MORPHOLOGY OF STATIC ORTHOTROPIC ISOCHROMATIC CRACK-TIP FRINGE PATTERNS

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ABSTRACT

This paper discusses recent developments towards a classification of static orthotropic crack-tip fringe patterns as obtained from stress-coating techniques. The influence of the degree of orthotropy as well as the role of regular stress fields on the shape of the overall patterns is analyzed in detail. Procedures for determination of stress intensity factors are discussed.

KEYWORDS

Linear-elastic fracture; isochromatic fringe pattern; orthotropic material; energy release rate.

INTRODUCTION

During the past two decades ever-increasing emphasis has been given to elasticity problems in anisotropic materials, primarely due to the important application of fibre-reinforced materials in engineering structures. Many engineering composites, like plastics, laminates and multicellular structures exhibit strong directional elastic effects associated with elastic symmetry with respect to three mutually perpendicular planes - the class of orthogonally-anisotropic, or simply orthotropic materials.

Considerable effort has been devoted to the determination of stress intensity factors and strain energy release rates for a variety of loading situations in finite specimens weakened by a crack /1/. A potential and effective method for experimental determination of stress intensity factors is offered by photoelasticity. An exhaustive classification scheme of isochromatic crack tip stress patterns for isotropic materials may be found in Ref./2/. A survey of K-determination procedures based on isochromatic fringe pattern data reductions is due to Rossmanith and Chona /3/. In engineering practice considerable complexities incurred by orthotropic photoelasticity are most often circumvented by employing isotropic birefringent coatings on the orthotropic composite materials. This technique allows for the separation of normal-mode and shearing-mode stress intensity factors as is commonly done for isotropic materials.

This study involves a classification scheme of analytically generated pure mode and mixed-mode isochromatic crack-tip fringe patterns for static cracks in orthotropic composite materials. Direct application within the framework of brittle coating stress analysis technique as applied to unidirectional or multidirectional structural materials weakened by a sharp crack subjected to uniaxial and biaxial stress fields is developed. The joint influence of changing parameters such as ratio of Young's moduli, prestressing parallel to the crack etc on the geometrical configuration of the isochromatic lobes can easily be identified from orthotropic fringe loop diagrams.

ANALYSIS

Consider a semi-infinite straight crack in an orthotropic, homogeneous, elastic plate specimen subjected to time-independent in-plane stress loading as shown in Fig.1.

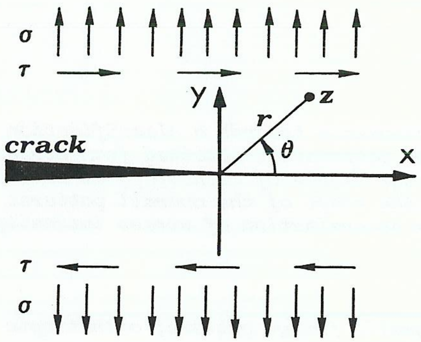


Fig.1 Geometry and loading configuration of a semi-infinite crack in an orthotropic material

Employing a complex stress function approach for combined-mode loading /4,5/

$$F = F_I + F_{II} \tag{1}$$

$$\left.\begin{aligned} F_I &= \frac{1}{2}\,\mathrm{Re}\{\bar{\bar{Z}}_I(z_1)+\bar{\bar{Z}}_I(z_2)\} + \frac{\beta}{2\alpha}\,\mathrm{Im}\{\bar{\bar{Z}}_I(z_1)-\bar{\bar{Z}}_I(z_2)\} \\ F_{II} &= -\frac{1}{2\alpha}\,\mathrm{Re}\{\bar{\bar{Z}}_{II}(z_1)-\bar{\bar{Z}}_{II}(z_2)\} \end{aligned}\right\}\ \alpha^2>0 \tag{2}$$

$$\left.\begin{aligned} F_I &= \frac{1}{2}\,\mathrm{Re}\{\bar{\bar{Z}}_I(z_1)+\bar{\bar{Z}}_I(z_2)\} - \frac{\beta}{2\alpha}\,\mathrm{Re}\{\bar{\bar{Z}}_I(z_1)-\bar{\bar{Z}}_I(z_2)\} \\ F_{II} &= -\frac{1}{2\alpha}\,\mathrm{Im}\{\bar{\bar{Z}}_{II}(z_1)-\bar{\bar{Z}}_{II}(z_2)\} \end{aligned}\right\}\ \alpha^2<0 \tag{3}$$

the stresses σ_x, σ_y, and τ_{xy} may be represented in the form

$$\left.\begin{aligned} \sigma_x^I &= (\alpha^2+\beta^2)/2\ \{\mathrm{Re}(Z_{I1}+Z_{I2})-\beta/\alpha\ \mathrm{Im}(Z_{I1}-Z_{I2})\} + \sigma_{ox} \\ \sigma_y^I &= 1/2\ \mathrm{Re}(Z_{I1}+Z_{I2}) + \beta/(2\alpha)\ \mathrm{Im}(Z_{I1}-Z_{I2}) \\ \tau_{xy}^I &= -(\alpha^2+\beta^2)/2\alpha\ \mathrm{Re}(Z_{I1}-Z_{I2}) \end{aligned}\right\} \tag{4}$$

for $\alpha^2>0$, and

$$\left.\begin{aligned} \sigma_x^{II} &= -\frac{1}{2\alpha}\ \{(\alpha^2-\beta^2)\ \mathrm{Re}(Z_{II1}-Z_{II2}) - 2\alpha\beta\ \mathrm{Im}(Z_{II1}+Z_{II2})\} \\ \sigma_y^{II} &= -\frac{1}{2\alpha}\ \mathrm{Re}(Z_{II1}-Z_{II2}) \\ \tau_{xy}^{II} &= 1/2\ \{\mathrm{Re}(Z_{II1}+Z_{II2}) - \beta/\alpha\ \mathrm{Im}(Z_{II1}-Z_{II2})\} \end{aligned}\right\} \tag{5}$$

for $\alpha^2>0$, and

$$\left.\begin{aligned} \sigma_x^I &= -(\alpha^2-\beta^2)/2\alpha\{(\alpha+\beta)\mathrm{Re}Z_{I1} + (\alpha-\beta)\mathrm{Re}Z_{I2}\} + \sigma_{ox} \\ \sigma_y^I &= 1/2\alpha\{(\alpha-\beta)\mathrm{Re}Z_{I1} + (\alpha+\beta)\mathrm{Re}Z_{I2}\} \\ \tau_{xy}^I &= (\alpha^2-\beta^2)/2\alpha\ \ \mathrm{Im}(Z_{I1}-Z_{I2}) \end{aligned}\right\} \tag{6}$$

for $\alpha^2<0$, and

$$\left.\begin{aligned} \sigma_x^{II} &= 1/2\alpha\ \{(\alpha+\beta)^2\,\mathrm{Im}Z_{II1} - (\beta-\alpha)^2\,\mathrm{Im}Z_{II2}\} \\ \sigma_y^{II} &= -1/2\alpha\ \mathrm{Im}(Z_{II1}-Z_{II2}) \\ \tau_{xy}^{II} &= 1/2\alpha\ \{(\alpha+\beta)\mathrm{Re}Z_{II1} - (\beta-\alpha)\mathrm{Re}Z_{II2}\} \end{aligned}\right\} \tag{7}$$

for $\alpha^2<0$. Re and Im denote real and imaginary parts of a complex function and it holds:

$$Z'_{kj} = d\bar{Z}_{kj}/dz = d^2\bar{\bar{Z}}_{kj}/dz^2 \quad (k=I,II;\ j=1,2) \tag{8}$$

Following the classical procedure and factoring out $K_k/\sqrt{2\pi z_j}$, the stress field in the vicinity of the orthotropic crack tip may be approximated by selecting Westergaard-type stress functions of the form

$$Z_{kj}(z_j) = K_k/\sqrt{2\pi z_j}\ \{\ 1 + O(z_j/r_s)\}\ (k=I,II;\ j=1,2) \tag{9}$$

where the symbol O() stands for higher order term regular stress fields and r_s is a reference scaling length. The subscript j refers to auxiliary coordinate transforms:

$$z_j = \begin{cases} (x \pm i\alpha y) + \beta y & \text{for } \alpha^2>0\ , \\ x \pm i(\alpha \pm \beta)y & \text{for } \alpha^2<0\ . \end{cases} \tag{10}$$

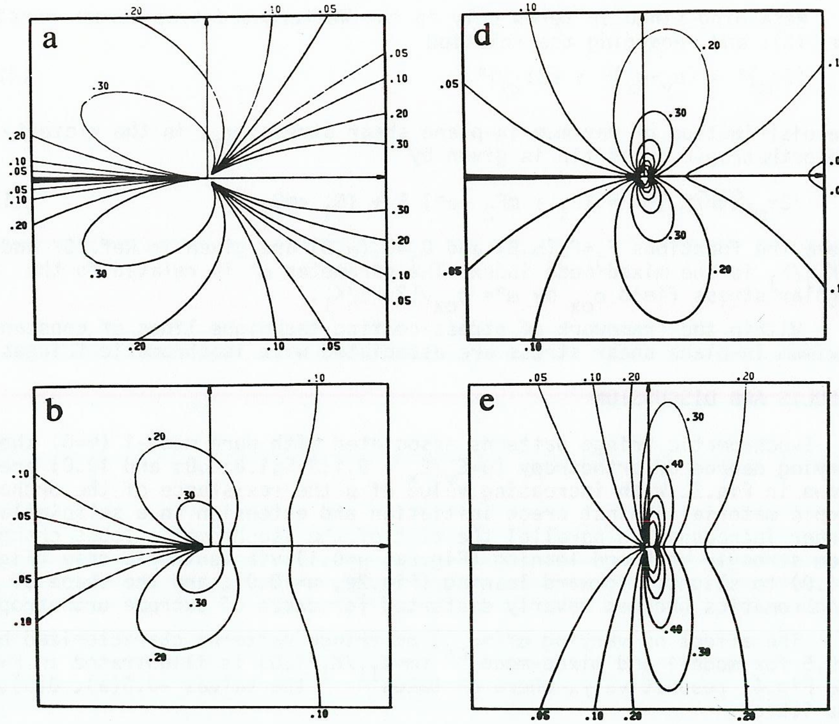
Upper (lower) signs refer to the subscript j=1 (j=2).

The effect of orthotropy is manifested in the parameters α and β which are linked with the elastic constants of a rectilinearly orthotropic material. Employing Hooke's law equations with only four elastic constants present

$$\left.\begin{aligned} \alpha^2 &= 1/(2\sqrt{\mu}) - \{1+\mu-2\nu_{xy}(1-\nu)\}/4\mu \\ \beta^2 &= 1/(2\sqrt{\mu}) + \{1+\mu-2\nu_{xy}(1-\nu)\}/4\mu\ . \end{aligned}\right\} \tag{11}$$

The parameter α^2 can attain positive and negative values, whereas β remains always non-negative. α^2 becomes negative whenever $\mu=E_y/E_x>1$, i.e. when the crack line is normal to the direction of maximum fracture toughness (usually the direction of fibre reinforcement). The isotropic limiting case is recovered upon setting $\mu=1$, i.e. $\alpha=0$ and $\beta=1$.

The additional homogeneous stress field σ_{ox} in equs(4) and (7) is representative for the load biaxiality. $\sigma_{ox}=0$ corresponds to uniform



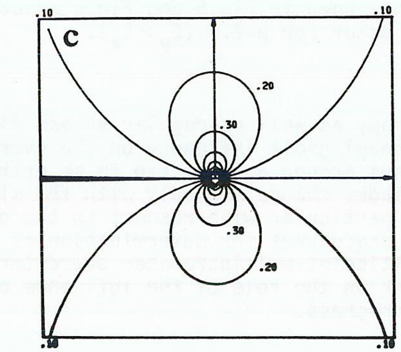


Fig.2 Analytically generated isochromatic mode-1 crack tip fringe patterns (lines τ_{max} = const) for orthotropic material showing the influence of direction of orthotropy:

$\mu = E_y/E_x$, $\sigma_{ox} = 0$,

a) μ = 0.1
b) μ = 0.5
c) μ = 1.0 (isotropic case)
d) μ = 2.0
e) μ = 10.0

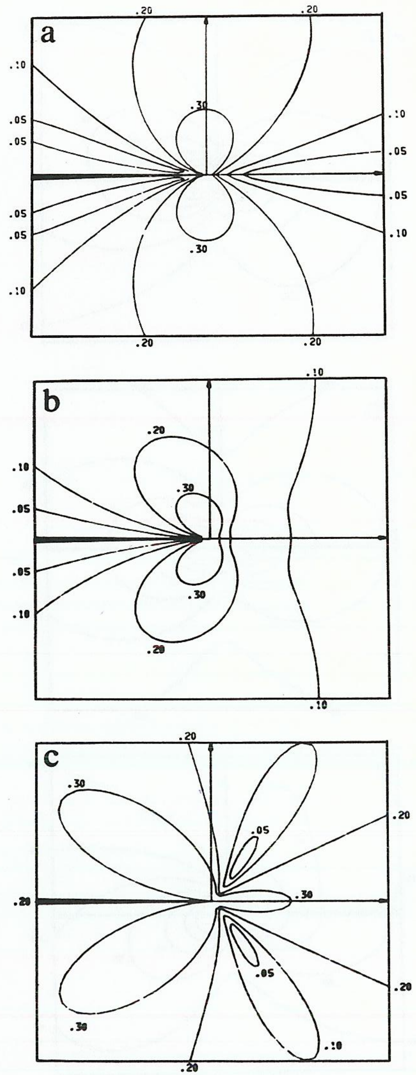


Fig.3 Analytically generated isochromatic mode-1 fringe patterns in orthotropic material μ=0.5 showing the effect of varying α^*: a) α^*=-0.5; b) α^*=0.; c) α^*= 1.0.

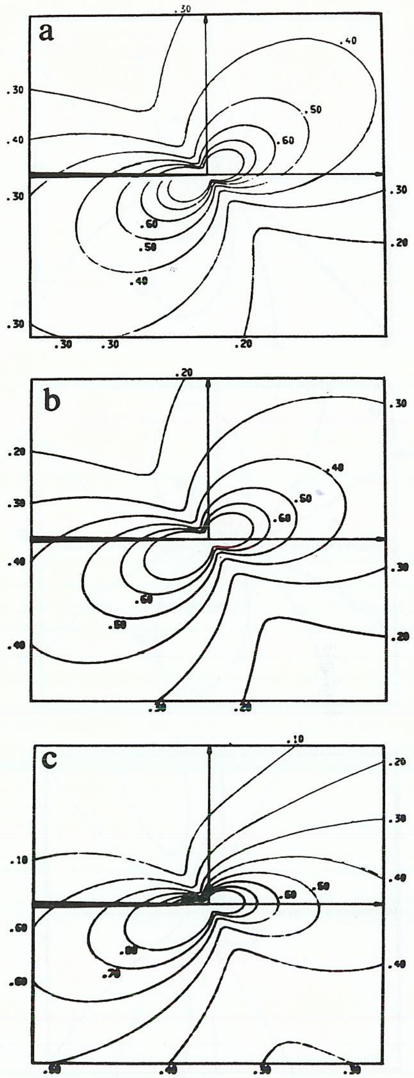


Fig.4 Analytically generated isochromatic mixed-mode fringe patterns in orthotropic material μ=0.5 showing the effect of varying α^*: a) α^*=-0.5; b) α^*=0.; c) α^*=1.0.

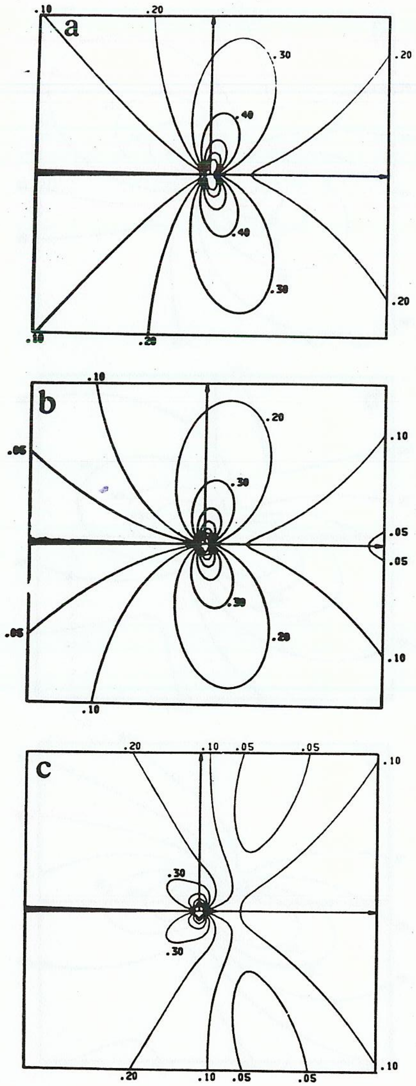


Fig.5 Analytically generated isochromatic mode-1 fringe patterns in orthotropic materials μ=0.2 showing the effect of varying α^*: a) α^*=-0.5; b) α^*=0.; c) α^*=1.0.

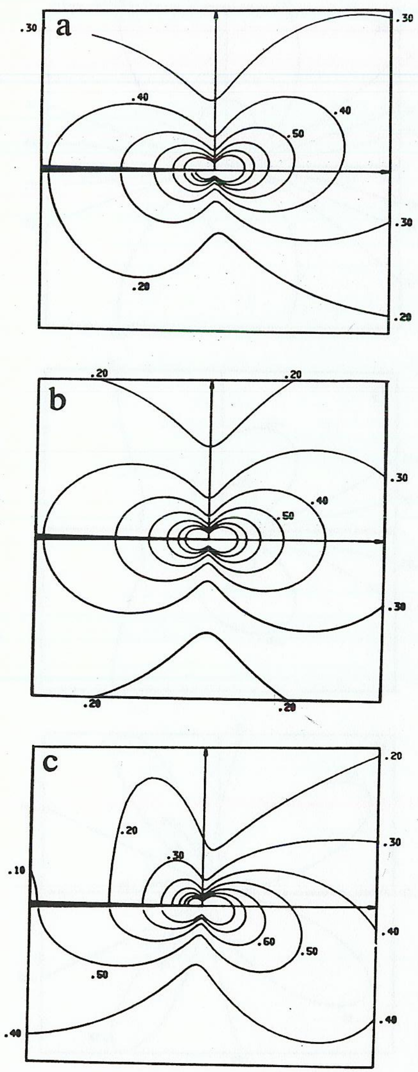


Fig.6 Analytically generated isochromatic mode-2 fringe patterns in orthotropic material μ=0.2 showing the effect of varying α^*: a) α^*=-0.5; b) α^*=0.; c) α^*=1.0.

biaxial tension, whereas positive (negative) values of σ_{ox} are associated with a homogeneous tension (compression) field acting parallel to the crack.

Retaining singular terms only in the Westergaard-type stress function, equ.(11), and regarding the relation

$$(2\tau_m)^2 = (\sigma_y-\sigma_x)^2 + (2\tau_{xy})^2, \qquad (12)$$

the distribution of maximum in-plane shear stress, τ_m, in the vicinity of the orthotropic crack tip is given by

$$\{2\tau_m\sqrt{2\pi r}/K_I\}^2 = \{F_1 + mF_2 - \alpha^*\}^2 + \{G_1 + mG_2\}^2 \qquad (15)$$

where the functions $F_j=F_j(\alpha,\beta)$ and $G_j=G_j(\alpha,\beta)$ are given in Ref./6/ and $m=K_{II}/K_I$ is the mixed-mode index. The parameter α^* is related to the regular stress field σ_{ox} by $\alpha^*=\sigma_{ox}\sqrt{(2\pi r)}/K_I$.

Within the framework of stress-coating technique lines of constant maximum in-plane shear stress are associated with isochromatic fringes.

RESULTS AND DISCUSSION

Isochromatic fringe patterns associated with pure mode-1 (m=0) showing varying degree of orthotropy ($\mu=E_y/E_x$ = 0.1;0.5;1.0;2.0; and 10.0) are shown in Fig.2. With increasing value of μ the resistance of the orthotropic material against crack initiation and extension in a selfsimilar manner increases. In parallel the tilt of the isochromatic lobes changes from strongly backward leaning (Fig.2a; μ=0.1) via isotropic case (Fig.2c, μ=1.0) to slightly forward leaning (Fig.2e, μ=10.0), and the shape of the isochromatics becomes severly distorted for cases of extreme orthotropy.

The effect of varying $\alpha^*(\sim\sigma_{ox})$ on fringe patterns characterized by μ=0.5 for mode-1 and mixed-mode ($m=K_{II}/K_I$=1.0) is illustrated in Fig.3 and Fig.4, respectively, where α^* takes the values -0.5(a), 0(b), and 1.0(c).

Similar fringe pattern distortions shown in Fig.5 and Fig.6 associated with mode-1 and mode-2, respectively, occur for μ=2.0 ($E_y > E_x$).

CONCLUSION

In general, the degree of orthotropy as well as regular stress fields in the stress functions may have a nonnegligible influence on the overall appearance of maximum shear stress lines around a crack tip in an orthotropic material. Size and shape of fringes change markedly with the state of stress at the tip of the crack, in particular with respect to the direction of orthotrpy. Advanced numerical procedures for determination of stress intensity factors such as a multipoint-multiparameter overdeterministic method /7/ may be employed. Work on the role of the influence of orthotropy on crack initiation is in progress.

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