A SIMPLE MODEL FOR SHEAR CRACKING AND FAILURE IN COMPOSITE MASONRY

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ABSTRACT

A two-dimensional finite element model developed earlier by the authors to compute stresses and strains in composite masonry walls subjected to inplane external loads under linear elastic conditions is developed further in this paper to incorporate the cracking and failure of the collar joint. A simple failure criterion is proposed which mobilizes resisting friction forces due to yielding of the horizontal reinforcement. It is shown with the help of an example that the growth of the interface crack can be arrested once enough resisting forces have been mobilized. The proposed model could be utilized to estimate strengths of composite masonry walls.

KEYWORDS

Composite masonry; two-dimensional finite element model; collar joint cracking; failure criterion; bed reinforcement; shear friction.

INTRODUCTION

Composite action in a masonry wall is obtained when the two independent wythes are connected together by metal ties and the cavity between the two wythes is grouted as shown in Fig. 1. For the two wythes to act together, it is important that the collar joint is strong enough to resist the shearing stresses induced in it. A quasi two-dimensional finite element model has previously been proposed by the senior author that is capable of predicting the shear stresses in the collar joint (Anand and Young, 1982). In that model, shearing stresses caused by inplane loads applied at one wythe were felt to be most significant and were only considered. No attempts were made to model the phenomenon of cracking in the collar joint.

A large amount of effort has been directed to modelling masonry and reinforced concrete with the finite element method. Specifically, the phenomena of cracking and crack propagation have received special attention (Chen, 1982; Page, 1978). There is no evidence, however, that analytical models have as yet been developed to predict cracking in composite masonry walls. A simple

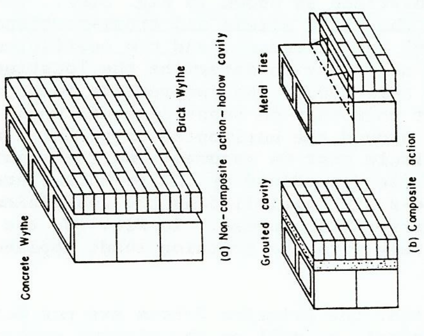

Fig. 1. Composite masonry wall.

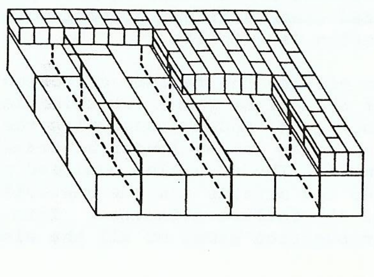

Fig. 2. Ladder reinforcement.

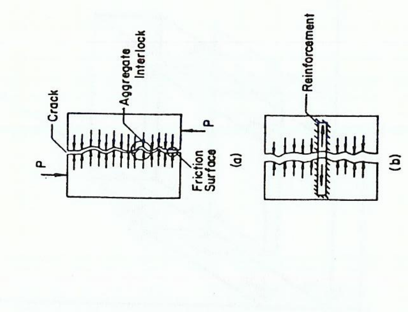

Fig. 3. Shear friction effects.

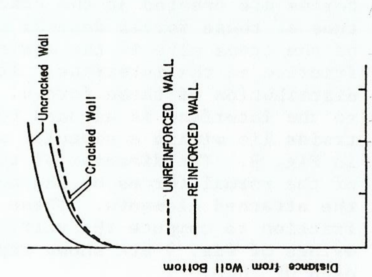

Fig. 4. Collar joint shear stresses

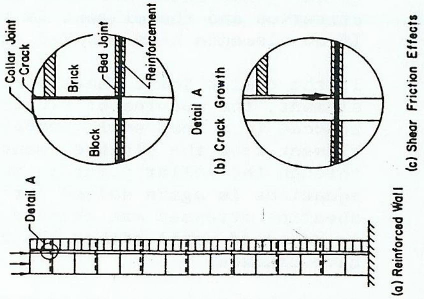

Fig. 5. Cracked wall.

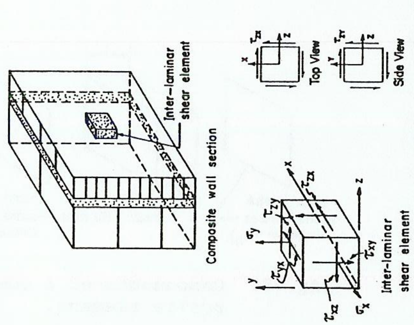

Fig. 6. Shear element.

model to investigate the phenomenon of cracking which includes the effects of horizontal reinforcement is proposed in this paper.

GENERAL CONSIDERATIONS

Composite masonry walls are often reinforced with the ladder type of reinforcement that lays across the bed joints of the block and brick wythes and is placed in every other layer of concrete block as shown in Fig. 2. The diameter of the steel wire varies from 9 gage to 3/16 in.(4.75 mm) and the cross pieces of wire are spaced 15 in.(38.1 cm) on center.

Frictional Forces at a Crack. Shear friction in reinforced concrete occurs when compressive stresses act normal to a crack surface, keeping the surfaces on either side of the crack in contact, as shown in Fig. 3(a). Any tangential movement of one face with respect to the other is resisted by the friction and aggregate interlock. The compressive normal stresses can be created by either the imposed loading or the presence of the reinforcement. Fig. 3(b) shows a length of reinforcing steel that is in tension due to the opening of the crack. Alphonso and Brown (1979) measured this crack distance for grouted reinforced masonry prisms and determined that the crack width was large enough in their specimens to bring the reinforcing steel to yield. Thus, the magnitude of the compressive force could be assumed to be equal to the yield stress of the steel multiplied by its cross-sectional area. Knowing the compressive force applied by the yielded steel and assuming an appropriate coefficient of friction, the frictional forces that resist any tangential movement can be calculated.

Description of Crack Growth. The shear stress distribution in the collar joint due to loads applied to the block wythe of an unreinforced uncracked wall is shown in Fig. 4. At some increased load, the peak shear stress becomes larger than the bond strength of the two interfaces and a crack forms at either the collar joint-brick interface or the collar joint-block interface depending upon the relative bond strength. The shear stress distribution for the unreinforced cracked wall is also shown in Fig. 4. The peak shear stress value remains the same since the load must still be transferred to the brick wythe. The only change is that the shear stress distribution has moved downwards with the maximum shear stress starting at the crack tip. As the peak shear stress value remains the same, the crack propagates downwards in the wall until a wythe is completely separated from the collar joint.

Walls that contain reinforcement can possibly resist crack growth. A reinforced wall in which the crack has reached the reinforcement is shown in Fig. 5. As the crack crosses the reinforcement, the frictional forces as described in the previous section are activated to resist the crack. These equal and opposite forces applied to both sides of the crack are shown in Fig. 5 and the corresponding shear stress distribution is shown in Fig. 4. The shape of the shear stress diagram remains the same but the peak shear stress value is lower because of the action of the friction force. If enough friction force can be developed to drop the peak shear stress below the maximum shear strength of the interface, the crack will stop growing.

ELASTIC ANALYSIS OF AN UNCRACKED WALL USING THE COMPOSITE ELEMENT

In general, the external loads are applied to the inner concrete block wythe. A portion of these loads are transferred to the outer brick wythe through the collar joint creating shearing stresses as shown in Fig. 6.

Stiffness Matrix of a Composite Element

The finite element method, in general, is well known and no attempts are made here to explain it in detail. Nevertheless, the development of the element stiffness matrix for a composite element consisting of the two wythe faces and a collar joint is presented. Formulation of a composite element stiffness matrix is accomplished by first developing the inplane stiffness matrix representing each of the masonry wythes, then developing the shearing stiffness matrix representing the collar joint, and finally combining these stiffness matrices to yield the total composite element stiffness matrix. It is assumed that all materials are homogeneous and linearly elastic, and out-of-plane bending effects are neglected.

Determination of displacements and stresses in wythes due to inplane loads can be accomplished by the plane stress finite element analysis in which the governing stiffness matrix $\underset{\sim}{k}$ relates forces and displacements in an element. The three components of a composite element are shown in Fig. 7. The wythe element stiffness matrices, $\underset{\sim}{k}_f$ for the front wythe and $\underset{\sim}{k}_b$ for the back wythe, are standard inplane stiffness matrices which may be found in any finite element text. Shear deformation of the collar joint element is shown in Fig. 8 and is composed of displacements in the x and y directions. Force-displacement relations for the collar joint shear element are given by the shear stiffness matrix $\underset{\sim}{k}_{sh}$ which may be found in Anand and Young (1982).

The superposition of the two wythe element stiffness matrices and the collar joint shear element stiffness matrix results in a composite element stiffness matrix which is given by

$$\underset{\sim}{k}_{ce} = \begin{bmatrix} \underset{\sim}{k}_f & 0 \\ 0 & \underset{\sim}{k}_b \end{bmatrix} + \underset{\sim}{k}_{sh} \; . \qquad (1)$$

Calculation of Displacements, Strains, and Stresses

Using the stiffness matrix of a composite element, the stiffness matrix for a finite element model of the complete wall can be assembled leading to equilibrium equations which are solved for the nodal point displacements. Shearing strains in each element across the collar joint are calculated by using the shearing element strain-displacement relations (Steven, 1983). Inplane stresses in the wythe elements are calculated from inplane strains by using the plane-stress stress-strain relations. Shearing stresses in the collar joint elements, on the other hand, are calculated from the corresponding shearing strains.

CRACK MODELLING

Unreinforced Composite Masonry Walls

Failure Criterion. To keep the model simple, the failure criterion for cracking of the collar joint is based only on the two shear stresses τ_{zx} and τ_{zy}

$$\tau_{max} \leq \sqrt{(\tau_{zx})^2 + (\tau_{zy})^2} \qquad (2)$$

where τ_{max} is the failure shear stress determined experimentally.

Solution Procedure. The finite element model of the initially uncracked wall is solved for the strains and stresses due to the applied loads. The

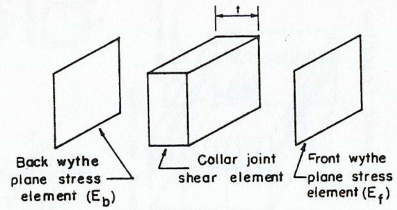

Fig. 7. Components of a composite element.

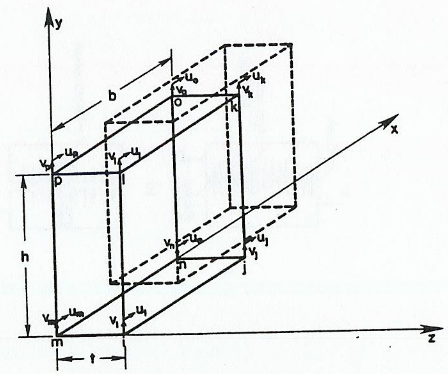

Fig. 8. Displacements in a collar joint element.

collar joint shear stresses in each element are checked against the failure criterion and the element is considered cracked if the criterion is violated. If no elements have cracked, the solution is complete.

If the collar joint shear stresses do violate the failure criterion in an element, the modulus of elasticity of the collar joint for this element is reduced to a very small number. Reducing the modulus uncouples the brick element from the block element. Thus, loads can no longer be transferred through the collar joint in this element. The resulting modified system of equations is again solved for stresses and strains and the new collar joint shearing stresses are checked against the failure criterion. This procedure is repeated until either the crack propagation stops or all the elements have cracked.

Reinforced Composite Masonry Walls

Application of Frictional Forces. Due to friction, equal and opposite forces are created at the crack interface as shown in Fig. 5(c). The magnitude of these forces depends upon the yield stress and cross-sectional area of the cross wire in the horizontal reinforcement, and the coefficient of friction at the interface. It is necessary to determine the location and distribution of these forces. In this model, the compressive force normal to the interface is assumed to act uniformly on those elements whose centroids lie within a circular area around the horizontal cross wire, as shown in Fig. 9. The diameter of the circle must be assumed, and the magnitudes of the normal forces on the nodes are determined by the tributary areas of the attached elements. These forces are multiplied by the coefficient of friction to compute the vertical frictional forces. In Fig. 10, the two wythes of Fig. 9 are shown separately with the friction loads applied to each of them.

An additional assumption is made that the friction forces are not activated until 50% of the area inside the circle as well as the element containing the reinforcement have cracked. If a crack approaches the reinforced section of a wall at an angle, the activated frictional forces will try to resist the progress of the crack by acting in a direcion opposite to that of the crack growth.

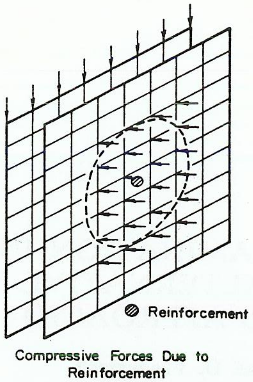

Fig. 9. Assumed circle of compressive force.

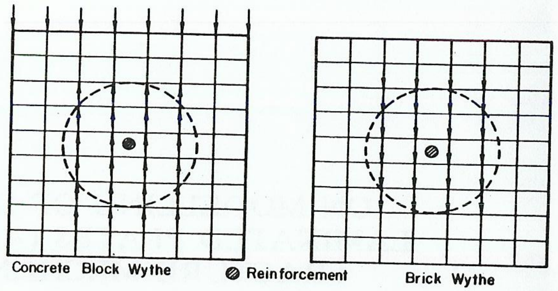

Fig. 10. Friction forces in brick and block wythes.

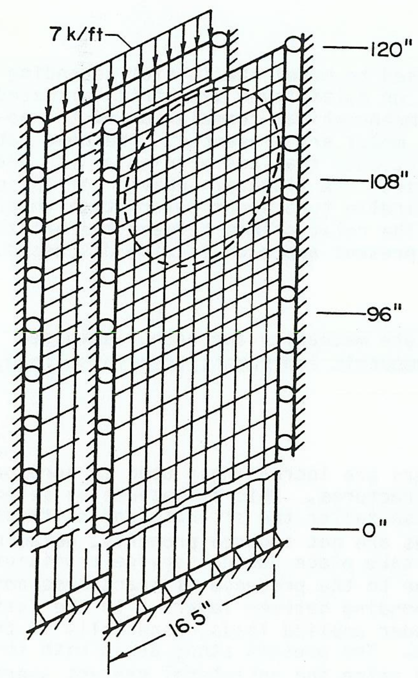

Fig. 11. Finite element mesh.

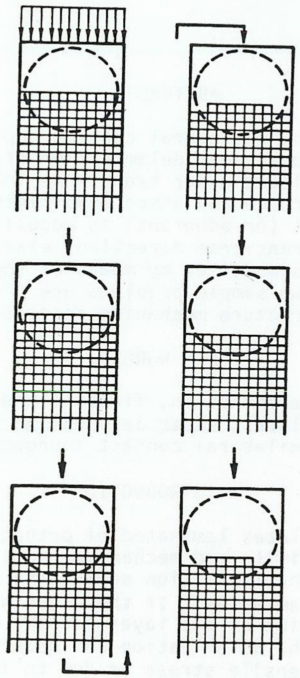

Fig. 12. Crack growth in a reinforced wall.

The solution procedure for a reinforced composite masonry wall is similar to that for an unreinforced wall except that the frictional forces are activated due to cracking. As before, the computations are stopped when no additional elements crack.

EXAMPLE PROBLEM

To examine the performance of the proposed cracking model, an example of a reinforced composite masonry wall is presented. The mesh selected for the model is shown in Fig. 11. The moduli of elasticity of the brick and block wythes, and grout used in the analysis are given collectively in Table 1. The cracking shear stress in the collar joint is assumed to be 50 psi (344.5×10^3 Nm^{-2}). The steel reinforcement has a diameter of 0.1483 in (3.75 mm) and a yield stress of 60 ksi (413.4×10^6 Nm^{-2}). The cracking occurs at the collar joint-brick interface where the coefficient of friction is 0.40. The block wythe is fully loaded with a uniform load which was determined a priori to insure that the cracks would extend past the reinforcement. The diameter of the circle over which the friction forces act is assumed to be 16 inches (40.64 cm).

TABLE 1. Material Properties

Masonry Type	Thickness		Elastic Modulus		Poisson's Ratio
	in	cm	ksi	Nm^{-2}	
Brick	4	10.16	2,000	13.78×10^9	0.25
Concrete Block	8	20.32	1,000	6.89×10^9	0.25
Grout	3/8	0.95	1,800	12.40×10^9	0.20

Results

The resulting crack growth is shown in Fig. 12. Because the crack will extend freely down the wall until it meets some resistance, the elements above the reinforcement are shown initially cracked in the first figure of the sequence shown in Fig. 12. At this stage, 50% of the area within the circle is cracked and the crack has progressed past the center of the circle. Consequently, friction forces are activated at the nodes of the cracked elements during further solutions. This procedure is continued, leading to additional cracked elements as shown in the sequential steps in the figure. Cracking in this example stops when sufficient friction forces have been mobilized and the maximum shear stress falls below the defined value of the cracking shear stress.

CONCLUSIONS AND RECOMMENDATIONS

A simple crack modelling technique that uses the composite element has been developed in this paper which can be applied to reinforced and unreinforced composite masonry walls. It is shown that the presence of horizontal reinforcement in a composite masonry wall can arrest the growth of a crack in the collar joint. By changing the diameter of the circle representing shear friction effects, the reinforcement size and strength, and the coefficient of friction, the user can approximate the actual behavior of any reinforced composite masonry wall.

ACKNOWLEDGEMENTS

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