FATIGUE LIFE PREDICTION BY STATISTICAL APPROACH UNDER CONSTANT AMPLITUDE LOADING

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ABSTRACT

In general, the experimental data of fatigue crack growth rates scatter very much even under identical experimental conditions with constant amplitude loading condition. It is ,thus, essential to take into account the data scatter of crack growth rates by using statistical approach for a reliable fatigue crack propagation analysis. In this study, fatigue crack propagation tests were conducted on a very thin 2024-T3 aluminum alloy under constant amplitude loading. The distribution of the fatigue crack propagation life is estimated by using the stochastic Markov Chain model based on a modified Paris-Erdogon equation. The fatigue lives under different loading conditions are estimated by using this model.

KEYWORDS

Markov Chain, fatigue life, DC(Duty Cycle), CD(Cumulative Damage), Markov Chain model, fatigue crack growth rate

INTRODUCTION

For the airplane structures, the fatigue damage analysis is the essential technique, since the airplane structures are generally under random variable loading condition. The airplane structures mostly consist of very thin 2024-T3 Al alloy. Therefore, it is find out of the fatigue crack growth behavior in very thin plate and data-base should be constructed based on the fatigue crack growth behavior of very thin specimens. The variability of fatigue crack growth rate, however, needs to have the statistical model. Generally, the following three sources of variability in experimentally obtained fatigue crack growth data are commonly regarded as the most decisive(Sobczyk and Spencer, 1992): (1) the difference in material behavior among identically prepared specimens (due to difference in stress concentration at grain boundaries, effects of thermal processing,etc); (2) uncertainty in the fatigue and fracture process itself;

(3) difference in environment among tests at the same load conditions and with the same materials.

There are two ways to consider the variability of the fatigue crack growth. One is estimation of crack growth life distribution from the Paris-Erdogon differential equation model(Ishikawa and Tsurui, 1987). The other is the Markov Chain model that is proposed by Bogdanoff-Kozin(1981, 1983) as an example of the evolutionary probabilistic approach. In this study, we used the Markov Chain model based on a modified Paris-Erdogon equation.

A modified Paris-Erdogon equation(Euw and Hertzberg and Roberts, 1972) used in this study is

$$da/dN = C(\Delta K_{eff})^m \qquad (1)$$

where da/dN : crack growth rate

C , m : random variables

$$\Delta K_{eff} = \Delta\sigma\sqrt{\pi a \sec\left(\frac{\pi a}{W}\right)}\,(0.5+0.4 \times R)$$

$$R : \text{stress ratio } \left(= \frac{\text{Min sress}}{\text{Max sress}}\right)$$

The principal purpose of this paper is to find an appropriate stochastic model and to evaluate reliability of this model for the fatigue crack growth analysis of a thin 2024-T3 aluminum alloy.

BACKGROUND

Bogdonoff and Kozin(1985) used the Markov Chain model so as to analyze statistically the fatigue cumulative damage process. They defined a duty cycle(DC) to be a repetitive period of operation in the life of a component during which damage can accumulate. And, they maked the following assumptions ;

1. Damage states are discrete and labeled j=1,2,...,b
   where state b denotes replacement, or failure.
2. Increment in damage at the end of a DC depends in a probabilistic manner only on the amount of damage present at the start of the DC, on that DC itself, and is dependent of how damage was accumulated up to the start of that DC.
3. Damage can only increase in a DC from the state occupied at the start of that DC to the state one unit higher.

If we define that P_j is the probability of remaining in state j during one step and q_j is the probability that in one step damage goes from state j to state j+1;

p_j = Prob{remain in state j | initially in state j}

q_j = Prob{go to state j+1 | initially in state j}.

The (1 × b) row vector

$$p_0 = \{\ \pi_1, \pi_2, \ldots, \pi_{b-1}, 0\ \}$$

specifies the initial distribution of damage, where π_j=Prob{damage is in state j at x=0}, and $\sum_{1}^{b-1}\pi_j = 1, \pi_b = 0$. The assumption that $\pi_b = 0$, means that no component is in the failed state b initially. For this simple version of the model, the DC severity is defined by the (b × b) probability transition matrix.

$$P = \begin{bmatrix} p_1 & q_1 & 0 & 0 & \cdots & \cdots & \cdots & 0 & 0 \\ 0 & p_2 & q_2 & 0 & \cdots & \cdots & \cdots & 0 & 0 \\ 0 & 0 & p_3 & q_3 & \cdots & \cdots & \cdots & 0 & 0 \\ 0 & 0 & p_4 & q_4 & \cdots & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & p_{b-1} & q_{b-1} \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 & 1 \end{bmatrix}$$

where $p_j \geq 0$, $p_j + q_j = 1$

In this matrix, we can know that all states are transient except for the last which is absorbing. The probability of being in state j at the x is given by the (1 × b) row vector.

$$p_x = \{\ p_x(1), p_x(2), p_x(3), \ldots, p_x(b)\ \}$$

where p_x (j) = Prob { damage is in state j at time x }

$$\sum_{1}^{b} p_x(j) = 1, p_x(j) \geq 0$$

We then have Markov property,

$$p_x = p_0 P^x \qquad (2)$$

For the fatigue cumulative damage, however, the Markov Chain model of Bogdanoff and Kozin is based simply on probabilistic process. Therefore, the model is not definite physical meaning of fatigue damage. Because of this reason, the crack growth law of the Paris-Erdogon is imported and its weakness can be maked up for. In the Markov Chain model, it is assumed that crack length δa increases by stage. Therefore, damage state i defines as

$$a_i = a_0 + i\delta a \quad i = 0,1,2,\ldots,n \qquad (3)$$

where a_i = crack length in state i(mm)

a_0 = initial crack length(mm)

The probability of going to next state, q_i, can be defined as stress intensity factor function.

$$q_i = q(\Delta K_i) \qquad (4)$$

In this approach, transient probability q_i can be obtained by using a modified Paris-Erdogon equation. The modified Paris-Erdogon equation is

$$da/dN = C(\Delta K_{eff})^m \qquad (5)$$

It is assumed that m and C are random variables, so m and C are taken as random variables independent of each other.

$$\frac{\delta a}{E[\delta N]} = C(\Delta K_{eff})^m \qquad (6)$$

where E[variable] = mean value of variable

Crack didn't propagate during duty cycles (δn-1). But if δn-th cycle acts, then crack would propagate, and that probability is q. Therefore probability distribution of δN is

$$P[\delta N = \delta n] = f_{\delta N}(\delta n) = qp^{\delta n - 1} \qquad (7)$$

Mean and variation of duty cycles number are first order and second order moment, respectively.

$$E[\delta N] = \sum_{\delta n=0}^{\infty} \delta n\, f_{\delta N} = \sum_{\delta n=0}^{\infty} \delta n\, q(1-q)^{\delta n-1} = \frac{1}{q} \qquad (8)$$

$$\mathrm{Var}[\delta N] = E[\delta N^2] - (E[\delta N])^2 = \frac{1-q}{q^2} \qquad (9)$$

As this two equations, transient probability q is

$$q = \frac{C}{\delta a}(\Delta K_{eff})^m \qquad (10)$$

Using this eq. (10), it can be obtained transient probability that considers scatter of fatigue crack growth.

EXPERIMENT

Specimen and Experimental method

The material used was 2024-T3 Al alloy plates of 1.02 mm thickness. Its chemical composition is shown in Table 1 and mechanical property is shown in Table 2.

Table 1. Chemical composition of 2024-T3 Al alloy (wt %)

Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti
0.11	0.23	4.46	0.58	1.44	0.04	0.03	0.02

Table 2. Mechanical properties of 2024-T3 Al alloy

Yield strength (MPa)	Tensile strength (MPa)	Elongation (%)
324	442	16.7

The geometry of specimen is CCT(Center Cracked Tension) as shown in Fig.1. The longitudinal direction of the specimen coincides with the rolling direction. All fatigue crack growth tests were carried out under axial loading using a servo hydraulic testing machine of 10 Ton capacity. The repeating frequency was 10 Hz. The stress range was $\Delta\sigma = 58.8$ MPa, and the mean stress was $\sigma_m = 39.2$ MPa. Hence the stress ratio was R = 0.25. The temperature of the specimen was room temperature. Crack growth was monitored using a traveling microscope, it can measure with accuracy of 0.01 mm. The crack length was measured at the two tips of the crack on both sides of the specimen. The time interval of the measurement was 5000 cycles. Fifteen specimens were tested under identical experimental conditions.

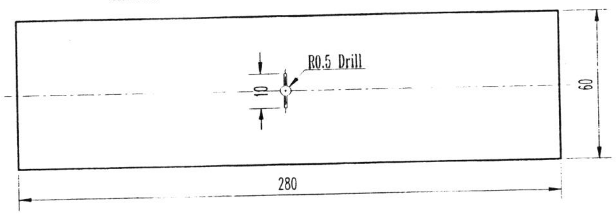

Fig. 1 Geometry of specimen.

EXPERIMENTAL RESULT AND DISCUSSION

Fatigue crack growth under constant amplitude loading

The crack length(a) is plotted against the number of cycles(N) in Fig.2. Measurements were started at different initial crack lengths. Therefore, we obtained the data by interpolation after translating all curves initial crack length into a_o=7mm.

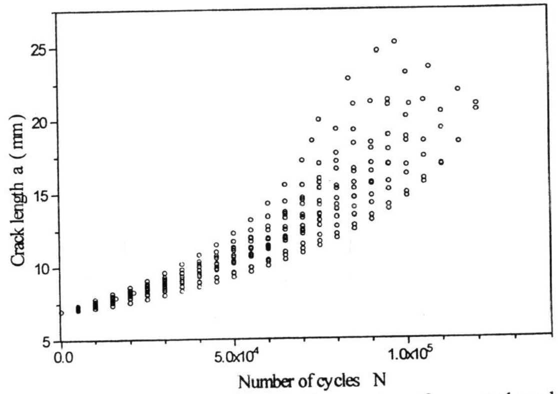

Fig. 2 Crack length plotted against the number of repeated cycles in Al 2024-T3 , R=1/4

The relation between the crack growth rate,da/dN, and the effective stress intensity factor range, ΔK_{eff} ,is shown in Fig.3. da/dN was evaluated as

$$\frac{da}{dN} = \frac{a_{i+1} - a_i}{N_{i+1} + N_i} \qquad (11)$$

where N_i and N_{i+1} = the number of cycles at which ith and (i+1)th measurement, respectively.
a_i and a_{i+1} are the value of a at $N=N_i$ and $N=N_{i+1}$, respectively.

ΔK_{eff} was evaluated as

$$\Delta K_{eff} = U\,\Delta K_{app} = U\,\Delta\sigma\sqrt{\pi a \cdot \sec(\pi a / W)} \qquad (12)$$

where ΔK_{eff} = effective stress intensity factor range,
ΔK_{app} = applied stress intensity factor range
U = crack closure parameter
W = specimen width

Elber showed empirically for 2024-T3 Al alloy that

$$U = 0.5 + 0.4R \qquad (13)$$

where $R = \dfrac{\text{Min.load}}{\text{Max.load}}$

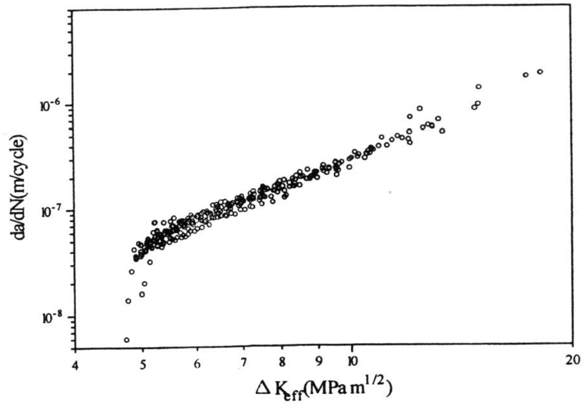

Fig. 3 Relationship between fatigue crack growth rate and stress intensity factor range, R=1/4

Substituting eqs. 9 and 10 into Paris-Erdogon equation, it is possible to correlate crack growth rates with effective stress intensity factor range for different stress values.

$$\frac{da}{dN} = C(\Delta K_{eff})^m = C[(0.5+0.4R)\Delta\sigma\sqrt{\pi a}\cdot\sec(\pi a/W)] \quad \text{———} \quad (14)$$

The modified Paris-Erdogon equation was applied to the data points of each specimen using the method of least squares. It is assumed that m and logC are random variables since the specimen-to-specimen variability of m and logC were known. They both show approximately normal distribution. They follow 2-parameter Weibull distribution, such as

$$F(t) = 1 - \exp[-(\frac{t}{\theta})^\beta] \quad \text{———} \quad (15)$$

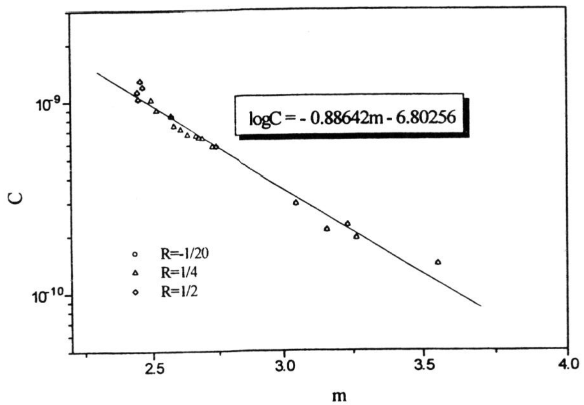

Fig. 4 Correlation between m and log C by using effective ΔK_{eff}, R=1/4

Figure 4 shows the correlation between m and logC. It is seen that a strong negative correlation exists between m and logC. Therefore, if m(or logC) is generated as random variables by following 2-parameter Weibull distribution, then we could obtain values of m and logC to take account their strong negative correlation .

Fatigue life estimation

Markov Chain model was constructed that duty cycles are 1000 cycles and δa=0.2mm. Figure 5 shows a edf's(empirical distribute function) of the cycle number to reach a=11mm, and the corresponding estimated result obtained from the proposed model. Figure 6 shows a edf's and estimated result at a=17mm. The agreement between edf's and estimated result is excellent.

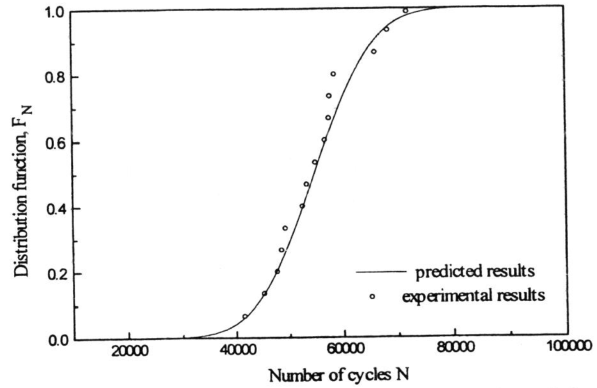

Fig. 5 Comparison between empirical results and the predicted fatigue life distribution using Markov Chain Model, a=11mm

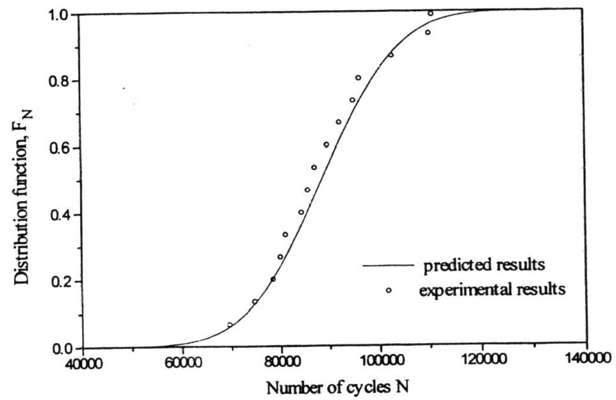

Fig. 6 Comparison between empirical results and the predicted fatigue life distribution using Markov Chain Model, a=17mm

Figure 7 show comparison between edf's and estimated results of different loading conditions. The agreement between edf's and estimated result seems to be excellent. This model can propose reliable fatigue life prediction.

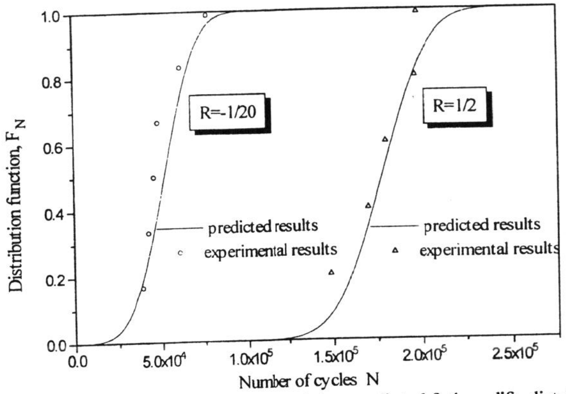

Fig. 7 Comparison between edf's and the predicted fatigue life distribution using Markov Chain Model under different loading condition, a=11mm

CONCLUSION

In this study, fatigue crack propagation tests were conducted and data scatter of fatigue crack propagation was considered by using statistical model. Random variables m and C are imported. Transient probability ,q, that considered scatter of fatigue crack growth is

$$q = \frac{C}{\delta a}(\Delta K_{eff})^m$$

In this equation, ΔK_{eff} is value that considered stress ratio. The distribution of fatigue crack propagation life is estimated by using the stochastic Markov Chain model based on a modified Paris-Erdogon equation. As a consequence, result of experiment and that of estimation are coincided very well.

REFERENCES

Bogdanoff J. L., F. Kozin (1981). A critical analysis of some probabilistic models of fatigue crack growth. *Engineering Fracture Mech.*, **14**, 59-89.

Bogdanoff J. L., F. Kozin (1983). On the probabilistic modeling of fatigue crack growth. *Engineering Fracture Mech.*, **18**, 623-632.

Bogdanoff J. L., F. Kozin (1985). Probabilistic models of cumulative damage. In: *B-models of cumulative damage* , pp. 63-162. John Wiley & Sons, USA.

Ishikawa H., A. Tsurui (1987). Stochastic fatigue crack growth model and its wide applicability in reliability-based design. *Current Japanese Material Research.*, **2**, 45-58.

Sobczyk, K., Spencer, B. F. (1992). Random Fatigue from data to theory. In: *Scatter in Fatigue Data* , pp. 42-43. Academic Press, San Diego.

Von Euw E. F. J., R. W. Hertzberg and Richard Roberts (1972). Delay effects in fatigue crack propagation. *ASTM STP 513.*, 230-259.