SHORT CRACK PROBLEMS IN GAS TURBINE DISKS

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ABSTRACT

Gas turbine disks operating under low-cycle fatigue conditions represent typical machine parts with short crack systems. Herewith, the authors aim at presenting some methods being good for solving real problems with short cracks in the process of disk airworthiness certification.

KEYWORDS

Mechanical systems; machine parts; gas turbine disks; low-cycle fatigue; crack detecting and development; short crack problems.

BACKGROUND

Němec, Drexler (1984) stated that there is no universal method which would enable solving short crack problems in machine parts under all possible varieties of structural configurations and corresponding operational conditions. Obviously, the engineering solutions of the mentioned problems are to be searched for a specific machine type and a representative sample of its working field characteristics, only. In these relations, short crack problems in gas turbine disks involve the following typical subproblems of

1. detecting a short crack at its first occurence,
2. an adequate description of a short crack system aiming at the relation of short crack characteristics to disk reliability parameters,
3. estimating the safe life or life to safe crack occurence in the worst damaged fir-tree blade attachment on the disk taken for being the least reliable disk in the whole envisaged production series.

In this relation, it is the authors' task to present some possible solutions of the subproblems mentioned above.

PROBLEM OF DETECTING A SHORT CRACK AT ITS FIRST OCCURENCE

The main problem in examining a disk under test - should any of the known detection method be used - is the human operator taking the following two final resolutions:

i) the crack is (is not) present in the investigated zone,
ii) if present, the crack length estimation is L mm.

The presence of the human operator in crack detecting procedure forces the necessity of carefully verifying outside reference data on crack detecting probability before applying them to one's own purpose. One of the reliable (robust) methods often used independently on others under laboratory conditions is visual detection supported by several component penetrants and a microscope. In this connection, a method is necessitated how to quantify the crack detection ability level of a human operator: such quantification is required by Civil Aviation Authorities when an airworthiness certificate is applied.

First, let us examine the information structure of the results of human operator's activity in detecting cracks: these results are in form of registered crack length data referring to critical zones, e.g. edges of disk blade attachments, see Fig. 1. Such a crack length datum forcibly involves the following information :

a) the one of the type of critical machine part under investigation,
b) the one of the type of the detection method used, i.e. on human ability (one operator or operator group) to crack detection,
c) the one of a finished programme test unit number,
d) the one of crack trace length as accessible to visual investigation,
e) the one on all other operating conditions which were not explicitely mentioned.

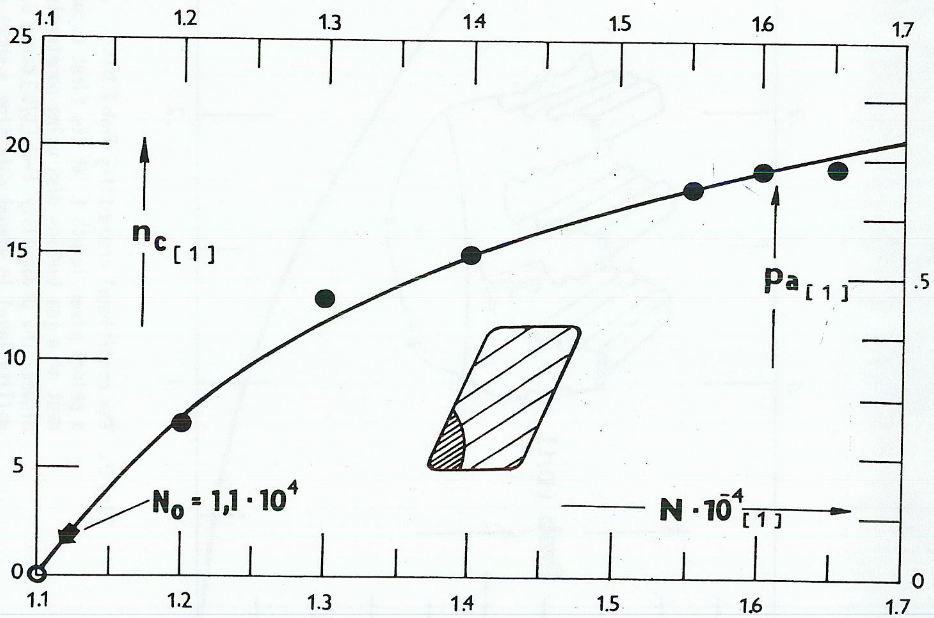

Fig. 1. Crack number n_c in function of the programme test unit number N as found on disk blade attachment edges, back side (Němec, Drexler, 1984). p_a - crack occurence probability in a fir-tree blade attachment.

The whole information content of a crack length datum may be depleted by the following elementary statement definitions :

C : " The critical machine part under investigation is a gas turbine disk "
N : " The programme test unit number finished by the disk up to this inspection time instant is N "
D : " Cracks are visually detected by a selected human operator whose ability level is to be quantified "
L : " The crack trace length is L mm "
O : " Other operating condition present when detecting cracks "

In our case, we have limited considerations of a specific machine part (disk), of an a priori agreed programme test unit number and of standard laboratory conditions. Hence, the fact that a crack length datum involves information expressed through statements C to O can be registered by a composed statement (D. L/C.N.O) being of random character due to the problem nature.

Therefore, our task to quantify the operator's ability level in respect to crack detection is transferred to that one of establishing an adequate probabilistic characteristic. Using the product rule, we may write

$$\mathrm{Prob}\,[\,D.L/C.N.O\,] = \mathrm{Prob}\,[D|L.C.N.O\,] . \mathrm{Prob}\,[\,L/C.N.O\,] \qquad (1)$$

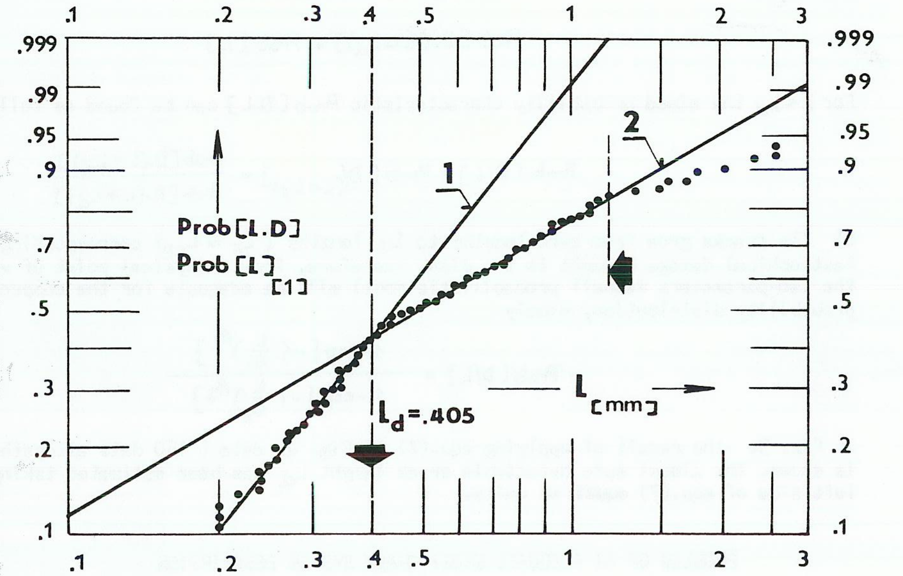

Fig. 2. Crack length first occurence experimental data L as investigated in Weibull probability paper in respect to human operator's ability to crack detection. 1, 2 component probability distributions (see right side of equ.(6), ◆ sample upper limit due to an a priori agreed test unit number N .

Omitting – for brevity sake – the limitation symbols C . N . D, we get

$$Prob\,[D.L] = Prob\,[D/L] \cdot Prob\,[L] \qquad (2)$$

where $Prob[D.L]$ is the joint occurence probability of the event that a crack is detected at its first occurence and has a length of L mm, $Prob\,[D/L]$ means the conditional probability of a crack being detected given its length L mm, $Prob\,[L]$ means the probability of a crack having a lenght $y < L$ mm independently of crack detection method used. It is evident that the conditional probability $Prob\,[D/L]$ presents an adequate characteristic to be found as qualifying the ability level of a human operator in respect to crack detection. From equ.(2), we obtain the basic formula

$$Prob\,[D/L] = \frac{Prob\,[D.L]}{Prob\,[L]} \qquad (3)$$

For estimating $Prob\,[D/L]$ from experimental crack length data, see Fig. 2., two basic physical boundary conditions are to be taken into account :

A) Given an almost sure detectable crack length L_d , then for all crack lengths $L \geq L_d$ (see straight line segment 2 in Fig. 2.) holds

$$Prob\,[D/(L \geq L_d)] = 1{,}0 \qquad (4)$$

and hence

$$Prob\,[D.(L \geq L_d)] = Prob\,[L] \qquad (5)$$

For $L < L_d$ the aimed probability characteristic $Prob\,[D.L]$ can be found as follows

$$Prob\,[D/L] \equiv Prob\,[D/(L<L_d)] = \frac{Prob\,[D.(L<L_d)]}{Prob\,[D.(L \geq L_d)]} \qquad (6)$$

B) The cracks grow from zero lengths to L_F lengths ($L_F \equiv L_{CR}$) corresponding to catastrophical damage brought to the disk. Therefore, from a physical point of view , the two-parameters Weibull probabilistic model will be adequate for the component probability distribution, namely

$$Prob[D/L] = \frac{1-\exp\{-(\frac{L}{\theta_1})^{\beta_1}\}}{1-\exp\{-(\frac{L}{\theta_2})^{\beta_2}\}} \qquad (7)$$

In Fig. 3. the result of applying equ.(7) to Fig. 2. data (160 data altogether) is shown. The almost sure detectable crack lenght L_d has been estimated taking the left side of equ.(7) equal to unity.

PROBLEM OF AN ADEQUATE SHORT CRACK SYSTEM DESCRIPTION

Drexler, Statečný (1983) presented one of the possible assessments of this problem using the quantile crack length L_Q for $Q = 0.05$ and the instantaneous total number of cracks n_c in the fir-tree blade attachments of the disk as parameters describing the short crack system as a whole. In Fig. 1. , the n_c total number of cracks is referred to the programme test unit number by the following formula

$$n_c = (m+1)\cdot(1-\exp\{-(\frac{N-N_0}{\theta})^{\alpha}\}) \qquad (8)$$

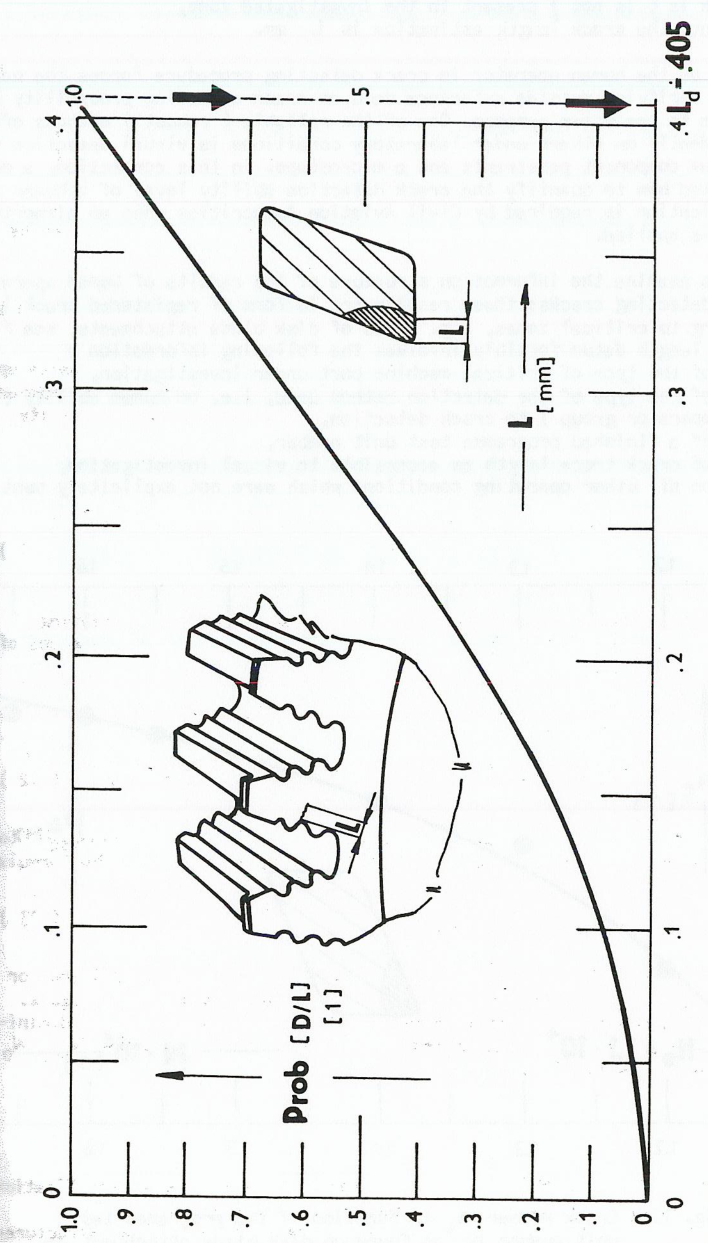

Fig. 3. The conditional probability $Prob\,[D/L]$ of detecting a short crack of an a priori given length L at its first occurence in a blade fir-tree attachment of a gas turbine disk using penetrants and microscope detection techniques. The probability $Prob\,[D/L]$ curve demonstrates the human operator ability level in visual detecting a short crack on basis of equ.(7) applied to Fig. 2. data.

m being the number of fir-tree attachments to the disk under investigation, whereby the crack occurence probability in one of the blade attachments is given by

$$p_a = \frac{n_F}{m+1} \qquad (9)$$

PROBLEM OF ESTIMATING LIFE TO SAFE CRACK OCCURENCE

Referring to airworthiness requirements for a hazard rate the life estimation in test unit number to safe crack occurence within a single disk has been derived by Němec, Drexler (1984) as follows

$$N_{FSC} \geq \lambda^{-1}.\{-\ln.(1-Prob[n_{FSC}/n_F; m; M=1])\} \qquad (10)$$

where n_F, n_{FSC} are total numbers of cracks in the disk blade fir-tree attachments up to the safe crack length L_{FSC} and to the critical length L_{CR} , M is the number of disks accounted for in the life estimation, whereby the conditional probability

$$Prob[n_{FSC}/n_F; m; M=1] = \frac{Prob[(x<n_{FSC}).n_F; m; M=1]}{Prob[x<n_F; n_F; m; M=1]} = \frac{\sum_{j=1}^{n_{FSC}} C_j^m \cdot p_a^j \cdot (1-p_a)^{m-j}}{\sum_{j=1}^{n_F} C_j^m \cdot p_a^j \cdot (1-p_a)^{m-j}} \qquad (11)$$

Considering the $M \gg 1$ disk production series, the probability (11) of meeting $x< n_{FSC}$ cracked ones of m fir-tree blade attachments in the least reliable one of M produced disks changes to

$$Prob[n_{FSC}/n_F; m; M\gg 1] = 1-(1-Prob[n_{FSC}/n_F; m; M=1])^M \doteq 1-\exp\{-M.Prob[(x<n_{FSC}).n_F; m; M=1]\} \qquad (12)$$

whereby $Prob[x<n_F; n_F; m; M\gg 1]$ approaches unity. Hence, the life estimation $N_{FSC}(M)$ to safe crack occurence for the least reliable one of M disks is given by the formula

$$N_{FSC}(M) \doteq \lambda^{-1}.M.Prob[(x<n_{FSC}).n_F; m; M=1] \qquad (13)$$

In our example case, we had $\lambda = 1.10^{-8}$, $n_F = 24$, $m = 28$, $1 \leq M \leq 5000$ and for $p_a = 0.82759$. The result when applying equ.(13) to these data is shown in Fig. 4. Therefrom we see that production series increase in M from 1 to 5000 disks diminishes in our example case the possible programme test unit number gain $\Delta N = N_{FSC} - N_0$ to about a half, when $N_0 = 1.1x10^4$.

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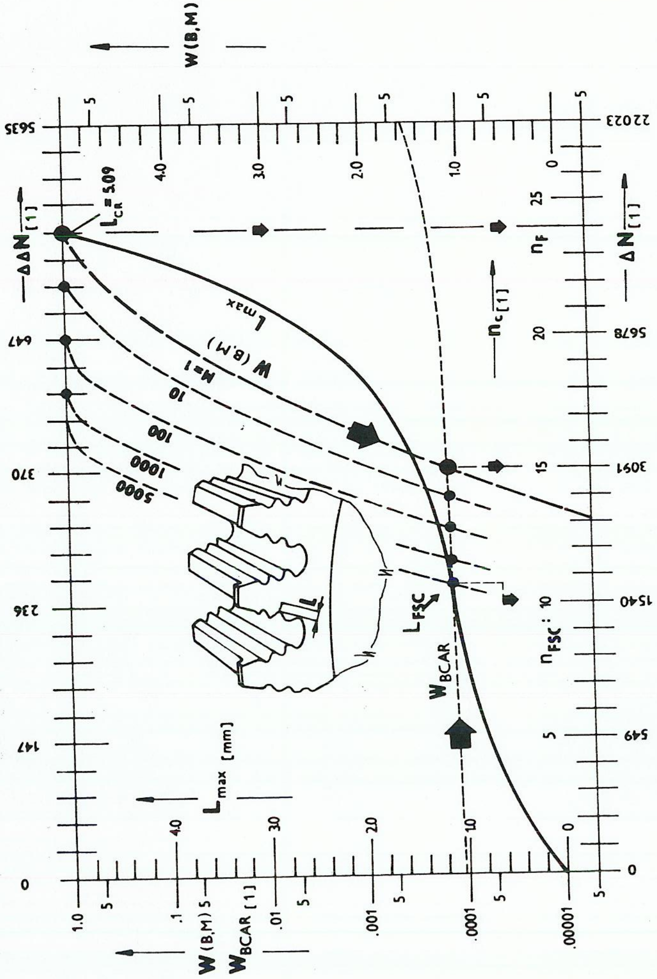

Fig. 4. Example of estimated life gain $\Delta N = N_{FSC} - N_0$ to safe crack ($L_{FSC} \doteq 1.1$ mm) occurence in the least reliable one of the M disks with short cracks according to equ.(13), given $\lambda = 1.10^{-8}$, $n_F = 24$ ($L_{CR} \doteq 5.09$ mm), $m = 28$, $N_0 = 1.1x10^4$. $W(B,M)$ corresponds to equ.(12), W_{BCAR} is the BCAR admissible probability of catastrophical damage brought to the disk, $\Delta\Delta N$ being the test unit number between individual crack occurences.