927	1166400
2	23146	22572	-22192	-18,1	3,6	-0,94	1,030	68400
3	18910	25605	-21384	-21,1	28,3	-0,97	0,872	54000
4	15845	18938	-16510	-16,1	-3,7	-0,95	0,922	102000
5	15368	18084	-16111	-8,6	-0,7	-0,96	0,922	195000
6	10262	16601	-12450	-21,8	-0,9	-0,95	0,781	141800
7	10374	15670	-12130	-14,5	17,7	-0,94	0,831	311700
8	11646	14844	-12668	-20,9	1,1	-0,96	0,883	302400
9	14112	11375	-12280	-9,3	-20,4	-0,97	1,118	463600
10	16008	13701	-7266	-6,0	-0,9	-0,49	1,114	331200
11	9124	10246	-5267	-2,2	20,8	-0,54	0,993	939600
12	11348	11527	-6062	11,4	24,5	-0,54	0,992	633600
13	13592	16276	-7653	-18,6	-17,4	-0,51	0,911	334800
14	17796	18929	-8156	-8,3	26,6	-0,60	0,959	101400
15	9161	6858	8558	-2,76	4,07	0,73	1,054	>2592000
16	4753	7332	4756	4,65	4,22	0,95	0,954	>6172500
17°	37812	37812	37812	0	0	1	1	>20000000
18°	84872	84872	84872	0	0	1	1	246800
19°	65522	65522	65522	0	0	1	1	619200
20°	47740	47740	47740	0	0	1	1	>20000000
21*°	68450	14450	31450	0	0	1	2.168	1532000
22°	104882	104882	104882	0	0	1	1	220400

° cyclic tests

* uncertain results

where: $r_{\sigma_{xx}\sigma_{yy}} = \frac{\mu_{xx,yy}}{\sqrt{\mu_{xx,xx} \cdot \mu_{yy,yy}}}$ $\qquad U_{x/y} = \sqrt{\frac{\mu_{xx}}{\mu_{yy}}}$

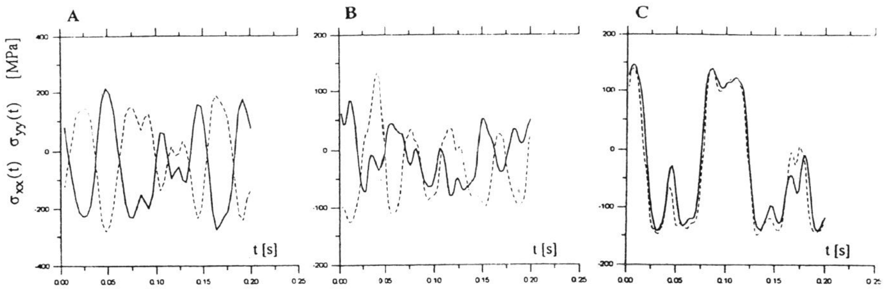

Fig.1 Fragments of stress histories in x (——— σ_{xx}) and y (- - - - - - σ_{yy}) directions for correlations of loadings: (A) - $r_{F_{xx}F_{yy}} \approx -1$; (B) - $r_{F_{xx}F_{yy}} \approx 0$; (C) - $r_{F_{xx}F_{yy}} \approx +1$

THE ALGORITHM OF FATIGUE LIFE CALCULATIONS

The biaxial stress state is reduced to the uniaxial one with a chosen fatigue criterion. Next, cycles and half-cycles are counted from the history of the equivalent stress, $\sigma_{eq}(t)$, with use of the rain-flow algorithm. The reduction takes place in the critical plane determined with the variance method (Będkowski et al., 1989).
In the generalized criterion of multiaxial random fatigue it is assumed that
1.Fatigue fracture occurs under the influence of a combination of the normal stress, $\sigma_\eta(t)$ and the shear stress, $\tau_{\eta s}(t)$ in $\bar{s}$ direction in the fracture plane with the normal $\bar{\eta}$,
2.Direction $\bar{s}$ coincides with the mean direction of maximum shear stress, $\tau_{\eta s\,max}$,
3.The maximum value of linear combination of stresses $\tau_{\eta s}(t)$ and $\sigma_\eta(t)$ under multiaxial random loadings satisfies the following equation

$$\max_t \{ B\, \tau_{\eta s}(t) + K\, \sigma_\eta(t) \} = F \tag{4}$$

where B, K, F - constants.
The criterion was analysed, among others, by Łagoda and Macha (1993) and by Macha (1984).
In specific cases for the biaxial stress state we obtain criteria of
-the maximum normal stress in the fracture plane (B = 0, K = 1)

CI - $$\sigma_{eqCI}(t) = [\hat{l}_\eta^2 \sigma_{xx}(t) + \hat{m}_\eta^2 \sigma_{yy}(t)] \tag{5}$$

-the maximum strain in the fracture plane (B = 0, K = 1)

CII - $$\sigma_{eqCII}(t) = [\hat{l}_\eta^2(1+\upsilon) - \upsilon]\sigma_{xx}(t) + [\hat{m}_\eta^2(1+\upsilon) - \upsilon]\sigma_{yy}(t)] \tag{6}$$

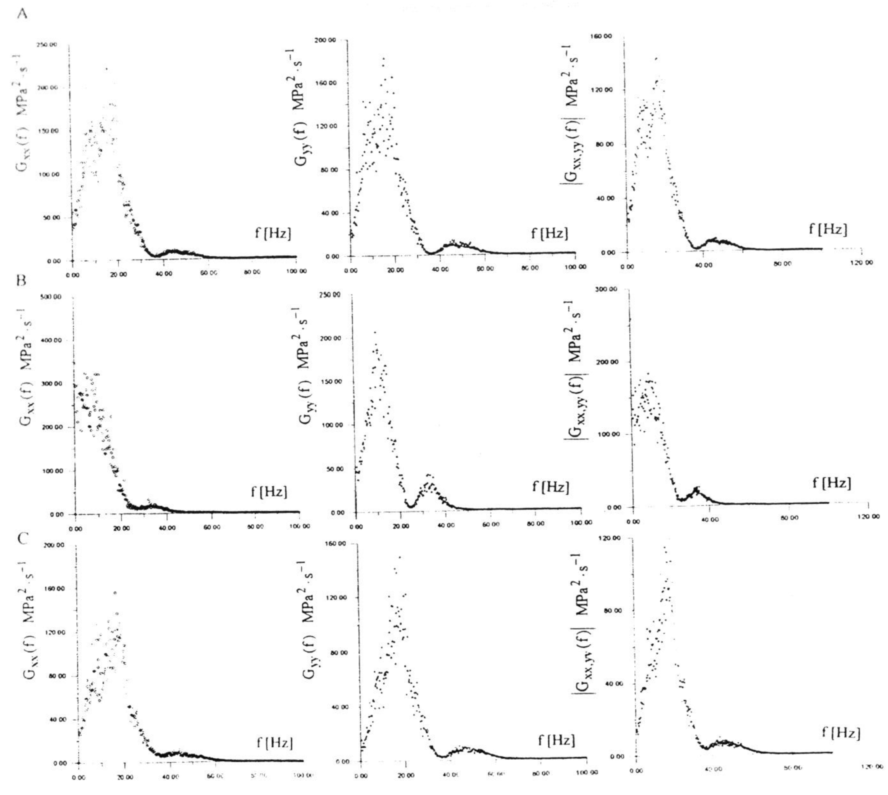

Fig.2 Autospectral power densities and modulus of cross-spectral power densities for loading correlations: (A) - $r_{F_{xx}F_{yy}} \approx -1$; (B) - $r_{F_{xx}F_{yy}} \approx 0$; (C) - $r_{F_{xx}F_{yy}} \approx +1$

-the maximum shear stress in the fracture plane (B = 2, K = 0)

CIII - $$\sigma_{eqCIII}(t) = 2[\hat{l}_\eta \hat{l}_s \sigma_{xx}(t) + \hat{m}_\eta \hat{m}_s \sigma_{yy}(t)] \qquad (7)$$

-the maximum shear and normal stresses in the fracture plane (B = 2, K ≠ 1)

CIV - $$\sigma_{eqCIV}(t) = \frac{1}{1+K}\{K[\hat{l}_\eta^2 \sigma_{xx}(t) + \hat{m}_\eta^2 \sigma_{yy}(t)] + 2[\hat{l}_\eta \hat{l}_s \sigma_{xx}(t) + \hat{m}_\eta \hat{m}_s \sigma_{yy}(t)]\} \qquad (8)$$

where $\hat{l}_\eta$, $\hat{m}_\eta$, $\hat{n}_\eta$, $\hat{l}_s$, $\hat{m}_s$, $\hat{n}_s$ - direction cosines of vectors $\hat{\eta}$ and $\hat{s}$, ν - Poisson ratio.
In the plane stress state a position of the vector normal to the fracture plane can be expressed with one angle α in relation to 0x-axis. Hence, the direction cosines of the stress axis are

$$\hat{l}_\eta = \cos\alpha, \qquad \hat{m}_\eta = \sin\alpha, \qquad \hat{l}_s = -\sin\alpha, \qquad \hat{m}_s = \cos\alpha \qquad (9)$$

THE ANALYSIS OF TEST RESULTS

The fatigue life was calculated according to four criteria (CI, CII, CIII, CIV). In criterion IV constant K assumed various numbers. Unfortunately, none of the analysed criteria gave satisfactory results. The results obtained according to CII seem to be the relatively best. But if the stress correlation is negative, the fatigue lives calculated according to the criterion are less than the experimental ones. If the stress correlation is positive, the opposite situation can be observed. Thus, the authors tried to modify the considered criteria (5) - (8). The first favourable results were obtained while modifying criterion CI in the following way

CIM - $$\sigma_{eqCIM}(t) = \left(\frac{0.5\sigma_{af}}{\tau_{af}}\right)^{r/2} \sigma_{eqCI}(t) \qquad (10)$$

The modification consists in multiplication of instantaneous values of the equivalent stress, $\sigma_{eqCI}(t)$, by the constant depending on the ratio of the fatigue limits under tension-compression, σ_{af} and torsion, τ_{af} and the cross-correlation coefficient of normal stresses, r. A greater part of the results is included in the scatter band with the factor of 3 (the scatter was greater only in two specimens with the loading correlation -1). From the analysis of Eq.(10) it results that under constant variances of normal stresses decrease of the cross-correlation coefficient, r, causes increase of variance of the modified equivalent stress ($0.5\,\sigma_{af} / \tau_{af} < 1$) and it means decrease of the fatigue life.
In the case of criterion CIV the modification consists in assumption of the following equation expressing the equivalent stress

CIVM - $$\sigma_{eqCIVM}(t) = B\,[\hat{l}_\eta \hat{l}_s \sigma_{xx}(t) + \hat{m}_\eta \hat{m}_s \sigma_{yy}(t)] + K[\hat{l}_\eta^2 \sigma_{xx}(t) + \hat{m}_\eta^2 \sigma_{yy}(t)] \qquad (11)$$

B = 0.7752 and K = 0.8490 were assumed from experiments. A good agreement with the experimental results was obtained in the scatter band with the factor of 3 (a greater part of the

results was included in the scatter band of the factor 2); only two specimens were exceptions. In all the cases we assumed σ_{af} = 182.2 MPa and τ_{af} = 252.3 MPa.
Fig.3 shows the results of fatigue tests under biaxial and uniaxial random loadings in the scatter band with the factor of 3, determined on the basis of experimental data obtained under uniaxial cyclic loadings. It can be seen that the weighed average stress amplitudes under uniaxial random loadings do not exceed the determined scatter band of the results for cyclic loadings. In the case of biaxial random loadings the weighed average amplitudes of the equivalent stress according to criteria CI and CII exceed the given scatter band. It should be stated that the weighed average amplitudes of the equivalent stress according to the modified criterion of maximum normal stress in the critical plane (CIM) and according to the modified criterion of maximum shear and normal stresses in the fracture plane with the weight participation of both stresses (CIVM) are included in the scatter band with the factor of 3 together with the test results obtained under uniaxial cyclic and random loadings.

CONCLUSIONS

1.Under biaxial tension-compression the coefficient of cross-correlation between normal stresses strongly influences the fatigue life and causes its increase in the case of positive correlation and its decrease under negative correlation.
2.Using the weighed average stress amplitude we can show, in one graph, the results of tests of 10HNAP steel obtained under uniaxial random loadings and under biaxial random loadings in the scatter band of the results corresponding to uniaxial cyclic loadings.
3.Biaxial random stress state with non-Gaussian probability distribution in 10HNAP steel can be efficiently reduced to the equivalent uniaxial one with the modified criterion of maximum normal stresses in the critical plane or with the modified criterion of maximum shear and normal stresses in the fracture plane with the weight participation of both stresses.

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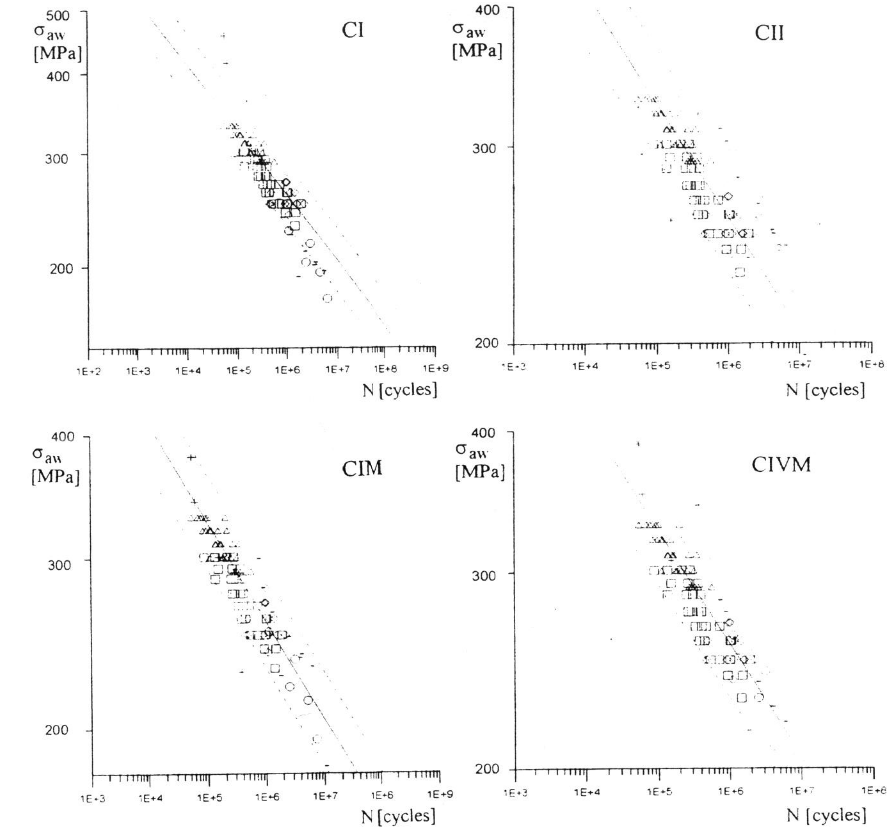

Fig.3 Weighed average stress amplitudes and corresponding numbers of destructive cycles
Δ - uniaxial cyclic fatigue; $\Diamond$ - uniaxial cyclic fatigue at the level of fatigue limit; $\Box$ - uniaxial random fatigue; — - biaxial random fatigue $(F_{xx}, F_{yy} \approx -1)$; O - biaxial random fatigue $(F_{xx}, F_{yy} \approx 0)$; + - biaxial random fatigue $(F_{xx}, F_{yy} \approx +1)$

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