METHOD FOR SPECIMEN LIFE PREDICTION UNDER PROGRAM LOADING TAKING INTO ACCOUNT THE SCATTER IN MATERIALS PROPERTIES

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ABSTRACT
A method of life prediction for individual specimens subjected to program loading, involving the use of strain criteria, is proposed. The study of steels 45 and 15 KP is carried out at different regimes of the program loading, and the conformity of calculated and experimental results is shown.

KEYWORDS
Fatigue; inelastic strain; program loading; life prediction; individual specimen.

INTRODUCTION
For metal fatigue problems much attention is paid to the methods of life prediction under program loading. The linear rule of damage accumulation (Palmgren-Miner's rule) is mostly used for this aim:

\[ \sum_{i=1}^{n} \frac{N_i}{N_{ti}} = 1, \quad (1) \]

where \( N_i \) is the number of loading cycles at the stress \( \sigma_{ai} \);
\( N_{ti} \) is the life at the stress \( \sigma_{ai} \);
\( n \) is the number of stress stages at the program loading.

The relation (1) may be put down in a more general way as follows:

\[ \sum_{i=1}^{n} \frac{N_i}{N_{ti}} = \alpha, \quad (2) \]

where \( \alpha \) is an experimentally defined value.

Numerous experimental data show that quite a limited number of these data corresponds to relation (1), in most cases \( \alpha \) value/relies on the interpretation of the linear rule of damage accumulation in its usual interpretation is also in the fact that this rule provides the possibility of determining only average values of lives, which may
differ substantially from the individual specimens' lives. It is accounted for by the fact that when using the linear rule of damage accumulation not sufficient attention is paid to the allowance for scatter of specimens and components lives.

The usual procedure is to count the number of cycles to complete failure under program loading at each stage and to divide this number of cycles \( N_i \) by the number of cycles to failure \( N_{cr} \), determined according to the \( \alpha_a - N_i \) diagram, that corresponds to 50 percent probability of failure.

This discrepancy may be eliminated by the use of not diagram with 50 percent probability of failure but diagrams corresponding to each individual specimen tested under program loading.

This paper presents the method of obtaining \( \alpha_a - N_i \) diagrams for individual specimens involving strain criteria of high-cycle fatigue failure described elsewhere (Troshchenko, 1979, 1981), as well as the results of life studies for steel 45 and 15 KP under program loading, and the analysis of the correspondence of experimental data to the linear rule of damage accumulation with account for individual properties of a specimen tested under program loading.

**EXPERIMENTAL PROCEDURE AND MATERIALS**

Steels 45 (0.42+0.55%, 0.5+0.6%Mn; 0.17+0.37%Si; 0.25%C) and 15 KP (0.12+0.19%, 0.25+0.6%Mn; 0.07%Cr; 0.025%) with ultimate strength 716 and 413 MPa, yield strength 468 and 267 MPa, relative elongation 22.8 and 40.1 and relative reduction 46.9 and 68.9, respectively, were studied.

Tests were carried out under fully-reversed push-pull loading with the frequency of 36Hz on the testing set described in the work by Troshchenko and co-workers (1979).

The testing set is equipped with a computer and allows constant stress amplitude test, as well as program loading to be carried out and inelastic strain per cycle to be measured with high accuracy in the course of testing. The diameter of specimens studied was 5 mm.

Figure 1 presents examples of inelastic strain change per cycle, strain being equal to the width of the hysteresis loop in the "stress-strain" coordinates, depending on the number of loading cycles for steels studied under constant stress amplitude loading.

In conformity with the results obtained earlier (Troshchenko, 1981), values of \( \Delta \varepsilon \) corresponding to point \( M \) on the \( \Delta \varepsilon - N \) curve were taken as a characteristic of fatigue damage accumulation intensity in a specimen at a given level of stress. Only these values of \( \Delta \varepsilon \) are used in the course of further investigation. The number of cycles corresponding to point \( P \) was taken for the number of cycles to failure.

As the analysis showed, this point for the steels and stresses studied corresponds to a fatigue crack on a specimen surface, the crack length being equal to 1.1.5mm, and the stage of this crack propagation to complete failure ranged from 1 to 10 percent of the general life.

**Fig. 2. Loading regimes.**

Figure 2 shows the program loading regimes for the steels studied. Stresses for some loading stages are represented in the figure as well.

To plot \( \alpha_a - N_i \) diagrams for individual specimens (the method will be described further on) block loading was preceded by control stage loading (K), at which inelastic strains per cycle were measured for undamaged specimens.
EXPERIMENTAL RESULTS AND LIFE PREDICTION PROCEDURE.

Figure 3 presents fatigue diagrams for steel 45 in \( \Delta \sigma - \Delta \varepsilon \) and \( \Delta \varepsilon - N_e \) coordinates, same marks on these curves correspond to the same series of specimens.

It follows from Fig. 3 that the scatter in experimental data, when they are presented in \( \Delta \varepsilon - N_e \) coordinates, is substantially less than when they are presented in the conventional \( \Delta \sigma - N_e \) coordinates. The specimens tested at the same values of \( \Delta \sigma \) have different values of \( \Delta \varepsilon \) and as \( \Delta \varepsilon \) increases, the number of cycles to failure decreases. Similar results were obtained for steel 15 KP as well. The results of experimental data evaluation in conformity with formula (2) are listed in the Table.

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>Loading regime</th>
<th>Values range</th>
<th>Arithmetic mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel 45 50% probability fatigue curve</td>
<td>A 0.32 + 7.77</td>
<td>2.19</td>
<td></td>
</tr>
<tr>
<td>B 0.43 + 3.77</td>
<td>1.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C 0.32 + 6.9</td>
<td>2.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel 15 KP</td>
<td>D 0.41 + 1.06</td>
<td>0.69</td>
<td></td>
</tr>
</tbody>
</table>

The method of plotting \( \Delta \sigma - N_e \) diagrams for individual specimens is illustrated by Fig. 4 with steel 45 used as an example. \( \Delta \sigma - N_e \) diagrams are obtained on the basis of inelastic strain measurement per cycle at the control stages of the program loading with the use of constant stress amplitude tests data.

Quadrant I contains \( \Delta \varepsilon - \Delta \sigma \) relations experimentally obtained at stage by stage increase of stress value for different specimens of the same series. The endurance limit for these specimens as suggested by Troschenko and co-workers (1979) corresponds to \( \Delta \varepsilon_\text{th} \), value, that may be assumed to be constant for certain classes of materials.

![Fig. 4. Schematic of plotting \( \Delta \sigma - N_e \) diagrams for individual specimens made of steel 45.](image)

Taking into account everything mentioned above, \( \Delta \sigma - N_e \) diagrams for individual specimens tested at regime A (Fig. 2) were plotted in the quadrant II. The slopes of these \( \Delta \sigma - N_e \) diagrams and \( N_e \) value were found in accordance with \( \Delta \varepsilon_\text{th} \) diagram in Fig. 3.

In case the value of inelastic strain is known only at the
control stage of program loading (regimes B, C, D in Fig. 3),

$G_0 - N_x$ diagrams for individual specimens have been plotted
in the following way.

Knowing the endurance limit for specimens with different $\Delta \varepsilon - G_0$
relations (quadrant I), relationships between the inelastic
strain value at the control stage of block loading and the en-
durance limit may be obtained like in quadrant III.

For every stress level at the control stage there will be a
particular $\Delta \varepsilon - G_0$ relation.

With such relations and known $G_0$ and $\Delta \varepsilon$ values for a spe-
cimen tested under program loading appropriate $G_0 - N_x$
diagrams may be plotted like in quadrant IV.

For reliability verification of endurance limit values obtained
by the procedure mentioned above, tests according to the
"up-down" method were carried out. Those tests demonstrated
good agreement of both methods. According to those methods the
average endurance limit value for steel 45 amounted to 271.9
MPa and 272.5 MPa, and root mean-square deviations came up to
11.9 and 7.2 MPa, respectively.

The fatigue curves plotted in the above way for individual
specimens were used in the analysis of test results obtained
by formula (2) are listed in the Table where $N_{x_i}$ values
were determined using fatigue curves for individual specimens.
The data presented shows that if the scatter in properties of
individual specimens is taken into account using the proposed
procedure, the scatter in the value $a$ to the unity and the scatter of $a$
decreases considerably.

To evaluate the effect of loading history on the accuracy of
prediction, the strain approach has been used in the present
study (Troshchenko, 1981). The calculation of an individual
specimen life was made using formula (1) in which $N_{x_i}$ values
were determined from the $\Delta \varepsilon - N_x$ curve equation of the type

$$\Delta \varepsilon - N_x = C$$

The magnitudes of $\Delta \varepsilon$ were measured in the course of the exper-
iment at each step of loading regimes A, B and C up to the
moment of specimen failure. Different periods in the process
of inelastic deformation (softening, strengthening) observed
at constant amplitude stress (see Fig. 1) are present under
conditions of program loading as well. Therefore, the magni-
tudes of inelastic strain corresponding to the same stresses
in different blocks vary depending on the number of blocks.
And at the same time, the corresponding magnitudes of $N_x$
determined from equation (3) will vary, as well as the magni-
tude factor $\Delta \varepsilon$ implies that in the process of loading the position of
the $G_0 - N_x$ curve changes continually, and as a result, the
damage introduced by the stress below the initial endurance
limit is taken into account. The variation of $\Delta \varepsilon$ in the
process of program loading described above takes place in each
specimen, but it differs for each specimen. That is why the
approach used makes it possible to simultaneously take into
account both the loading history and the scatter in fatigue
strength characteristics of individual specimens.

![Fig. 5. Comparison of calculated and experimental values of lives for individual specimens. A, B, C steel 45; D steel 15 KF.](image)

Figure 5 presents a comparison of experimental $N_{x_i}$ and
calculated $N_{x_c}$ values of the number of cycles to fail-
ure for the steels and loading regimes studied.

The use of equation (1) in its conventional interpretation,
that is for determining $N_{x_i}$ value which is a part of the
equation, using 50 percent fatigue curve, allows to define
only one value of $N_{x_i}$ for each loading regime. These values
are marked by dashed lines in Fig. 5.

Dark points in Fig. 5 represent data in calculation of which
only the scatter of individual specimens properties was taken
into account, while hollow points represent data for which
the effect of both loading history and properties scatter
was considered.
Curves 2 and 3 were obtained by processing each of the above mentioned groups of points using the polynomial regression method. Curve 1 in Fig. 5 corresponds to the case of ideal coincidence of experimental and calculated lives. The dashed lines inclined at the angle of 45° are graphical representation of the range in which the difference between calculated and experimental lives does not exceed 2.

It follows from the analysis of the results shown in Fig. 5 that for the steels and loading regimes studied errors observed when using Palmgren-Miner's rule result mainly from the fact that the scatter in fatigue strength characteristics of individual specimens was not taken into account. The position of curves 2 and 3 in respect to curve 1 proves that simultaneous allowance for both the loading history and specimen properties that scatter in accordance with the procedure proposed makes it possible to improve the accuracy of the prediction of the number of cycles to failure. The effect of the loading history for the steels and loading regimes studied is more pronounced in case of long lifetimes.

CONCLUSION

The results obtained gave grounds for the following conclusions.

The procedure is proposed for plotting fatigue curves for individual specimens involving the measurement of inelasticity characteristics at the control level of the stress amplitude. Allowance for the scatter in fatigue strength characteristics of individual specimens with the help of the procedure mentioned ensures better agreement of experimental data with the Palmgren-Miner's rule.

For the steels and loading regimes studied the main error, when Palmgren-Miner's rule is used, is caused by the fact that the scatter in individual specimens properties is not taken into account in calculations of lives. The best agreement of experimental and calculated number of cycles to failure is observed in the region of short lifetimes.

REFERENCES

