OBSERVATIONS ON R-CURVE DETERMINATION

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ABSTRACT

R-curves may be obtained from various specimen geometries using multiple or single specimen testing techniques. Furthermore, various methods for measuring crack length can be utilised. The results obtained for tests on compact tension and three-point bend specimens for two different thicknesses using a variety of crack growth measuring techniques are presented for a specific material. J_{IC} values are computed in accordance with ASTM E813-81 and comments made regarding the use of this procedure.

INTRODUCTION

Under situations where LEFM conditions are contravened, due to excessive plasticity at the crack tip, K_{Ic} is no longer a valid parameter and recourse must be made to the application of elastic-plastic fracture mechanics. There are two fracture techniques widely applied to situations where yielding predominates, namely crack opening displacement (COD) (Wells, 1961) and the J contour integral (Rice, 1968). In order to assess the performance of a cracked structure using the J integral approach it is necessary to determine the critical value of J corresponding to the onset of stable crack growth. This is achieved by experimentally determining the relationship between the J-integral and crack extension, termed its Resistance curve. ASTM E813 (1981) describes a standard test procedure for the determination of J_{IC}, the critical value of J, considered a material property. This procedure requires obtaining a minimum of four data points within a specified range of physical crack growth and allows for the use of either a single or multi specimen test approach.

EXPERIMENTAL METHODS

The multiple specimen technique involves testing four to six pre-fatigue cracked test pieces to different specific values of displacement and obtaining an accurate record of corresponding measurements of load and load line displacement. Final crack length is obtained from physical measurement stations. In order to aid differentiation between the regime of stable
crack growth and that of final overload fracture, measures such as heat
tinting prior to final failure are often employed. Integrating the load-
load point displacement curves and applying appropriate geometric correction factors enables J-integral to be calculated. The plot of the resulting J- da points defines the R-resistance curve.

The single specimen technique involves testing one pre-crack specimen by
loading incrementally throughout a load increasing test, and at various
stations determining the load, load point displacement, crack length and
the area under the appropriate section of the load – load point displacement
curve. The crack length can be determined by such techniques as unloading
elastic compliance or by direct current potential drop. Typically
some ten-thirty stations are employed giving a corresponding number of
J- da points on the R-resistance curve.

TESTING AND ANALYSIS

The material studied is a high strength age-hardening wrought copper nickel
alloy to DGS Specification 337, of typical chemical composition and mechanical properties as shown in Table 1.

| Table 1 Chemical Composition and Mechanical Properties |
|-------------|-----|-----|-----|-----|
| Cu | Ni | Al | Mn | Fe |
| Rem | 14.4 | 1.5 | 4.4 | 1.0 | 0.05 | 0.1 |


Tensile Strength 744 MPa
0.2% Proof Stress 552 MPa
Reduction in Area 16%
Young's Modulus 143 x 10^3 MPa

Compact tension and three-point bend specimens were manufactured to standard
dimensions, two sizes being tested of thicknesses W = 30 mm and W = 50 mm.
Pre-cracking was carried out using a load shedding technique to give a
crease length to width ratio (a/W) of 0.6 and a final stress intensity range of 25 MPa \sqrt{m}. Crack length was controlled by fractomat surface gauges. Fracture toughness testing was undertaken at a displacement rate of 4 x 10^{-3} mm/sec the load and load point displacement being displayed on a X-Y chart recorder. Crack length, which is probably the most important measurement to be made, was determined by various methods.

Physical measurement of crack length following heat tinting is a widespread
standard technique used to identify the regimes of fatigue growth, stable
crack growth and final overload failure. However, for this material there
was a difficulty in determining the boundary between fatigue and stable

crack growth since both were essentially intergranular (Duggan, Dimbylow and Jones, 1980). Secondary cracking on heat tinting was also observed, giving an apparent increase in the length of stable crack growth.

During fatigue pre-cracking, surface crack length measurements were made
using Fractomat gauges. In some instances, before carrying out the R-
resistance test the Fractomat gauges were removed, the specimen surface polished and direct measurements of the surface crack length made optically and using replica techniques. These confirmed the accuracy of the

Fractomat gauges. Further surface measurements were made, both directly
and with replicas, at the end of the R-resistance test. Following heat

tinting, the surfaces were re-polished and surface measurements again made
before finally fracturing the specimens. This technique provided
confirmation that secondary cracking resulted from the thermal treatment. The heat

tinting method and its associated problems have been discussed elsewhere
(Jones, Duggan, Spence and Barnes, 1982).

Unloading elastic compliance can also be utilised in determining crack length since a/W is a function of the normalised compliance (CEB). Various
relationships may be found in the literature (Towers, 1981) relating CEB as
a function of a/W. In this work a polynomial expression is used, i.e.

\[
(a/W) = C_0 + C_1 U + C_2 U^2 + C_3 U^3 + C_4 U^4 + C_5 U^5
\]

where

\[
U = 1/(CEB) + 1
\]

The coefficients for the compact tension specimen based on load-line clip
gauge measurements (Jahloinski, 1983) and for three-point bend specimens based on load-line displacements measured using a linear displacement transducer, are listed in Table 2.

| Table 2 Coefficients for Determining Crack Length from Compliance |
|-------------|-----|-----|-----|-----|-----|
| Co | C1 | C2 | C3 | C4 | C5 |
| Compact Tension | 1.0002 | -4.06319 | 11.242 | -106.043 | 464.335 | -650.677 |
| 3-point Bend | 1.013 | -4.498 | 19.708 | -361.706 | 2692.97 | -7941.8 |

For the 3-point bend specimens the linear displacement transducer was generally
directed directly beneath the specimen notch. However an additional test was also undertaken using a comparator bar technique (Schwalbe, 1983).

At each station in the unloading compliance test the displacement was held for
a period sufficient to allow the crack to stabilise. The extent of unloading was determined to be within the range 10-20%, little difference being detected within these limits.

Crack length can also be measured during the R-resistance test by means of
dc potential difference. This has the advantage of not requiring unloading
during the test and associated implications on J. Two methods were examined
for obtaining crack length from the potential difference measured across the notch. The first was a calibration curve obtained from a simul-
ated electro-static paper specimen arrangement (Anciti, Kula, DiCesaro, 1963). The second relied on the use of a theoretical calibration equation
suggested by Johnson (1965) such that for a centre crack panel of width 2W
with an initial crack length a0 exhibiting a potential drop U0, the crack length corresponding to potential drop U can be calculated from:
\[
\alpha = \frac{(2W/L) \arccos \left( \frac{\cosh(\Pi y/2W)}{\cosh(\Pi y/2W) \arccosh \left( \frac{\cosh(\Pi y/2W)}{\cos(\Pi a_0/2W)} \right)} \right)}{
\cosh(\Pi y/2W) \arccosh \left( \frac{\cosh(\Pi y/2W)}{\cos(\Pi a_0/2W)} \right)}
\] (3)

In each case a constant current of 40 amperes was applied to the specimen and insulators were contained in the load chain to isolate the specimen from the load frame.

On completion of the R-resistance test J was calculated using the relationship (Clark, Andrews, Begley, Donald, Emblay, Landes, McCabe and Underwood, 1979):

\[
J = (A/\text{Eb}) f(a/W)
\] (4)

where

- A = area under load, load-point displacement record
- B = specimen thickness
- b = uncracked ligament length

For the three-point bend specimen \( f(a/W) = 2 \), whilst for the compact tension specimen a tensile correction is also included such that:

\[
f(a/W) = 2 \left[ \frac{1 + \alpha}{1 + \alpha^2} \right] \] (5)

where

\[
\alpha = \left[ \left( \frac{2a_0}{W-a_0} \right)^2 + 4a_0(W-a_0) + 2 \right]^{\frac{1}{2}} - \left[ \left( \frac{2a_0}{W-a_0} \right) + 1 \right]
\] (6)

Since the linear displacement transducer used to measure load-point displacement during the 3-point bend tests was referenced to the loading jig, calibration trials were undertaken (Buzzard and Fisher, 1978) to ascertain the extent of errors introduced due to elastic compression of the jig and plastic indentation of the specimen. These errors if significant will contribute to the area under the load displacement curve and need to be taken into account.

Using the testing techniques and methods of analysis described, R-resistance curves, in the form of J-\( \Delta a \) plots, have been obtained for both 3-point bend and compact tension specimens, examples of each are shown in Fig. 1. The data points were validated in accordance with ASTM E813-81, Fig. 2 showing the linearly regressed data for three point bend and compact tension tests. The values of J_{Ic} obtained using the various test procedures are compared in Table 3.

**Fig. 1. Experimental R-Resistance Data**

**Fig. 2. Linear Regression of Experimental Data**

**TABLE 3 Summary of Results**

<table>
<thead>
<tr>
<th></th>
<th>Three Point Bend</th>
<th>Compact Tension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 PB S</td>
<td>3 PB L</td>
</tr>
<tr>
<td>W = 30</td>
<td>W = 50</td>
<td></td>
</tr>
<tr>
<td>J_{Ic} MN/m</td>
<td>J_{Ic} MN/m</td>
<td>J_{Ic} MN/m</td>
</tr>
<tr>
<td>U.C.P.D. (PD)</td>
<td>0.14</td>
<td>0.109</td>
</tr>
<tr>
<td>Unloading (UC)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Compliance</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
DISCUSSION OF RESULTS

The most critical measurement required in obtaining R-resistance curves is that of crack length. In the case of the material of this study physical measurements of the fatigue and fracture zones are extremely difficult to make. The crack growth mechanisms under fatigue is predominantly intergranular and the distinction between the fatigue pre-cracking and the crack extension during a load increasing test is not easy to determine. Further, heat tinting to distinguish the crack extension from final fracture has been shown to result in secondary cracking (Jones, Duggan, Spence and Barnes, 1982). This behaviour is not to be totally unexpected since copper base super alloys, and particularly nickel in particular are subject to ductility trough behaviour at intermediate temperatures (Chubb, Billingham, Hancock, Dibbayol and Newcombe, 1978). The secondary cracking appears to be dependent on the extent of plastic deformation ahead of the crack tip and it is postulated that the differing degrees of restraint associated with the compact tension and 3-point bend specimen may account for the observed heat tinting characteristics.

Unloading elastic compliance methods for the determination of crack extension proved to be unreliable. This is particularly true for initial crack extension, where frictional effects and grip arrangements are important. In fact, in some instances there are indications that the crack was actually reducing in length with increase in load. The phenomenon was more pronounced for three-point bend when measuring the compliance using a displacement transducer below the specimen. Although the comparator bar technique showed advantages in this respect, it was considered that compliance could not be determined with sufficient accuracy to be reliable from an analogue load-displacement plot.

Greater accuracy was attempted for the compact tension tests by digitising the load and displacement values at the maximum and minimum positions of the unloading sequence. However, although this gave greater apparent accuracy, any non-linear response between load and displacement, which could result in an erroneous prediction of compliance, could not be detected. This is a particular problem with the material of this study, where significant relaxation is observed at each station.

The use of the direct current potential drop technique for determining crack length was found to be generally the most reliable method for the compact tension specimens. The application of equation (3) was found to give good agreement with experimentally determined calibration curves but was not applicable to the compact tension specimen. The application of equation (3) is based on the presence of a finite width slot in a uniform potential field and it is probable that this condition is not achieved for the compact tension geometry.

The resultant J-Δa plots show differences in behaviour between compact tension and three-point bend and also between the two sizes of specimen examined. Different R-curve characteristics have been observed between compact tension and centre cracked tension specimens (Simpson, 1981) and the variations were attributed to the lower restraint of the centre cracked tension geometry. However, the difference in restraint between compact tension and three point bend specimens is not as significant and would not give rise to the observed results. An increase in slope of the R-curve can also be attributed to larger shear lips, although this feature was not observed in the present series of tests. It is perhaps significant that this material shows a tendency for intergranular failure and it is possible that the tensile component of stress present during the compact tension test accentuates this feature giving rise to a lower crack growth resistance. Probably of more significance, however, is the scatter associated with the crack extension measurement. No absolute measurement was available which could act as a base-line for the various measuring techniques and doubt must exist for the values obtained.

For each of the single specimen J-Δa curves, the final crack length determined by either direct current potential drop or unloading compliance was within 15% of the physically measured length. The end points of each of the single specimen tests used as data points for a multi-specimen analysis give a different JIC from the average of the single specimen tests. Since ASTM E813-81 does not include the provision of an F test or some other similar statistical indicator of degree of fit such variations in data are apparently acceptable.

![Fig. 3. Scatter Associated with Typical Set of Data](image)

By depicting the results as a single line it is implied that a unique linear relationship exists between the J-Δa data points. However, if a variance analysis is undertaken the degree of scatter associated with the data can be shown. Such an analysis is illustrated in Fig. 3, the shaded area depicts the 95% confidence limits and incorporates the standard error of the slope and intercept. Statistically this suggests that due to the nature of the data, JIC lies within the range 0.09–0.19 MN/m as opposed to the unique value given by the regression of 0.14 MN/m. The unique value given by the regression analysis is also very dependent on the data position, minor changes resulting in quite different values of JIC. The data grouping rules of ASTM E813-81 are therefore themselves not sufficient and support the view that a variance analysis should be undertaken in conjunction with the regression.

The arbitrary exclusion of data lying outside the 1.5 mm offset line requires further consideration. Undoubtedly as Δa increases there is a departure from linearity in the J-Δa curve and if the data is to be regressed in a linear manner then some method of exclusion needs to be applied. However, when the data can be shown to conform, this predefined exclusion would appear to be unduly restrictive, particularly since there is often greater
confidence in these $\Delta a$ measurements. An alternative method would be to apply a correlation coefficient limit to the regressed line. An iterative procedure could then be undertaken to sequentially exclude the furthest most $J-\Delta a$ points until the condition was achieved. If the condition is not achieved then the test could be designated invalid. Also for consideration is the use of a power law fit through the data points since this appears a more appropriate description of material crack growth resistance. A least square analysis comparing linear, quadratic and cubic fits to the valid data from the 3-point bend multi-specimen tests is shown in Fig. 4.

Fig. 4. Power Law Fitting Through Valid Data Points

A similar analysis using all data points is given in Fig. 5. As can be seen the value of $J_{IC}$ obtained is dependent on the form of analysis adopted. The least square power law analysis does not necessarily result in a better representation of the materials $R$-resistance curve, and of particular concern is the possibility of an inflection point at low values of $\Delta a$ giving an upward turn in the resistance curve as it is extrapolated back to the blunting line. Again the data grouping rules would need to be revised if a power law fit is to be adopted.

CONCLUSIONS

1. Data obtained using multiple-specimen testing is not very reliable for the material studied due to the difficulty in physically measuring the crack extension.
2. Unloading elastic compliance cannot be determined with sufficient accuracy from the load-displacement trace.
3. Johnson’s equation relating normalised crack extension with normalised voltage was found to apply to the three-point bend test specimens, but not to the compact tension specimens.
4. $J_{IC}$ values obtained from the three point bend geometry are different to those obtained from the compact tension geometry.
5. The ASTM E813 data reduction guidelines are not sufficient to produce a unique value of $J_{IC}$ and an alternative procedure is required.
REFERENCES