IMPROVED METHOD FOR THE PHOTOELASTIC DETERMINATION OF MIXED MODE STRESS INTENSITY FACTORS

M. M. Prabhu*, P. B. Godbole*, S. K. Bhave* and L. S. Srinath**

*Bharat Heavy Electricals Limited, Corporate R & D Division, Hyderabad, India
**Mechanical Engineering Department, Indian Institute of Science, Bangalore, India

ABSTRACT

Most of the existing methods of photoelastic determination of stress intensity factors (SIF) employ data from an extremely small measurement zone very close to the crack tip, thus presenting measurement difficulties and uncertainties. A modified photoelastic method was suggested by the authors for the determination of mode I SIF using data from a convenient measurement zone slightly away from the crack tip. This was extended to a mixed mode case by Srinath and others. In this paper some of the limitations of the method of Srinath and others are pointed out and improvements suggested which give reliable values of mixed mode SIF's.

KEYWORDS

Photoelasticity; stress intensity factors; mixed mode fracture; isochromatics; 'not too near' zone; SEN specimen.

INTRODUCTION

Most of the practical fracture problems exhibit mixed mode situations involving the opening and the sliding modes of fracture characterised by the stress intensity factors (SIF) $K_I$ and $K_{II}$ respectively. Photoelasticity gives an easy indication whether a crack in a two dimensional problem will propagate in mode I or mixed mode.

In an uncracked photoelastic model if the portion where the crack is to be made is marked by a line $AA$ as shown in Fig. 1 oriented at an angle $\theta$ with reference to some fixed direction and the model is studied in a plane polariscope for isoclinics, in case the crack is in mode I situation the $(\theta \pm 90^\circ)$ isoclinic will cover
the line AA completely. Otherwise the crack will be in a mixed mode situation.

![Fig. 1.](image)

Almost all the existing photoelastic techniques to determine SIF make use of data obtained from a very limited measurement zone near the crack tip. The measurements outside this zone give highly inconsistent & erroneous values of SIF because the stress singularity equations used in these methods are nearly valid only in the near neighborhood of the crack, barring the yield zone and away from this zone the farfield stress distribution affects the near field stress distribution. The extent of the data zone to obtain reliable results is also a matter of opinion yet and with all the measurement difficulties involved in a small zone one is at a loss in relying on a single method to obtain correct values of SIF.

The authors (1982) presented a modified method wherein photoelastic data obtained from a convenient 'not too near' zone can be used to determine consistent and accurate SIF in the case of mode I loading configuration. This paper also gave a long list of reference literature dealing with the gradual development in the area of photoelastic stress analysis applied to fracture mechanics studies. An extension of the modified method for mixed mode situation involving $K_I$ and $K_{II}$ was tried on a cracked cylindrical shell subjected to pure torsion by Srinath and others (1983) and was reported to give accurate results.

The equations used in the mixed mode case were:

$$\sigma_1 - \sigma_2 = K/\sqrt{r/a} + B_1 \left( r/a \right)$$  \hspace{1cm} (1)

where $K = \frac{1}{\sqrt{2\pi a}} \left[ (K_{I} \sin \theta + 2K_{II} \cos \theta)^2 + (K_{II} \sin \theta)^2 \right]^{1/2}$  \hspace{1cm} (2)

This is the same equation developed by the authors (1982) with suitable modifications to accommodate both $K_I$ and $K_{II}$ in K and truncated to the first term of the polynomial $\sum_{m} B_m (r/a)^m$. It should be noted that $K$ and $B_1$ are functions of $\theta$, whereas $K_{I}$ and $K_{II}$ should be independent of $\theta$.

The method involves determination of fringe orders at two points 'not too near' the crack tip and located along a same arbitrary $\theta$. Using the stress optic relation:

$$\sigma_1 - \sigma_2 = N\theta/t$$  \hspace{1cm} (3)

In conjunction with Eq. (1), the values of $K$ and $B_1$ can be calculated for this particular $\theta$, say $\theta_1$. This is repeated for another arbitrary angle $\theta_2$.

Using Eq. (2) and the values of $K$ calculated for the two angles $\theta_1$ and $\theta_2$, $K_{I}$ and $K_{II}$ can be computed. To check these values, these are substituted in Eq. (2) and the value of $K$ is calculated for a third arbitrary angle $\theta_3$. For this angle, $K$ is again separately calculated using fringe order data at two points in Eq. (1) as was done for $\theta_1$ and $\theta_2$. The two values of $K$ thus obtained are compared.

The authors observed that $K$ is not so sensitive to the various parameters in comparison to the sensitiveness of $K_{I}$ and $K_{II}$ values. As the parameters of interest are $K_{I}$ and $K_{II}$ and not $K$, the above method of checking and obtaining the so-called unique values of $K_{I}$ and $K_{II}$ is felt to be unsatisfactory. To elaborate this, if one checks the values of $K_{I}$ and $K_{II}$ obtained from two arbitrary angles as given above, one may find that the deviation in $K$ is well within the accepted experimental error of 5%. Alternatively, if one takes sets of two angles at a time and calculates $K_{I}$ and $K_{II}$, one may get three sets of values of $K_{I}$ and $K_{II}$ and these may be quite different and the deviation may be beyond the accepted experimental error of 5%.

Hence, to study the validity of the above method for a mixed mode situation in obtaining unique values of $K_{I}$ and $K_{II}$, the present work was undertaken. In this work, single edge notched (SEN) specimens with inclined notches and subjected to uniaxial tension were studied. Srinath and others (1983) reported that the fringe loops on one side of the crack were very prominent in comparison to those on the other side and the analysis was carried out on these prominent loops. In the mixed mode SEN specimen, the difference in size of the loops on the two sides of the crack is not considerable. So, another objective of this study was to check whether the above method gives comparable values of $K_{I}$ and $K_{II}$ for both the fringe loops on the two sides of the crack.

**Experiments**

SEN specimens of 250 x 50 x 6.76 (mm) size with crack length "a" of approximately 15 mm (a/W < 0.3) and subjected to uniaxial...
tension as shown in Fig. 2 were studied. Three angles of inclina-
tion of the crack each under two different loads of 48.3 Kg.
and 64.36 Kg were studied. The cracks were simulated by saw cuts
using a fine blade of 0.35 mm thickness. The slot tips were left
untouched (without any deliberate rounding off). Similar slots
had given accurate results by the modified method for the mode I
SEN (Prabhu and others, 1982). Live loads were employed to avoid
the effect of Poisson’s ratio difference in stress freezing tech-
nique and also the possible crack tip blunting and plastic zone
errors.

\[
\begin{array}{c|c}
\theta & K_I \\
\hline
22.5^\circ & 15.5 \\
30.0^\circ & 16.0 \\
45.0^\circ & 15.0
\end{array}
\]

Fig. 2. SEN specimen

Both dark field and bright field fringe loops were photographed
for each specimen under each load. These photographs were magni-
fied in the range of 10 x to 15 x and the fringe order and dis-
tance measurements were carried out. As the method was found to
be extremely sensitive to r, the combination of positive error in
the smaller r and negative error in the larger r being the worst,
the clear boundaries of the fringes were traced and the midpoints
were measured accurately. The midpoints, thus measured, were found
to give better results in comparison to other r values in the
fringe width.

The approximate $\theta_m$ value, corresponding to the maximum r for a
fringe loop were determined and measurements were made on a number
of lines 10º apart on either side of $\theta_m$. The theoretical results
of Wilson reproduced by Smith & Smith (1972) were taken for com-
parison.

Typical crack tip isochromatic fringe loops observed in the mixed
mode SEN specimen is shown in Fig. 3.

RESULTS AND DISCUSSION

Typical results obtained on one of the specimens are given in
Table 1. It can be seen that the backward tilting loop (BTL)
which tilts behind the crack, gives very high $K_I$ value and very
low $K_{II}$ value, not comparable with the theoretical values. Also,
the scatter in $K_{II}$ values is extremely high.

![Fig. 3. Crack tip isochromatics](image)

In the forward tilting loop (FTL), which tilts ahead of the crack
the zone $\theta \leq \theta_m$ tends to give lower $K_I$ values and higher $K_{II}$ values
in comparison to the zone $\theta > \theta_m$. The average over the entire FTL
zone results into $K_I$ and $K_{II}$ values quite comparable with the
theoretical values.

As can be seen from the above table, for the FTL, $K_I$ value ranges
from 55.42 to 63.27 and $K_{II}$ value from 13.55 to 20.34. But if $K_I$
and $K_{II}$ values obtained from any two arbitrary angles is used to
obtain value for any arbitrary third angle, it generally does not
exceed 5%. For example, consider $\theta_A = 60^\circ$, $\theta_B = 80^\circ$ and $\theta_C = 70^\circ$.
The $60^\circ$ and $80^\circ$ lines give $K_I$ and $K_{II}$ values of 61.82 Kg/cm$^3$/2
and 16.00 Kg/cm$^3$/2 respectively. If these values are substituted
in Eq. (2) for the $70^\circ$ line, the resulting $K$, in non dimensional
form is 1.188, which in comparison to the value of 1.186 obtained
from Eq. (1) for the $70^\circ$ line deviates only by about 0.2%. Now,
if the $70^\circ$ line is taken in combination with the $60^\circ$ and $80^\circ$ lines
by turn and $K_I$ and $K_{II}$ obtained using Eq. (2), the resulting
values are as in Table 2.

It can be seen that the scatter in $K_I$ is 2.3% and in $K_{II}$ 9.3%,
whereas in $K$ it is only 0.2%. The difference in the scatter of $K_I$ and $K_{II}$ values in comparison to that of $K$ is worse when the
lines are in different zones.

For the entire FTL zone, the maximum scatter as a percentage of
the average value is 13.8% in $K_I$ and 38.8% in $K_{II}$, though that
in $K$ is only about 9%. If the average values are determined over
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REFERENCES

