SHORT CRACK PROBLEMS IN GAS TURBINE DISKS

J. Drexler and J. Statečny
Aeronautical Research and Test Institute, Prague, Czechoslovakia

ABSTRACT

Gas turbine disks operating under low-cycle fatigue conditions represent typical machine parts with short crack systems. Herewith, the authors aim at presenting some methods being good for solving real problems with short cracks in the process of disk airworthiness certification.

KEYWORDS

Mechanical systems; machine parts; gas turbine disks; low-cycle fatigue; crack detecting and development; short crack problems.

BACKGROUND

According to Drexler (1984) stated that there is no universal method which would enable solving short crack problems in machine parts under all possible varieties of structural configurations and corresponding operational conditions. Obviously, the engineering solutions of the mentioned problems are to be searched for a specific machine type and a representative sample of its working field characteristics, only. In these relations, short crack problems in gas turbine disks involve the following typical subproblems of

1. detecting a short crack at its first occurrence,
2. an adequate description of a short crack system aiming at the relation of short crack characteristics to disk reliability parameters,
3. estimating the safe life or life to safe crack occurrence in the worst damaged fir-tree blade attachment on the disk taken for being the least reliable disk in the whole envisaged production series.

In this relation, it is the authors' task to present some possible solutions of the subproblems mentioned above.

PROBLEM OF DETECTING A SHORT CRACK AT ITS FIRST OCCURRENCE

The main problem in examining a disk under test - should any of the known detection method be used - is the human operator taking the following two final resolutions:
The whole information content of a crack length datum may be depleted by the following elementary statement definitions:

C: "The critical machine part under investigation is a gas turbine disk".

N: "The programme test unit number finished by the disk up to this inspection time instant is N".

D: "Cracks are visually detected by a selected human operator whose ability level is to be quantified".

L: "The crack trace length is L mm".

O: "Other operating condition present when detecting cracks".

In our case, we have limited considerations of a specific machine part (disk), of an a priori agreed programme test unit number and of standard laboratory conditions. Hence, the fact that a crack length datum involves information expressed through statements C to O can be registered by a composed statement (D.L/L.C.N.O) being of random character due to the problem nature.

Therefore, our task to quantify the operator's ability level in respect to crack detection is transferred to that one of establishing an adequate probabilistic characteristic. Using the product rule, we may write

\[ \text{Prob} [ D.L/C.N.O] = \text{Prob} [D] \cdot \text{Prob} [L] \cdot \text{Prob} [C/N/O] \]

\[ (1) \]

**Fig. 1.** Crack number \( n_c \) in function of programme test unit number \( N \) as found on disk blade attachment edges, back side (Návec, Drexler, 1994). \( P_a \) - crack occurrence probability in a fir-tree blade attachment.

**Fig. 2.** Crack length first occurrence experimental data \( L \) as investigated in Neybull probability paper in respect to human operator's ability to crack detection. 1, 2 component probability distributions (see right side of equ.(6)). \# sample upper limit due to an a priori agreed test unit number \( N \).
Omitting - for brevity sake - the limitation symbols C, N, O, we get

\[
\text{Prob}[D,L] = \text{Prob}[D/L] \cdot \text{Prob}[L]
\]

(2)

where \(\text{Prob}[D,L]\) is the joint occurrence probability of the event that a crack is detected at its first occurrence and has a length of \(L\) mm, \(\text{Prob}[D/L]\) means the conditional probability of a crack being detected given its length \(L\) mm, \(\text{Prob}[L]\) means the probability of a crack having a length \(y < L\) mm independently of crack detection method used. It is evident that the conditional probability \(\text{Prob}[D,L]\) presents an adequate characteristic to be found as qualifying the ability level of a human operator in respect to crack detection. From eqn(2), we obtain the basic formula

\[
\text{Prob}[D/L] = \frac{\text{Prob}[D,L]}{\text{Prob}[L]}
\]

(3)

For estimating \(\text{Prob}[D/L]\) from experimental crack length data, see Fig. 2, two basic physical boundary conditions are to be taken into account:

A) Given an almost sure detectable crack length \(L_d\), then for all crack lengths \(L \geq L_d\) (see straight line segment \(A\) in Fig. 2) holds

\[
\text{Prob}[D/(L \geq L_d)] = 1.0
\]

(4)

and hence

\[
\text{Prob}[D/(L \geq L_d)] = \text{Prob}[L]
\]

(5)

For \(L < L_d\) the aimed probability characteristic \(\text{Prob}[D,L]\) can be found as follows

\[
\text{Prob}[D/(L < L_d)] = \frac{\text{Prob}[D/(L < L_d)]}{\text{Prob}[D/(L \geq L_d)]}
\]

(6)

B) The cracks grow from zero lengths to \(L_f\) lengths \((L_f = L_{cr})\) corresponding to catastrophic damage brought to the disk. Therefore, from a physical point of view, the two-parameters Weibull probabilistic model will be adequate for the component probability distribution, namely

\[
\text{Prob}[D/L] = \frac{1 - \exp \left\{ -\left( \frac{L}{\theta_1} \right)^{\theta_2} \right\}}{1 - \exp \left\{ -\left( \frac{L}{\theta_1} \right)^{\theta_2} \right\}}
\]

(7)

In Fig. 3, the result of applying eqn(7) to Fig. 2, data (160 data altogether) is shown. The almost sure detectable crack length \(L_{d}\) has been estimated taking the left side of eqn(7) equal to unity.

**Problem of an Adequate Short Crack System Description**

Drexler, Staček (1983) presented one of the possible assessments of this problem using the quantile crack length \(L_q\) for \(Q = 0.05\) and the simultaneous total number of cracks \(N_c\) in the first-tree blade attachments of the disk as parameters describing the short crack system as a whole. In Fig. 1, the \(N_c\) total number of cracks is referred to the programme test unit number by the following formula

\[
N_c = (m+1) \cdot \text{exp} \left\{ -\left( \frac{N_c \cdot m}{4} \right)^\theta \right\}
\]

(8)
\[ P_a = \frac{n_F}{m+1} \]  

(9)

PROBLEM OF ESTIMATING LIFE TO SAFE CRACK OCCURRENCE

Referring to airworthiness requirements for a hazard rate the life estimation in test unit number to safe crack occurrence within a single disk has been derived by Němec, Drexler (1984) as follows

\[ N_{FSC} = \lambda^{-1} \{- \ln(1 - \text{Prob}[n_{FSC}/n_F; m; M = 1])\} \]  

(10)

where \( n_F, n_{FSC} \) are total numbers of cracks in the disk blade fin-tree attachments up to the safe crack length \( L_{FSC} \) and to the critical length \( L_{CR} \), \( M \) is the number of disks accounted for in the life estimation, whereby the conditional probability

\[ \text{Prob}[n_{FSC}/n_F; m; M = 1] = \frac{\text{Prob}[x < n_{FSC}; n_F; m; M = 1]}{\text{Prob}[x < n_F; n_F; m; M = 1]} \]

\[ = \frac{\sum_{i=1}^{n_F} c_i^m \cdot p_i^m \cdot (1 - p_i)^{m-i}}{\sum_{i=1}^{n_F} c_i^m} \]  

(11)

Considering the \( M >> 1 \) disk production series, the probability (11) of meeting \( x < n_{FSC} \) cracked ones of \( m \) fin-tree blade attachments in the least reliable one of \( M \) produced disks changes to

\[ \text{Prob}[n_{FSC}/n_F; m; M >> 1] = 1 - (1 - \text{Prob}[n_{FSC}/n_F; m; M = 1])^M \approx 1 - \exp \{-M \cdot \text{Prob}[x < n_{FSC}; n_F; m; M = 1]\} \]  

(12)

whereby \( \text{Prob}[x < n_{FSC}; n_F; m; M = 1] \) approaches unity. Hence, the life estimation \( N_{FSC}(M) \) to safe crack occurrence for the least reliable one of \( M \) disks is given by the formula

\[ N_{FSC}(M) = \lambda^{-1} \text{Prob}[x < n_{FSC}; n_F; m; M = 1] \]  

(13)

In our example case, we had \( \lambda = 1.10^{-8}, n_F = 24, m = 28, 1 \leq M \leq 5000 \) and for \( P_a = 0.006759 \). The result when applying equ.(13) to these data is shown in Fig. 4. Therefrom we see that production series increase in \( M \) from 1 to 5000 disks diminishes in our example case the possible programme test unit number gain \( \Delta N = N_{FSC} - N_0 \) to about a half, when \( N_0 = 1.10^{-11} \).

REFERENCES
