FAILURE ANALYSIS OF STRUCTURES BY CONTINUUM DAMAGE MECHANICS

A. Benallal, R. Billardon and J. Lemaitre

Laboratoire de Mecanique et Technologie, ENSET Paris 6/CNRS, 61 Avenue du Président Wilson, 94230 Cachan, France

ABSTRACT

In this paper, principles of Continuum Damage Mechanics are applied to the failure analysis of structures. First, constitutive equations of elastic, plastic or viscoplastic materials coupled to damage laws are presented. Then, their implementation in a in-house finite element program is described. Within this framework, a numerically predicted macrocrack is the set of points in the structure where damage has reached its critical value. This procedure is applied to the prediction of macrocrack initiation and propagation.

KEYWORDS

Damage mechanics ; Crack initiation ; Crack growth ; Finite element analysis.

INTRODUCTION

The continuum damage theory first introduced by KACHANOV and developed within the framework of thermodynamics has already been applied to crack initiation analyses in structures under creep and fatigue loadings (LEMAITRE and CHABOCHE, 1979 - BENALLAL and MARQUIS, 1982) : in those applications the damage laws were not coupled to the constitutive equations, i.e. stress and strain fields are obtained with the constitutive equations of the undamaged material, and then introduced in damage laws.
A way of introducing the continuous evolution of damage in the finite element analysis of a precracked structure consists in implementing a damage criterion which must be fulfilled to relax the crack tip node (ABDOULI, 1982).
In the procedure adopted here, coupled strain damage constitutive equations are implemented in a finite element code : therefore, stress, strain and damage histories can be predicted from a single iterative calculation during which the redistribution of stresses due to damage evolution is fully taken into account.
The first section of this paper is devoted to a general presentation of coupled equations. It is assumed that the coupling of damage to the constitutive equations is made through the concept of effective stress. The evolution of the damage variable for different types of loadings (elasticity, plasticity and viscoplasticity) is given.

Numerical considerations are emphasized in the second section : the choice of the algorithms and their properties are underlined.
Eventually, these numerical procedures are applied to the prediction of crack initiation and growth in complex specimens ("Tulip" specimens) subjected to high cycle fatigue, full plasticity and creep loadings. The macrocracks numerically appear as the domain within the structure where elastic stiffness has completely vanished, that is where the damage has reached its critical value.

DAMAGE MODELS AND CONSTITUTIVE EQUATIONS

Damage variable and effective stress concept

The deterioration of the physical properties of a material (mainly due to micro-cracks, voids and their interactions) can be modelled by a continuum damage variable D as first suggested by KACHANOV. This quantity is zero for the virgin material and has a critical value D_c ($.2 \leqslant D_c \leqslant .8$ for metals) corresponding to the local failure in the continuum. When the defects are oriented, $\mathbb{D}$ is a tensor (CHABOCHE, 1979 - MURAKAMI, 1980 - CORDEBOIS and SIDOROFF, 1982 - LADEVEZE, 1983). If the defects are equally distributed in all directions, D is a scalar. Only this last isotropic approach is used throughout this paper.
The microcracks and cavities within the material give rise to an effective area (smaller than the nominal area), and hence an effective stress $\tilde{\sigma}$ related to the usual stress tensor σ by :

$$\tilde{\sigma} = \frac{\sigma}{1-D}$$

Strain-damage coupling and constitutive equations

To avoid microscopic measurements of damage, the strain equivalence assumption can be made : the behaviour of a damaged material is represented by the constitutive equations of the undamaged material in which the stress is just replaced by the effective stress.
Of course, different constitutive equations must be chosen according to the type of loading considered.

Elasticity

When dealing with high cycle fatigue, one can assume that the energy dissipated in plasticity is negligible when compared to the energy involved in elasticity and in the damage process, even in the crack tip zone. So the damaged elasticity constitutive equations which can be used are :

$$\varepsilon_{ij} = \frac{1+\nu_o}{E_o}\,\tilde{\sigma}_{ij} - \frac{\nu_o}{E_o}\,\tilde{\sigma}_{kk}\,\delta_{ij}$$

where E_o and ν_o are YOUNG's modulus and POISSON's ratio of the virgin material.

Plasticity

When plasticity can no longer ve ignored, it is necessary to consider equations taking into account the process of hardening and the associated ductile damage. When coupled to damage, the non linear kinematic and isotropic hardening model proposed by MARQUIS, 1979, becomes :

$$\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^p_{ij}$$

$$\varepsilon^e_{ij} = \frac{1+\nu_o}{E_o}\,\tilde{\sigma}_{ij} - \frac{\nu_o}{E_o}\,\tilde{\sigma}_{kk}\,\delta_{ij}$$

$$f(\tilde{\sigma}, \tilde{X}, \tilde{R}) = \sqrt{\frac{2}{3}(\tilde{s}_{ij} - \tilde{X}_{ij})(\tilde{s}_{ij} - \tilde{X}_{ij})} - \tilde{R} = 0$$

$$\varepsilon^p_{ij} = \lambda\frac{\partial f}{\partial \sigma_{ij}} \qquad \begin{array}{l} \lambda > 0 \text{ if } f = 0 \text{ and } \mathring{f} = 0 \\ \lambda = 0 \text{ if } f = 0 \text{ and } \mathring{f} < 0 \\ \quad\text{or if } f < 0 \end{array}$$

$$\mathring{X}_{ij} = c\left[a\,\mathring{\varepsilon}^p_{ij} - X_{ij}\,\mathring{p}\right]$$

$$\mathring{R} = \gamma\,(b - R)\,\mathring{p}$$

where ε_{ij}, ε^e_{ij} and ε^p_{ij} are respectively the total, elastic and plastic components of the strain tensor,

f	the yield surface (VON MISES' criterion)
s	the stress deviator with $\tilde{s} = s/(1-D)$
X	the back stress tensor with $\tilde{X} = X/(1-D)$
R	the radius of the yield surface with $\tilde{R} = R/(1-D)$
p	the accumulated plastic strain with $\dot{p} = \sqrt{2/3\,\dot{\varepsilon}^p_{ij}\,\dot{\varepsilon}^p_{ij}}$
c,a,b,	and γ material dependent parameters.

Viscoplasticity

In the range of high temperatures, another kind of damage appears. It can be described by the following modification of the viscoplastic constitutive equations first proposed by CHABOCHE (1978) :

$$\varepsilon_{ij} = \varepsilon^e + \varepsilon^{vp}_{ij}$$

$$\varepsilon^e_{ij} = \frac{1+\nu_o}{E_o}\,\tilde{\sigma}_{ij} - \frac{\nu_o}{E_o}\,\tilde{\sigma}_{kk}\,\delta_{ij}$$

$$\Omega(\tilde{\sigma}, \tilde{X}, \tilde{R}) = \frac{K}{n+1}\left\langle\frac{(\tilde{s}_{ij} - \tilde{X}_{ij})(\tilde{s}_{ij} - \tilde{X}_{ij}) - \tilde{R}}{K}\right\rangle^{n+1}$$

$$\mathring{\varepsilon}^{vp}_{ij} = \frac{\partial\Omega}{\partial\sigma_{ij}}$$

$$\dot{X}_{ij} = c\left[a\,\mathring{\varepsilon}^{vp}_{ij} - X_{ij}\,\mathring{p}\right]$$

$$\mathring{R} = \gamma\,(b - R)\,\mathring{p}$$

where ε^{vp}_{ij} are the viscoplastic components of the strain tensor,

Ω the viscoplastic potential with $\langle u \rangle = u$ if $u \geqslant 0$
or $\langle u \rangle = 0$ if $u < 0$

K and n material dependent parameters

Damage models

Thermodynamic considerations lead to the definition of the damage energy release rate :

$$-Y = \frac{\sigma^{*2}}{2E_o(1-D)^2}$$

with the damage equivalent stress (LEMAITRE and BAPTISTE, 1982) :

$$\sigma^* = \bar{\sigma}\left[\frac{2}{3}(1+\nu_o) + 3(1-2\nu_o)\left(\frac{\sigma m}{\bar{\sigma}}\right)^2\right]^{1/2}$$

Where $\bar{\sigma}$ is VON MISES' equivalent stress, and σm the hydrostatic stress. The damage energy release rate assumes a symmetrical behaviour in tension and compression and the damage equivalent stress takes into account the influence of triaxiality through the ratio $(\sigma m/\bar{\sigma})$.

Ductile damage

In the case of plasticity, damage evolution is given by :

$$\overset{\circ}{D} = \frac{-Y}{S}\overset{\circ}{p}$$

with S a material and temperature dependent parameter. In this expression $\overset{\circ}{p}$ can be replaced by its value taken from the hardening law of plasticity or viscoplasticity. Of course, this can be applied to any monotonic or cyclic loading in the plastic range.

Creep damage

In this case the damage law can be written :

$$\overset{\circ}{D} = \left[\frac{-Y}{S}\right]^{r/2}$$

with B and r material and temperature dependent.

High cycle fatigue

Fatigue in the elastic range involves fairly complex mechanisms. A few principal features of this phenomenon can be represented by the law (LEMAITRE, 1983) :

$$\overset{\circ}{D} = \frac{-Y}{S}\frac{M}{K^M}(\tilde{\sigma})^{M-1}\left|\overset{\circ}{\tilde{\sigma}}\right|$$

where symbol | | indicates the absolute value, and S , M , K are material dependent parameters.

STRAIN-DAMAGE CONSTITUTIVE EQUATIONS AND FAILURE ANALYSIS

The coupled constitutive equations described above are implemented in a in-house finite element program the main features of which are listed below :

- infinitesimal strain assumption ;
- plane stress/strain or axisymmetric conditions ;
- 3-node constant stress triangles and isoparametric elements (6-node and 8-node) ;
- equation solver using the frontal technique.

Macrocrack initiation and propagation

Whatever the strain-damage constitutive equations given in the previous section the macrocrack initiation is modelled in the same manner : a macrocrack initiates as soon as the damage variable has reached its critical value in any point of the structure (a "point" being a constant stress triangle or a Gaussian integration point in the case of isoparametric elements). It must be noticed that in all cases (elasticity, plasticity or viscoplasticity) iterative damage increase corresponds to iterative elastic stiffness decrease : hence the macrocrack propagation is but the expansion of the domain where the elastic stiffness has completely vanished. This approach was first adopted by MARQUIS et al (1981) for initiation problems. HAYHURST et al (1983) used a similar approach to predict creep crack growth : however in their numerical procedure, whereas damage has a continuous iterative evolution, elastic stiffness has a discontinuous evolution, constant as long as damage is lower than its critical value and zero when it has reached it.

Numerical procedures

Of course, different algorithms can be used to integrate with respect to time the strain-damage constitutive equations described in section 2 : those which have been chosen are reviewed below.

High cycle fatigue

In the case of constant amplitude cyclic loading and if proportional loading conditions can be assumed during one cycle the fatigue damage law (9) can be integrated over a cycle and :

$$\frac{\delta D}{\delta N} = \frac{2B}{\beta+1}\left[\frac{2}{3}(1+\nu_o) + 3(1-\nu_o)\left(\frac{\sigma m}{\bar{\sigma}}\right)^2\right]^{s_o}\left[\tilde{\sigma}_{Max}^{\beta+1} - \tilde{\sigma}_{Min}^{\beta+1}\right]$$

with B , β and s_o which are adjustable coefficients identified from "S-N" fatigue curves, and with subscripts Max and Min which indicate the extreme values of one cycle.
The numerical solution strategy first adopted is the standard EULER scheme. The size of the time steps is determined by the maximum value of the damage rate obtained in the whole structure and the stiffness matrix is reevaluated at each time step.

Plasticity

To allow the analysis of structures subjected to monotonic or cyclic loading with material behaviour governed by equations (3) and (7), the modified NEWTON-RAPHSON procedure called initial stiffness algorithm was chosen. In this method, the stiffness matrix is maintained constant and equal to its initial value during the iterative process. However, of course, if one point unloads elastically during the loading process, it unloads with its present damaged elastic stiffness.

Viscoplasticity

Because of the large computation times involved, strategies for optimum time step determination have been especially looked for in the case of viscoplasticity coupled to damage. A numerical study of three one-step algorithms was carried out : these algorithms are the standard EULER scheme, the forward gradient scheme and an implicite scheme. For each of them, a stability crite-

rion has been derived (BENALLAL, 1983) and a general accuracy criterion is under investigation.
For the implicit scheme, the stiffness matrix has to be reevaluated at each iteration inside each time step, whereas only one evaluation per time step is necessary for the other schemes. But, on the other hand it must be considered that larger time steps are allowed by the implicit scheme. In fact, the choice of any of the three algorithms implemented in the finite element program is mainly based on the type of the studied problem (monotonous or cyclic loading asymptotic or transient behaviour).

Applications

The high cycle fatigue procedure has been applied to model the initiation of the first macrocrack and its propagation in an axisymmetric specimen called "Tulip Specimen" because of its shape (see figure 1). This unusual shape was chosen to obtain a fairly complex stress field in the critical zone despite of a simple uniaxial loading. Figure 2 shows (from top to bottom) the evolution of the damaged zone in the case of a 3-node mesh.
The other example depicted in figure 3 is the case of a thick elasto-visco-plastic sphere under increasing internal pressure the mesh used is shown in figure 3 as well as the material parameters and the loading history. In figure 4, the stress and damage histories along a radius until the initiation are given.

CONCLUTION

Highly non linear and three dimensional problems involving effects of history of loading still constitute a challenge to classical fracture mechanics. Local approaches by means of continuum damage mechanics are promising but, of course, structure analysis implementing the time integration of coupled strain-damage constitutive equations are highly computer consuming.

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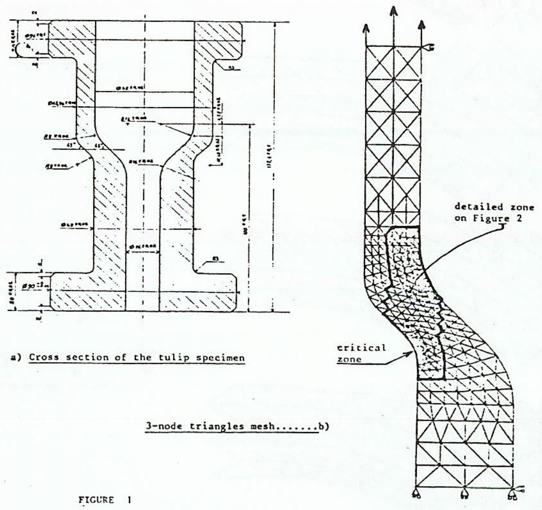

a) Cross section of the tulip specimen

3-node triangles mesh.......b)

FIGURE 1

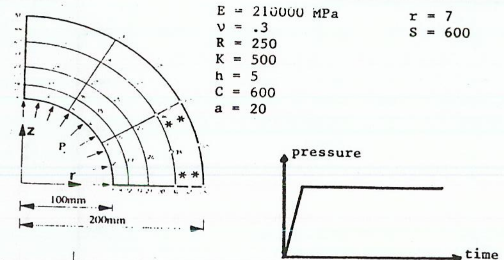

Fig. 3 Thick sphere : mesh, loading and material properties

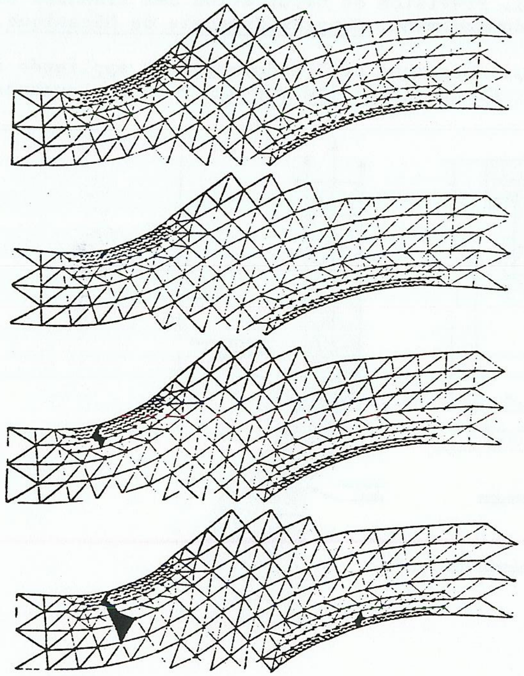

FIGURE 2 : Evolution of the damaged zone ($D = D_c$)

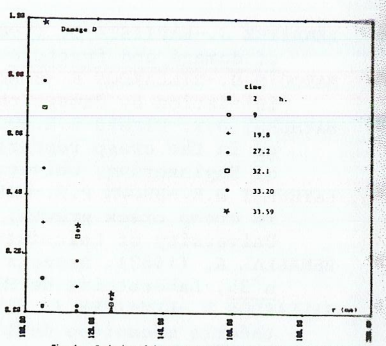

Fig. 4a : Evolution of damage along a radius with time until initiation

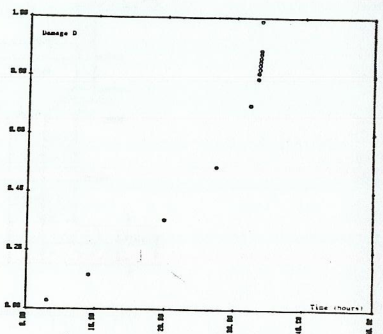

Fig. 4b : Evolution of damage with time for the most stressed Gauss Poi[nt]

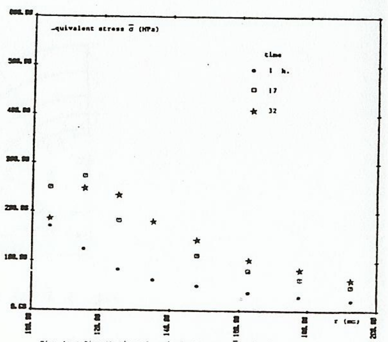

Fig. 4c : Distribution of equivalent stress $\bar{\sigma}$ with time