THE FRACTURE MECHANICS OF FATIGUE, A SYNTHESIS

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ABSTRACT

The fatigue stress limit for a plain specimen, σ_o, corresponds with the material transistion from elastic to plastic strain. Thus the threshold stress intensity at σ_o is a state of equal external and internal stress intensities. This is developed here for a transfer route between crack tip and nominal states of stresses and strains. A simple model of crack tip extension is then proposed which when transferred to LE-stress intensities shows the two known stages of crack propagation. For the EP-case a stage III also appears which falls outside the frame of the normal K_I concept. Apparently a foundation of fatigue can be established in this way. Cycles to fracture functions follows from summation of crack propagation, and for plain specimen a probable relation between such functions and the stages of crack propagation is shown.

KEYWORDS

Fracture, fatigue, crack propagation, crack tip extension, crack tip energy, threshold, potential stress, life-time functions, LE and EP fracture mechanics.

INTRODUCTION

The knowledge of fatigue has been steadily increasing during nearly 150 years of experimental studies and an overwhelming amount of detailed knowledge is now available. (Thompson, Wadsworth 1958, Plumbridge, Ryder 1969, Frost 1974, Laird 1975, Fine 1980, Klesnil, Lucas 1980 - and many others). Still the field has maintained empirical.

With the introduction of crack propagation as function of stress intensity, (Paris 1961), a division into two apparently different fields of work also started:

The traditional field which has cycles to fracture or fractional damage as prime parameters (still extensively used for design) and the more recent field with crack propagation and defect sizes as main variables (mainly used for defect assessments).

Although the two are different sides of the same problem, the lines between them have been diffuse. A rational basis for the different sides of the established fatigue empiricism is thus highly wanted, it is the author's hope that this work may contribute.

1. THE STRESS-INTENSITY LIMIT FOR FATIGUE, K_{th}.

This limit is in itself a fine demonstration of the impact of the stress intensity concept; stress and square root of the crack length are for a nominally elastic material demonstrated equal in effect upon the cracktip material. It is now well known that fatigue is plasticity dependent. For a single phase material it is beyond doubt that the fatigue (plain specimen) initiation stress limit $\Delta\sigma_o$ is coupled to a lower limit for plastic strain, ε_o, of magnitude 10^{-5}-10^{-4}, (Helgeland 1968, Klesnil, Lucas 1980). No surface dislocation activity is observed below this limit. Corresponding values of stress and plastic strain are given by:

$$\Delta\varepsilon_p = k_\varepsilon \cdot \Delta\sigma^n \tag{1}$$

Note the applied form of this equation which is used throughout here. The cyclic behaviour follows from the shape of the hysteresis loops as:

$$(d\Delta\sigma/d\Delta\varepsilon_p) \cdot \Delta\varepsilon_p = \Delta\sigma_B$$

where $\Delta\sigma_B$ is the long range part of the dislocation reaction stresses.

With: $\Delta\sigma_B = 1/n\ \Delta\sigma$ the stress-strain relation, (1), is given (Helgeland 1967).

It may be inferred that the fatigue initiation-limit indicates the energy necessary for dislocations to break through or form in the surface of the material.

It is known that the stress-intensity concept may be applied for internal dislocation stress fields as for the grain-size effect upon yield stress. It is relevant here then to introduce a model of the fatigue limit as:

- an internal lower stress intensity limit for cyclic dislocation activity.

This in effect means that there exists a characteristic internal "flawlength" for the build up of dislocation stresses. This length may be determined by grain-size, barrier distance or generally the wavelength of the internal stress pattern of the dislocations. It follows that the observed fatigue treshold is a state of equal internal and external stress intensities.

The external crack length at the fatigue limit, $a_o = \Delta K_{th}^2 / \Pi \cdot \Delta\sigma_o^2$, evidently has an internal synonymous flaw length for the dislocations.

With the principles of fracture mechanics this indicates that cyclic plasticity within a zone ahead of the crack tip of radius $r = a_o$ is necessary for crack propagation to proceed.

It is thus evident that the length a_o, which is bound to the transition from nominal elastic to nominal plastic material, is essential and acts as a reference both for stress and stress intensity in fatigue.

2. THE POTENTIAL STRESS.

The perception of the fatigue limit as an internal stress intensity limit to overcome dislocation barriers, opens for introduction of a potential stress, $\Delta\sigma_p$. This stress should then convey the effective available, elastic energy comprized by the stress intensity of a crack tip. At the fatigue limit (with a surface crack of length a_o) evidently: $\Delta\sigma_p = \Delta\sigma_o$. A more general definition then follows as:

$$\Delta\sigma_p = \Delta\sigma_o \cdot (\Delta K/\Delta K_{th}) = \Delta\sigma_N\ (a/a_o)^{\frac{1}{2}} \tag{2}$$

with N = nominal stressrange

and

$$\Delta K = (1/\theta\sqrt{\Pi} \cdot \sqrt{a_o}) \cdot \Delta\sigma_p = T \cdot \Delta\sigma_p \tag{3}$$

For the most simple case, Θ, the geometrical factor of the stress intensity is 1.12 and $\Theta\sqrt{\Pi}$ is 2 (long crack, no ligament effects) so:

$$T = 1/2\sqrt{a_o} \quad \text{which is used throughout here.} \tag{4}$$

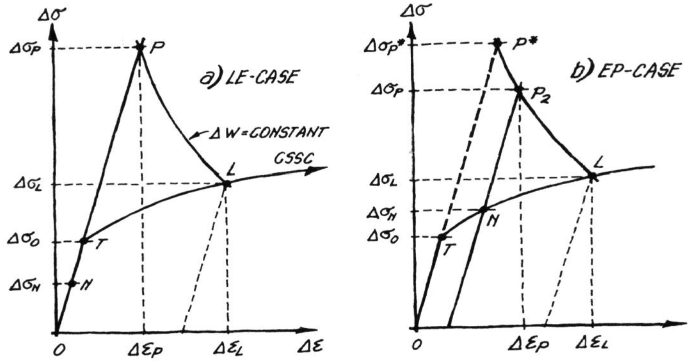
It may be noted that the definitions of the potenital stress as given, is based on a material behaviour which implies that a dislocation barrier energy is always involved.

3. ENERGY DENSITY, MECHANICAL WORK

For cyclic stressing the elastic potenital energy is evidently covered by:

$$\Delta\sigma_p^2 \cdot 1/E = \Delta W \tag{5}$$

ΔW is here energy density or work given either by an elastic line or a given stress strain curve (i.e for a half cycle of stress). This principle, (Neuber 1961), is applied in fig. 1 in a) for a nominally elastic material and in b) for a elastic-plastic loaded material. The line PL is here in accordance with (5) above.



ONTL - CYCLIC STRESS STRAIN CURVE.
PL - CURVE FOR CONSTANT ENERGY DENSITY.
OP - ELASTIC LINE.
N - NOMINAL WORKING POINT
$\Delta\sigma_p$ POTENTIAL STRESS = $\Delta\sigma_N \sqrt{\frac{a}{a_o}}$
$\Delta\sigma_p$* CORRESPONDING LE-POTENTIAL STRESS = $\Delta\sigma_N \sqrt{\frac{a^*}{a_o}}$

Fig. 1. ENERGY DETERMINED CRACK TIP STRESS AND STRAIN.

In terms of stress intensity eq. (5) yield for the LE-case:

$$\Delta K^2 = 4a_o(\Delta\sigma_L^2 + \alpha E k_\varepsilon \cdot \Delta\sigma_L^{n+1}) \tag{6}$$

L here stand for crack tip values and $\alpha = n-1/n+1$ is a correction for the shape of a hysteresis loop. The elastic-plastic case, Fig. 1 b), is slightly more comples. The normal stress intensity correspond with $\Delta\sigma_p$ and P_2 whereas the effective or virtual LE stress intensity ΔK^* is given by $\Delta\sigma^*$ and P^* Thus:

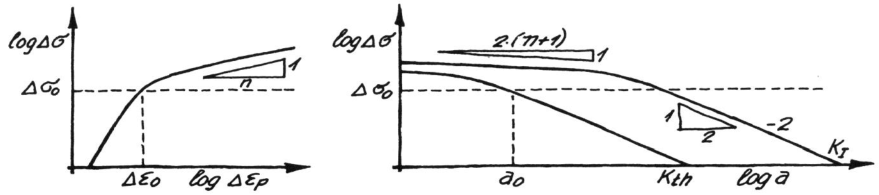
$$\Delta K_{eff}^2 = \Delta K^* = \Delta\sigma_N \cdot 2\sqrt{a^*} = \Delta\sigma_N \cdot 2(a(1+\Delta\varepsilon_p/\Delta\varepsilon_e)_{p2})^{\frac{1}{2}}$$

Splitting the stress intensity in elastic and plastic terms yields for eq. (5):

$$\Delta K_{eff} = \Delta K_{el}^2 + \Delta K_p^2 = \left[\Delta\sigma_N \cdot 2\sqrt{a}\right]^2 + \left[\Delta\sigma^{n+1/2} \cdot 2(E k_\varepsilon \sqrt{a a_o})^{\frac{1}{2}}\right]^2$$

$$= 4a_o(\Delta\sigma_L^2 + \alpha E k_\varepsilon \cdot \Delta\sigma_L^{n+1}) \tag{7}$$

Fig. 2 b) illustrates the effect of the plastic strain (above $\Delta\sigma_0$) in the material upon the effective stress intensity. The corresponding stress-strain curve is shown in a). Fig. 2 b) is well comparable with the known observations of the threshold conditions.



a) THE CYCLIC STRESS-STRAIN CURVE.

b) RESULTING PATH OF STRESS INTENSITY FUNCTIONS.
LE-RANGE $\Delta\sigma \leq \Delta\sigma_0$
EP-RANGE $\Delta\sigma > \Delta\sigma_0$

Fig. 2.

4. THE CYCLIC CRACK EXTENSION, Δa.

It is well established that fatigue is a commulative form of cyclic crack extension (Rankine 1843, Laird, Smith 1962, Richie 1983). Models for the cyclic crack extension have been proposed; none so far, however, seem to have the flexibility neccessary to cover different material behaviour, and information which could lead to a general fatigue crack tip model is lacking. Some simple energy consideration may, however, indicate a track to follow.

The specific surface energy for crack extension is defined as: $\gamma_s = \Delta U/\Delta a$ where ΔU is the actual work related to crack extension. For fatigue the plastic components are essential and $\Delta Up = \Delta Wp \cdot \Delta V$ may be applid with ΔV indicating the volume of the plastic zone. Thus:

$$\Delta a = \Delta W_p \cdot \Delta V/\gamma s \sim \Delta W_p^{1+\Delta} \qquad (8)$$

(Since the local stress and strain are the essential parameters it is assumed that both ΔV and γ_s can be expressed as f(ΔWp). Available information indicate that the normal range for Δ should be: $0 < \Delta < 1$.

Equation (8) may be rewritten as:

$$\Delta a/\Delta a_0 = (\Delta W/\Delta W_0)^z = (\Delta\sigma/\Delta\sigma_0)^b = (\Delta\varepsilon_p/\Delta\varepsilon_0)^c \qquad (9\ a,b,c)$$

with $z = 1 + \Delta$, $b = z(n+1) = n\cdot c$, and $a_0 = a^* \cdot n^*$,
a^* is the lattice unit and n^* a correcting number ≥ 1.

So without having a detailed understanding of fatigue crack extension equation (9) may be used as a working platform.

5. CRACK PROPAGATION

The combination of equations (9) and (6) og (7) give the cyclic crack extension in terms of the normally used experimental parameters.

The linear elastic case yields, (6) and (9):

$$(\Delta K/\Delta K_{th})^2 = (\Delta a/\Delta a_0)^{2/b} + (\Delta\varepsilon_0/\Delta\varepsilon_{eo})\cdot\alpha\cdot(\Delta a/\Delta a_0)^{n+1/b} \qquad (10)$$

From equation (7) and (9) the elastic plastic case give:

$$(\Delta K_{eff}/\Delta K_{th})^2 = (\Delta K_{el}^2 + \Delta K_p^2)/\Delta K_{th}^2 = (\frac{\Delta a}{\Delta a_0})^{2/b} + (\frac{\Delta\varepsilon_0}{\Delta\varepsilon e_0})\alpha\cdot(\frac{\Delta a}{\Delta a_0})^{n+1/b} \qquad (11)$$

With linear elasticity we then find two stages of crack propagation:

Stage I: $$\Delta a_I = \Delta a_0(\Delta K/\Delta K_{th})^b \qquad (12)$$

Stage II: $$\Delta a_{II} = \Delta a_0(\Delta\varepsilon_{eo}/\Delta\varepsilon_0\cdot\alpha)^{m/2}\cdot(\Delta K/\Delta K_{th})^m \qquad (13)$$

with $m = b \cdot 2/n+1 = 2z$.

The equation (10) of course gives a smoth transition between the stages. The transition between the two stages, i.e.

$$\Delta a_I = \Delta a_{II}, \text{ is given by } \Delta K = \Delta K_{th}(\Delta\varepsilon_{eo}/\alpha\cdot\Delta\varepsilon_0)^{1/n-1} \qquad (14)$$

For the elastic plastic case an additional stage III is present when the two plastic terms dominate:

Stage III: $$\Delta a_{III} = \Delta a_0(\Delta\sigma/\Delta\sigma_0)^b\cdot(a/a_0)^{m/4} \qquad (15)$$

The normal LEFM stress intensity concept here breaks down. From experimental evidence attempts have been made to define a strain intensity function, $\Delta\varepsilon_p\cdot\sqrt{a}$, for this type of crack propagation, (Boettner and colleagues 1965). Written in terms of plastic strains eq. (15) gives:

$$\Delta a = \Delta a_0(\Delta\varepsilon_p/\Delta\varepsilon_0)^c(a/a_0)^{c/2}\cdot(n/n+1) \qquad (16)$$

which, since $n/n+1 \sim 1$, is in accordance with this proposal. Fig. 3 is a schematic illustration of the different domains of fatigue with respect to nominal stress ranges and crack lengths. $a \leq a_0$ is here interpreted as the initiation domain, since the internal stress intensity then will dominate.

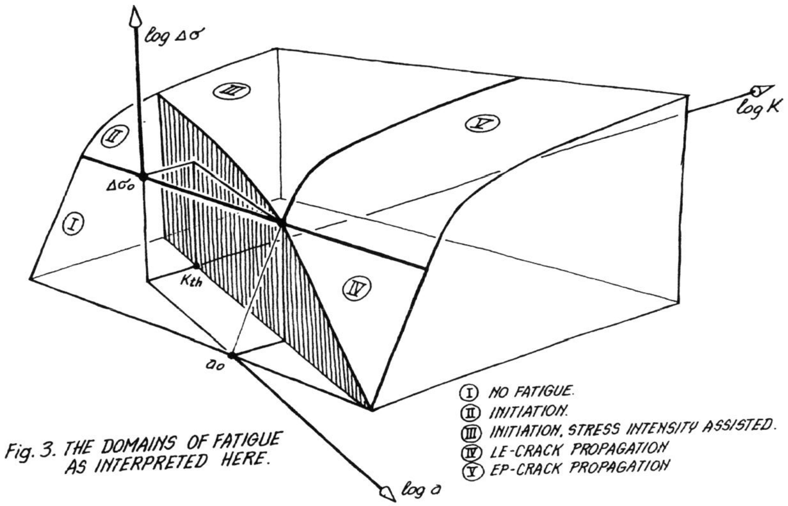


Fig. 3. THE DOMAINS OF FATIGUE AS INTERPRETED HERE.

6. CYCLES TO FRACTURE FUNCTIONS

6.1. Sn-curves (Wöhler-curves), constant nominal stress.

The traditional SN-curve is determined for plain specimens with a gauge length and a defined state of surface roughness. If this roughness is equal to a_o, (depending on material a_o is in the range: $0.01 \le a_o \le 1$ mm), the number of cycles to fracture should equal that for crack propagation from a_o to a_{crack} at the monitored test stress. The observed fatigue limit should be equal to σ_o, the stress for the transition from elastic to elastic plastic strain. (Steels with multiphase structure and strain ageing is evidently an exception; a fatigue limit slightly higher than the transition point have been repeatedly reported).

If the surface is defect free and more even than a_o, there will be a surface-slip initiation period until the effective roughening of the PSB's is equal to a_o. Crack propagation will from then on dominate. This initiation period will normally be relatively short, and the influence of the initiation upon the, logaritmically presented, total number of cycles may be presumed to be small.

Thus for most cases a material's "plain" specimen SN-curve will be equal to the summed up crack propagation from a_o to a_{crack}. Equation (11) is relevant for this case.

For surface conditions with roughness or defect size greater than a_o, the number of cycles to fracture decreases and the effective fatigue stress limit is lowered in accordance with threshold stress intensity, K_{th}. Tests are then also performed at stresses below σ_o, i.e. in the elastic range where equation (10) is relevant.

A simplified summation gives the following results for the three stages of crack propagation:

$$\sum^{N_B} \Delta a = \overline{\Delta a} \cdot N_B = a_c$$

a_c is the crack length at final fracture and $\overline{\Delta a}$ the mean crack extension in the range a_o to a_c:

$$\overline{\Delta a} \approx \Delta\sigma^{e1} \cdot \overline{a^{e2}}.$$

So that:

$$\Delta\sigma^{e1} \cdot N_B \approx a_c / \overline{a^{e2}} \sim \text{const.} \qquad (18)$$

For the three stages of crack propagation the exponent e_1 is

Stage I: b, Stage II: m and stage III: b

As stage I and stage III should be expected to dominate for the plain specimen whereas stage II may appear for specimen with $a > a_o$, Fig. 4a illustrate the results to be expected.

6.2. Constant plastic strain to fracture. (Travernelli, Coffin 1959, Manson 1953)

For the plain specimen, the SN-curve is changed to $\varepsilon_p \cdot N_B$-curve by way of the CSS-curve. As stage I and III should be expected to dominate the results will be:

$$\Delta\varepsilon_p{}^c \cdot N_B = \text{const. with } c = b/n.$$

The two stages may give rise to two parallel lines as indicated in fig. 4 b. Note that with a defect size (or an equal stress concentration effect) resulting in $a_{init} >> a_o$ crack propagation may proceed with nominally $\Delta\varepsilon_p = 0$.

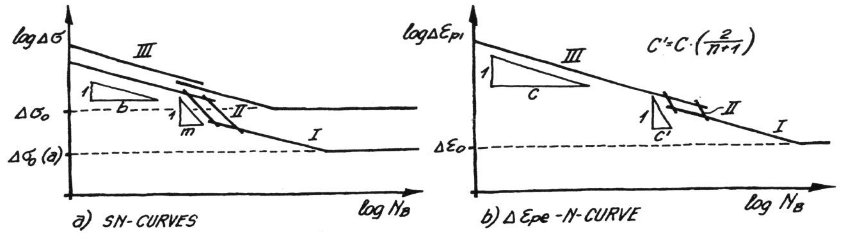


Fig. 4. LIFETIME CURVES AS ANTICIPATED FROM THIS WORK

6.3. Energy to fracture.

Fatigue fracture tests have also been presented as:

a) Plastic strain energy loss versus cycles to fracture: $\Delta W^z \cdot N_B = \text{const.}$

b) Total energy loss versus cycles to fracture: $(W \cdot N_B) = f(N_B) = \text{const.}\ N_B{}^r$.

Both cases are in accord with the results in 6.1 and 6.2 with $z = m/2$ and $r = (1\text{-}2/m)$. This is also well in accordance with reported results (Halford 1964; Klesnil, Lucas 1980) Recent results of crack propagation also confirm $z = m/2$ (Dowling, Begley 1976).

7. SOME COMMENTS

The cyclic extension mechanism including details of possible influence of mechanical, structural or environmental origin, is evidently the sole origin of fatigue fracture. Dislocation activity expressed by the elastic-plastic transition limit, as persistant slip bands or plastic strain generally, is arranging the ground for fracture mechanics to take over.

The three stages of crack propagation are evidently not fundamental, i.e. not connected with the crack extension directly. Rather they appear experimentally as result of the relation between the dominating forms of mechanical energy nominally and at the crack tip.

The Stage III of crack propagation disturbs somewhat the normal FM stress intensity picture. If, however, a plastic strain energy determined component of stress intensity is defined as suggested:

$$K_p = \sigma_N \cdot 2((\varepsilon_p/\varepsilon_e)_N \cdot a_p)^{\frac{1}{2}} \text{ with } a_p = (a \cdot a_o)^{\frac{1}{2}} \qquad (19)$$

a continous range of stress intensity is maintained also for the EPFM case. Stage III of crack propagation is then transformed to Stage II with logaritmic crack propagation rate equal to <u>m</u>.

A unified picture of fatigue emerges from this work, it should however be regarded at present as a working platform, further research is neccessary for assessment of its general value.

The exponent Δ introduced with equation (8) represents a new angle of attack, it reflects hints to crack tip fracture modes which so far have been without evident influence on fatigue.

Fig. 5 shows a statistical plot of m-values estimated from fatigue fracture data for 53 steels. Though it is too early to judge this kind of caclulations, it may indicate the applicability of the general frame for fatigue as put forth in this work.

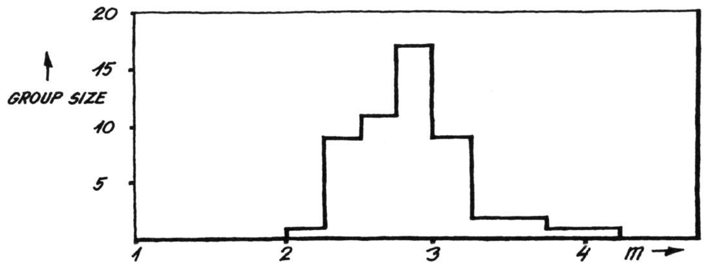


Fig.5. DISTRIBUTION OF m-VALUES FOR 53 STEELS ESTIMATED AS:

$$m = b \cdot \frac{n+1}{2} \quad , \quad n = \frac{b}{c}$$

(DATA: METALS HANDBOOK: 1, 680, 1978)

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