# DAMAGE AND FRACTURE MECHANICS FOR CONCRETE (A COMBINED APPROACH)

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ABSTRACT

In the first part we give the theoretical bases for an approach combining damage mechanics and fracture mechanics in order to apply it to concrete. In the second part we give an example of this approach and we analyse the performances we can expect from both theories.

## INTRODUCTION

The misreading of the phenomena of rupture in concrete structures leads to proscribe the use of it when it is not reinforced or prestressed. This problem is solved either by avoiding completely the initiation of cracks (prestressed concrete) or by distributing them (reinforced concrete). In the second case, the main objective is to limit the opening of the cracks in order to avoid the problems due to corrosion which affect the durability of works.

We think that a fine approach of the concrete failure, through the formulation of models based on the physics of the phenomena, is able to have an effect upon the design of the structures, in which it is possible to optimize strength and durability.

In this paper, we propose a reflection about the idea of damage and rupture, showing the relationship between damage mechanics and fracture mechanics from experimental results on the one hand and numerical approaches on the other hand ; we also analyse the performances we can expect of both theories.

## THEORETICAL ASPECT

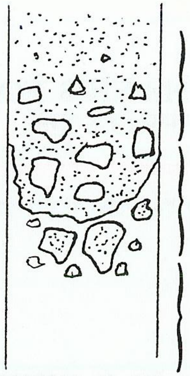
### Kinematics of crack propagation in concrete

Many authors have been interested in the study of crack propagation in a concrete structure (KAPLAN (1961), ENTOV (1975), HILLERBORG and al. (1976), MAZARS (1976) ).

Some interesting results have been obtained by BARON, FRANCOIS, CHHUY and BENKIRANE (1980,1981). They have shown that the damage of the material is found beyond the tip of the visible crack (several ten centimeters as it is shown by acoustic emission.

Some late observations (LOLAND (1980), BENKIRANE (1980), MAZARS (1984) ) have shown that the visible crack on the surface was not significant of the complete separation of the material in the structure thickness.

We propose a sketch of this internal degradation, which can be decomposed in three zones :



Damaged zone : some microcracks appear

Cracked-damaged zone : the crack is visible on the surface but a part of the internal section is still stress-active

Cracked zone : the fracture is complete

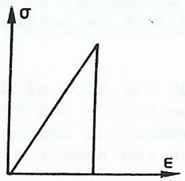
Two points can explain why the degradation is more important on the surface than inside the material :

- there are superficial tensions which result from shrinkage (ACKER, 1980)
- the solicitation on the surface approaches a plane-stress condition whereas the solicitation inside the material is near a condition of plane deformation.

## Relation between damage mechanics and fracture mechanics

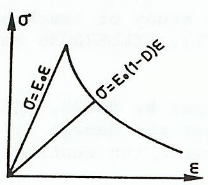
### General hypotheses

#### Fracture mechanics




The material is linear-elastic, perfectly brittle.

#### Damage mechanics




The material is linear-elastic, damageable ; the damage is isotropic, hence modelled by a scalar variable D defined through the concept of effective stress :

$$\tilde{\sigma}_{eff} = \frac{\sigma}{1 - D} \quad \text{(KACHANOV, 1958)}$$

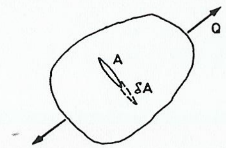


### Framework of the thermodynamics approach

We used here elements of the presentation made by LEMAITRE (1982)

#### Fracture mechanics

In the case of a cracked structure that supports a loading dependent on a single parameter, the usual classification of the variable is :

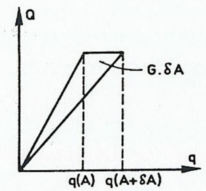



| Observable variable | Internal variable | Associated variable |
|---|---|---|
| displacement q | | load Q |
| | area of crack A | G |

The equations of state make it possible to give the expression of the variable associated to A :

$$G = -\frac{1}{2} q^2 \frac{\partial R}{\partial A}$$

(where $R = \frac{Q}{q}$ is the global stiffness of the structure)




Thus, for a constant load, an increment δA of the crack area gives a stiffness variation :

$$\frac{\delta R}{\delta A} = -\frac{Q}{q^2} \cdot \frac{\delta q}{\delta A}$$

which leads to the usual expression of the strain energy release rate :

$$G = \frac{1}{2} Q \cdot \frac{\partial q}{\partial A}$$

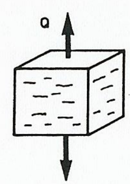
For a brittle material, the propagation law depends on the critical strain energy release rate through the relations :

$$G < Gc : \text{no propagation } (\dot{A} = 0)$$

$$G = Gc : \text{propagation } (\dot{A} > 0)$$

#### Damage mechanics

The classification of the variables is :




| observable variable | internal variable | associated variable |
|---|---|---|
| deformations $\varepsilon_{ij}$ | | stress $\sigma_{ij}$ |
| | damage D | Y |

The equations of state give :

$$\sigma_{ij} = \Lambda_{ijkl}\varepsilon_{ij}\varepsilon_{kl}\ (1-D)$$

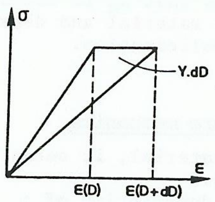
$$Y = \frac{1}{2}\Lambda_{ijkl}\varepsilon_{ij}\varepsilon_{kl}$$

(where $\Lambda_{ijkl}$ is the elastic tensor)

Consider a variation of the damage dD (for a constant stress). The expression of the internal energy U leads to the following expression :

$$\left(\frac{dU}{dD}\right)_{\sigma=cte} = \sigma_{ij} \cdot \frac{d\varepsilon_U}{dD} = (1 - D)\ \Lambda_{ijkl}\ \varepsilon_{kl}\ \frac{d\varepsilon_{ij}}{dD}$$

It can be demonstrated that in these conditions (LEMAITRE, MAZARS, 1982)




$$\left(\frac{dU}{dD}\right)_{\sigma=cte} = 2Y$$

Where Y is the damage energy release rate.

## Notion of equivalent crack

Consider a structure (S) with a crack A (the material is virgin everywhere else) (Fig. 1)

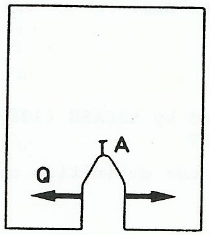



Fig. 1

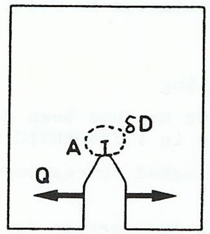



Fig. 2

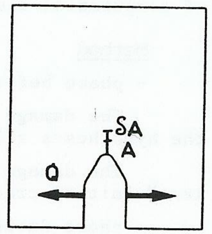



Fig. 3

In a real material, a loading Q creates a damage zone ; let δD the local evolution of damage (Fig. 2). As we have seen in § "Framework of the thermodynamics approach", the density of dissipation is : Y.δD. The total energy dissipation is :

$$\mathcal{D}(\delta D) = \int_v Y.\delta D\ dv = \frac{1}{2}\int_v \Lambda_{ijkl}\varepsilon_{kl}\varepsilon_{ij}\delta D\ dv$$

Taking into account the fact that the material is supposed to be elastic and perfectly brittle, an energy dissipation caused by a load Q corresponds to an increase δA in the pre-existent crack (Fig. 3) :

$$\mathcal{D}(\delta A) = Gc.\delta A$$

Thus, we will say that the crack propagation δA is equivalent to the damage evolution δD in the volume v when :

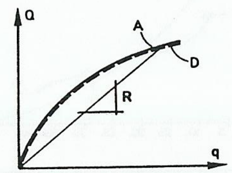
$$\mathcal{D}(\delta D) = \mathcal{D}(\delta A)$$



Therefore :

$$\delta A = \frac{\frac{1}{2}\int_v \Lambda_{ijkl}\varepsilon_{kl}\varepsilon_{ij}\delta D\ dv}{Gc}$$

Generally speaking, we will say that the crack "equivalent" to a damaged-cracked zone is the crack for which the energy necessary to its creation is equal to the energy dissipated in the formation of the damaged zone.




For a linear-elastic material, these considerations lead to say that the two structures have the same global behaviour :

$$R(D) = R(A)$$

## APPLICATIONS AND CONSEQUENCES

According to the theory developed above, the objectives are :

- to determine the propagation of the equivalent crack by means of experimental results
- to infer and discuss the values of $G_c$ (and $K_c$)
- to propose a numerical approach coupling damage mechanics to fracture mechanics

### Experimental details

The work employed C.T. specimen, the sizes of which are given on Fig. 4 The load is applied on both sides of the notch with an hydraulic jack controlled on the opening of this notch.

When the visible cracking begins, we note the propagation of the crack with photographies.

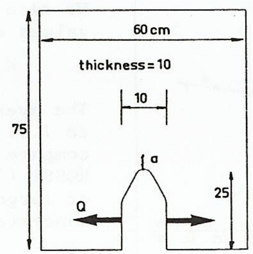
The measurement of the load and the opening of the notch allows to draw the curve given on Fig. 5.

### Numerical analysis

A finite element mesh (three-nodes constant stress elements) of the C.T. specimen has been performed and the calculation of the global stiffness $R=Q/q$ as a function of a fictituous crack $a$ has been obtained by suppressing the limit conditions on the displacement of the nodes along the symetry axis of the mesh.

### Analysis of results

Fig. 4 : C.T. specimen



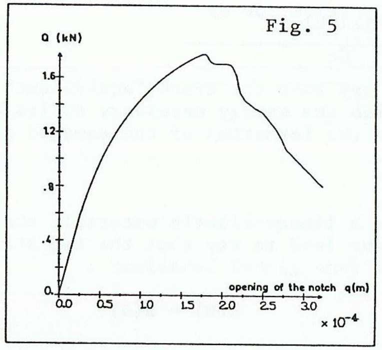


Fig. 5

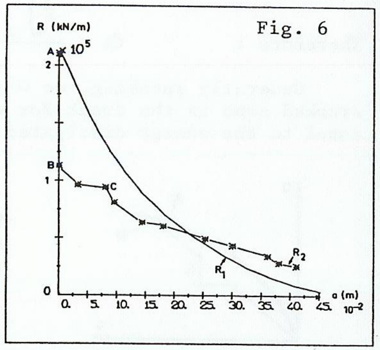


Fig. 6

We study the curves $R_1 = f(a)$ (where $a$ is the equivalent crack) and $R_2 = g(a)$ (where $a$ is the visible crack in the experiment) (Fig. 6).

It can be noticed that the initial stiffness (virgin material) is the same for both curves (point A).

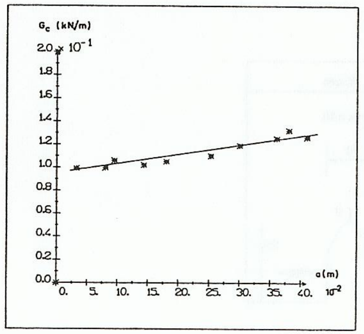
We first see a loss of stiffness (AB) before the coming out of the visible crack, which is probably due to the damage of the material.
The strong slope variation (point C) of the curve $R_2$ seems to show the formation of a through-crack, whereas the visible one is already 8 cm long. This fact confirms what we said in the theoretical part : the visible crack is not representative of the internal degradation of the structure. We can estimate an "equivalent crack" $\tilde{a}$, which is the threshold of propagation of a through-crack (in the proposed example, $\tilde{a}$ = 16 cm).

## Fracture mechanics analysis

We have calculated the tangent dR/da on the curve $R_1$ for several points and then calculated the value of critical rate $G_c$ by the relation :

$$\frac{1}{2}\left(\frac{Q_c}{R}\right)^2 \cdot \frac{dR}{dA} \qquad \text{(where } A = a \times \text{thickness)}$$

The curve of $G_c$ against $a$ (Fig. 7) shows a very light increase in $G_c$ when the crack length increases.




The average value of $G_c$ is 0,114 kN/m
We also have calculated the different values of $K_c$ with the formula :

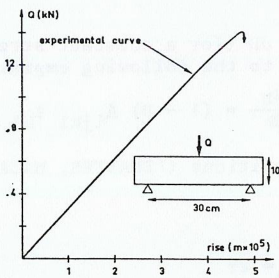
$$K_c = \sqrt{G_c . E} \qquad \text{(plane stress)}$$

The average value of $K_c$ can be estimated to 1.91 MPa.$\sqrt{m}$, that is a result we can compare with the one BENKIRANE (1980) and ROSSI (1983) obtained on a D.C.B. specimen of large size, manufactured with the same concrete.



## Values of $G_c$ and $K_c$ on an other type of test

Using the same process as for the C.T. specimen, we have studied the behaviour of a prismatic beam in three point bending.




Taking into account the fact that after the maximum value of the load the rupture is sudden (because the damaged zone is very confined) we have calculated $G_c$ only at the beginning of the visible crack.
$G_c$ is about 27.9 N/m and the corresponding $K_c$ is about 1.01 MPa.$\sqrt{m}$, i.e. half the $K_c$ value of the C.T. specimen.
It confirms that $K_c$ is not a characteristic of the material and depends on the type of solicitation.

## Approach by coupling damage mechanics to fracture mechanics

Though $G_c$ is not a characteristic of the material, it seems to have a nearly constant value when the crack progresses.
With this hypothesis, we can consider the description of :

- the behaviour before the cracking by the damage mechanics ;
- the behaviour during the cracking by the fracture mechanics.

The interest in coupling these approaches lies in the important sparing of computation time. Application to the C.T. specimen.

### Method

- phase before cracking

The damage model we use has been described by MAZARS (1982,1984) with the hypotheses alluded to in § THEORETICAL ASPECT

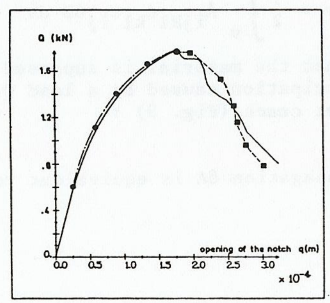
The damage thus created corresponds with the definition of an "equivalent" initial crack $a$.

- phase during crack propagation ($a > a_o$)

We suppose $G_c$ = constant

By using the curve of the Fig. 6, for a fixed value of $a$, we can calculate $R(a)$, $dR/da$, i.e. $Q_c$. We can therefore deduce the value of $q = Q_c/R$.

### Results




This figure shows :

The numerical result (finite elements method applied to the elastic-damageable material) ● ● ●

Experimental points ———

Calculation by fracture mechanics ■ ■ ■

## CONCLUSION

We have shown results on a C.T. specimen. Other tests have corroborated these results and shown the interest of coupling damage mechanics and fracture mechanics when $G_c$ can be considered as a constant.

These two techniques have specific application fields (for example, it is impossible to use fracture mechanics before the crack starts) but their joint uses permit to have the whole global behaviour of a structure while reducing time - and thereby the cost - of computer utilisation.

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