ANALYTICAL INVESTIGATIONS OF FRACTURE AND ACCOMPANYING THERMAL EFFECTS IN SUPERCONDUCTORS UNDER LORENTZ FORCES

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ABSTRACT

The mechanical disturbances in type I and II superconductors (SC) under a magnetic field and currents and some thermal effects as a result of local failure at low temperatures are studied. General statements are given by macroscopic approaches to superconductivity and brittle fracture. As for particular problems we focus our attention to stress concentration sites (the top of an angle, a contact zone, ...). Besides we find the solution for a source of heat into SC and use it for the calculation of temperature around a small quickly arisen defect.

KEYWORDS
Superconductor, current, magnetic field, fracture, heat

INTRODUCTION

The discovery of superconductivity in the metaloxyster at nitric temperatures by Bednorz and Müller (1986) has stimulated the new explosive of attention to this phenomenon. The most values of current density $10^9$ A/m² have been recorded in very thin films. They predict fantastic values of critical magnetic field of some hundred tesla. Nevertheless, low temperature superconductivity remains in good while the critical field $10^{-10}$ T and the current $10^9$ A/m² only. It keeps such kind of adventures as more technologies of materials. The large Lorentz forces created by the interaction between currents and magnetic field can cause stresses within SC more than limit stresses. Especially, the stress concentration points can be dangerous. In turn, the mechanical failure and fracture (plastic deformation, cracking,...) prevents to achieve the desirable currents and magnetic field because of heat emission. However, the stress-strain states of different SC specimens have not been studied enough. It is important not only for the prediction of fracture but also for the analysis of the SC-phase-stability because the critical temperature and
magnetic field depend on the deformation (Gurevich et al., 1987). Just the present paper is devoted to failure in SC under load. The possible original stress fields include, for example, by coalesce of composite SC can be taken into account by means of superposition. In contrast, the known statements are not stable and may need to be reconsidered. In order to understand the theory of both SC and magnetic field, we restrict to a simple approach. Plate with an elliptical hole is considered. These results yield prediction of fracture near the edge of a SC film strip placed contact zone of type II SC cylinder pressed by Lorentz force to surface of fracture in the space of magnetic induction-current. It is of great interest for SC stability. We receive some evaluations of non-cavitational potential flow around rigid conductor by ideal incompressible liquid. Only compressive pressures are on $\Gamma$ and shear stresses are dangerous for convex forms of SC. By known solution their small maxima for sphere and cylinder are reached in the center. For a SC angle $\beta$, the asymptotics are:

$$B = \rho^\alpha, \quad P = \rho^\beta, \quad \alpha = (\beta - 1)/(2 - \beta)$$

where $\rho$ is the distance at the angle top. At $\alpha > 1$, the unlimited growth of $B$ near the top fracture of superconductivity. At $\alpha < 1$ and $P$ vanish at $\rho = 0$ but within SC stretching singular stresses appear ($\sigma = \rho^\beta$, where $\sigma = 1/2$). The calculations are shown that even in case of special points and real value $\rho^\beta$, $\sigma^\beta$ breaking magnetic field $B$ seems to be much more in comparison with common operating fields (which is well known for common points). Therefore, the above consideration is of more practical interest for skin-effect.

**Type I Superconductor**

Let SC occupy a domain $\mathbb{R}^n, n = 2, 3, 4$, and be surrounded by a background magnetic field $B$. Each media in mechanical aspect is elastic, brittle and homogeneous. According to the known approach to current $I$; temperature $T$ and field $B$ satisfy the conditions $\phi^g = 0$ (2-$\beta$), $B^g = 0$ (2), $B^g = 0$ (2), $T < T_c$. The $B$ and $T$ are critical values. In other words, the current concentrates onto $\Gamma$ only, so Meissner effect. To point out that it is true under skin-effect rates into two more simple ones. The former is electromagnetic equation with respect to the potential of the disturbance field $2\mu_0\varepsilon = \rho^\beta$, where $\mu_0$ and $\varepsilon$ are magnetic permeability and constant.

The problem consists in the study of the stress state under stress $\sigma = \rho^\beta$. In regular stress points in $\Phi$ we consider usual criterion of strength including limit tension and shear stress parameters $\sigma$ and $\tau$. In singular points we make the same stress analysis but at the distance $\rho_0$ from these points, where $\rho_0$ is a core region radius. The radius is comparable with the characteristic size of structure to preclude the damage (for metals $\rho_0 < 10^{-2}, 10^{-3}$). Of course, the other energy or force criteria can be used in case of need. Now we proceed to some particular cases assuming that the matrix is much more weak in comparison with SC.

The Field $B^g(0, 0, B)$ and a Plane Domain $\Omega(x, y)$. Pressure is constant on $\Gamma$. Therefore, in case of finite $\Omega$ the state of SC is hydrostatically stable. Now let SC-plate contain an elliptic hole. Then the tension stresses appear and the limit value $B = 2\mu_0\overline{\sigma}^g / \rho_0$ if $\rho < \rho_0$. For thin ellipse $a/b = 1/2$ if $\rho = a/2b$ and $\rho > \rho_0$. As known the large tension stresses are produced within the skin-effect.

The Field $B^g(0, 0, 0)$. The former problem is equivalent to that of non cavitation infinite flow around rigid conductor by ideal incompressible liquid. Only compressive pressures are on $\Gamma$ and shear stresses are dangerous for convex forms of SC. By known solution their small maxima for sphere and cylinder are reached in the center. For a SC angle $\beta$ the asymptotics are:

$$B = \rho^\alpha, \quad P = \rho^\beta, \quad \alpha = (\beta - 1)/(2 - \beta)$$

**Type II Superconductor**

The following physical assumptions concerning the behaviour of the rigid type II SC can be advanced (Gurevich et al., 1987):

1. $I = I_{c1}B_{c2} B^g$, where $I_{c1}, I_{c2}$ are the upper and lower critical magnetic induction; at the operating field $B = B^g$ we deal with a normal conductor.

2. According to the concept of critical state by Bean and London a volt-ampere characteristic of rigid SC is $I = E^2/\rho^2$, where $E$ is the any small intensity of electric field and $\rho$ is a constant at $\rho = \rho_0$ (Bean concept).

3. As a rule, even in high pulsed magnetic field the quasielastic with respect to stress fields takes place. It can be broken only for very thin plates and shells.

4. Within a monocrystal oxide film on a dielectric substrate the normal and shear stresses are distributed according to
a linear law through thickness, if the inequality \( j \neq B \) takes place. On the contact surface they achieve the values \( J = j_c \), \( h \) is the thickness, \( B \) is a constant across the film section. We can ignore elasticity of the film because usually \( hs10^5 \). Hence, these stresses are the boundary conditions for an elastic problem for the dielectric. In composite and microcomposite type II SC the average electric and elastic parameters. So, \( j_c \) needs to be replaced by \( j_c = \kappa_s j_c \) where \( \kappa_s \) is a volume contents of the SC component.

6. The mentioned brittle fracture approach is attracted below.

Mathematical Model. According to physical assumptions 1-6 we give a general statement of problem to determine magnetic and flows within partial volume \( \Omega_{j_c} = \Omega_j \) and \( \Omega_j \) on \( \Gamma_j \). In both general case of SC-film and SC-volume the complete magnetic induction \( B \) Maxwell equations in the form of Biot-Savart law (contributions of currents \( ) \) and the background field \( B_{\text{const}} \).

\[
\nabla \cdot F = 0, \quad \nabla \times E = 1/2 (\nabla \times (\nabla \times (\nabla)) \quad F = j \cdot B
\]

\[
B(x,t) = \mu_0 \nabla \times \frac{1}{2 \pi n} \left( \frac{J}{r} - \frac{J}{r^3} \right), \quad \nabla \times F = 0
\]

\[
\nabla \cdot \sigma = -\tau B \cdot \nabla \times \mathbf{F} (\Gamma), \quad \mathbf{C} = \begin{cases} C_{ij} \end{cases}, \quad C = 2,4 (n = 2,3), \quad \mu_0 = 4\pi 10^{-7} \text{H/m}
\]

Here \( C, \nabla, E, U, F \) are the matrix of elastic moduli, the stress and deformation tensors, the vector of displacements and volume operation, \( n \) is the outward normal to \( \Gamma_j \), \( F \) is a vector of \( J \) and different relations for superconductivity are attracted. In this case we have a connected complex problem. However, often we can account \( j \)-const on \( \Omega_j \), \( j \)-const on \( \Gamma_j \). Then the system (1)-(3) is complete and, moreover, divided. have to solve an elastic problem, then we may utilize the fracture criterion and find the fracture surface in space of parameters \( B_s, j_c \).

**Elastic-Dielectric Angle or Semi-Space with SC Film Strip.**

Consider dielectric elastic angle \( \rho \neq 0 \), \( 0 \leq \theta \leq \pi \) in the magnetic field \( B_s \). Part of its surface \( \theta = \beta \), \( 0 < \rho < r \) is covered by a SC film strip with high current \( J \) which is parallel to the edge line. Asymptotics of stresses at \( \rho = 0 \) are completely defined by the local boundary conditions at \( \beta \) are because any available eigenfunction (stress) as the solution of uniform problem disappears at \( \rho = 0 \) as \( \rho^{-1} \) at \( \beta = 0 \). Guesing the particular solution of the singular problem, where non-zero conditions are the asymptotics of Lorentz tractions with the unusual in singularity of stress, we find the main terms of stresses at \( \rho = 0 \) with accuracy \( O(1) \):

\[
\sigma_{\rho \theta} = 2(a(\sin \theta - \sin \theta))a, \quad \sigma_{\theta \theta} = 2(a(\sin \theta - \sin \theta))a
\]

If \( \beta = 0 \) then \( -1/2 \alpha \leq 0 \) and the major eigenfunction with the coefficient depending on the problem as a whole must be added. The magnetic field \( B \) also has the same singularity. The character of singularities in stresses at \( \beta = 0 \) are the same as in classical theory of thin plates. On the other hand at \( \beta = \infty, \delta = 0 \) we see \( \rho^{-1} \), the limits at \( \delta = 0 \) are not regular and \( 1/n \) appears at \( \beta = 0 \) for the case of semi-space or near the other film edge:

\[
\sigma_{\rho \theta} = -1/2 \alpha \sin \theta, \quad \sigma_{\theta \theta} = -1/2 \alpha \sin \theta
\]

The fracture begins at having achieved one of the equalities \( \sigma_{\rho \theta} = \sigma_\theta, \sigma_{\theta \theta} = \sigma_\rho \) in case \( h(1) \). Else the above results are external expansions for future local analysis. The fracture surface depends on \( \rho_0, \beta, \sigma, \tau \) for an angle and it is difficult to give general analysis. If one of the above limit equalities fulfills for semi-space then opening crack can be produced along the normal to the surface or shear crack at \( \beta = \pi/2, \delta = \pi/2 \).

**Elastic-Dielectric Cylinder or Semi-Space with SC Film Strip.**

Consider dielectric elastic cylinder (cable of unit radius) rest on a dielectric elastic homogeneous semi-space. The rectangular coordinate system \( xy \) is connected with the center of the contact zone and the polar axis \( \rho, \theta \) with the center of the cylinder. The background magnetic field is \( B = (B, 0, 0) \) and the transport current flows with a constant density \( j \) through the whole section of the cylinder. Then Lorentz forces by volume \( F = B \cdot j \) press cylinder to dielectric. Another component \( F = -1/2 \mu_0 \rho \) from the current interaction directs to the center. Therefore, the main problem can be represented by superposition of two problems. The former consists in the determination of a stress state arising under the action of the internal forces \( f \) in the absence of any loading on the cylinder surface. It is similar to the problem of stress state in a rotating cylinder under the
Equilibrium equations in the latter problem should be solved in the absence of surface shear stresses and resolved by the method of asymptotic expansion matching at $R_0$. In areas far from the contact zone it is similar to one of self-weight balanced thin disk. Sum of the two solutions give a contact strip: shear stresses are not very large but become very high near the contact zone. Both bodies near the contact zone can be considered as two elastic semi-planes. The boundary problem is found from the solution of known known Hertzian problem. All of the stresses on the cylinder surface are evaluated at the Green's function for semi-plane and determined. The stresses near the small contact domain can be eliminated at the contact zone edge (contrary to 3-D case). Maximum shear stress is demonstrated:

$$\sigma_x = \frac{4P}{\pi R_0} \left[ \frac{1}{R_0} - \frac{1}{R_0} \right], \quad \sigma_y = \frac{2P}{\pi R_0} \left[ \frac{1}{R_0} - \frac{1}{R_0} \right], \quad \sigma_z = v (\sigma_x + \sigma_y)$$

$$T_x = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right], \quad T_y = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right], \quad T_z = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right]$$

$$\eta = \frac{y}{a}, \quad \nu = \frac{\gamma}{1-\nu}, \quad v = \frac{1-\nu}{1-2\nu} \frac{E}{2(1-\nu)}, \quad \nu = \frac{1-\nu}{3(1-2\nu)} \frac{E}{2(1-\nu)}$$

$$T_x = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right], \quad T_y = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right], \quad T_z = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right]$$

Here $E$, $E'$, $v$, $\nu'$ are the elastic parameters of SC and substrate. $T_x$ are dimensionless complete shear stresses. At $\eta = 1$ the values of $T_x$, $T_y$, $T_z$ will be maximal. If $\gamma$ is small then the maximum $T_x$ and $T_y, T_z$ are achieved at the center of contact strip. With increasing of $\gamma$ the second extremum of function $T_x(\eta)$ placing at $\eta = 0.8$ will dominate.

**Temperature Field by Formation of a Defect**

It is well known the crack propagation gives rise to large growth of temperature near its tip. The increase of hundred degree at low temperature because of degeneration of the heat capacity by known lattice and electronic asymptotics:

$$C_0 = \frac{12}{5} N \frac{A}{k} \left[ \frac{T}{T_0} \right]^{3}, \quad C_1 = \frac{12}{5} N \frac{A}{k} \left[ \frac{T}{T_0} \right]^{3}$$

where $N$, $A$, $k$, $N_0$, $T_f$ is Avogadro number, atomic weight, Boltzmann constant, Debye and Fermi temperatures. The simple estimation shows that at the adiabatic plastic deformation of the crack $10^{10}$ and $T_f = 4.2 K$ the increase of $T$ is approximately $10^{-10}$ to $10^{-10}$. The mean increase of $T$ through the plastic zone near a moving crack tip has the same order. The approximate increase of $T$ near the suddenly formed defect within a material (crack, failure domain, plastic zone near a pore) can be found from the solution of the problem about instantaneous concentrated point line, plane point: $T = 2\sqrt{\rho} C_0$, $n = 0, 1, 2$

$$\phi = x \left[ \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right] \right], \quad T = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right], \quad \phi = 2\sqrt{\rho} C_0, \quad n = 0, 1, 2$$

where $\rho$, $T$ is the density and time, $\rho$ is the density at the crack plane, $T$ is the crack plane, $\rho = 2\sqrt{\rho} C_0$, $n = 0, 1, 2$.

Some calculations of coordinate $x_a$, where $x_a$, where $x_a = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right], \quad x_a = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right], \quad x_a = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right], \quad x_a = \frac{a^2}{6} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{R_0}{a} \right) \right]$

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Here $r$ is radius of penny-shaped crack at the point source of heat. If $T_1 = 4.2$ and (usually) $T_2 = 0.1$, then $x_a$ can be found.
from classical fundamental solution of the linear equation with constant $C = C_0 r^3$ and $\lambda = \lambda_0 T$. It may be made because the zone of the great disturbances of $T$ at $T_{cr} = 4K$ and $r_0 < 10$mm localizes close by crack and $x_0(T_{cr} = 4.3') \times r_0$. Then we calculate $x_0 = 1.5, 7.0, 33$ mm at $r_0 = 0.1, 1, 10$mm, i.e. really $x_0 r_0$. The above estimations corroborate our assumptions about the essential influence of heat at the microfracture of SC.

CONCLUSIONS

The main idea was to develop the theory of mechanical disturbances in different superconductors by combination of the general approaches from physics and mechanics. Then the different canonical and practical problems have been considered. Some were suggested by physicists. The knowledge of the studied average stress fields are enough if a macroscopic failure theory is in force and are necessary before microscopic analysis for composites. The present results can be used for the choice of optimum configurations of SC, the estimations of their potential ability, the limit values of $j$ and $B$ and the temperature field near the small suddenly arisen defect and as the ground for future investigations.

REFERENCES.

