THE EFFECT OF CONTACT STRESS INTENSITY FACTOR ON FATIGUE CRACK PROPAGATION UNDER VARIABLE AMPLITUDE LOADING

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ABSTRACT

The phenomenon of crack closure is modelled by a "Contact Stress Intensity Factor" Kc measured by the Crack Arrest Method. Using this concept, it is predicted and demonstrated experimentally that there will be no load interaction effect during variable amplitude loading as long as K_{max} is kept constant. For a reduction of K_{max} in step-down loading, crack retardation will occur. Excellent agreement is obtained between the measured values of Kc and the experimentally observed retarded crack growth rates. The disagreement between the suggested relationship for the variation of Kc during crack retardation and the measured values can most probably be explained by the observed crack branching after step-down loading. From previous work by Lam and Williams (1982,1984), it is demonstrated that Kc can explain the R effect under constant amplitude loading. This implies that crack closure modelled on the basis of Kc can explain the effect of R, crack retardation and crack arrest and reveals that all these phenomena are intimately related to one another.

KEYWORDS

Crack closure; contact stress intensity factor; crack retardation; crack arrest; variable amplitude loading.

INTRODUCTION

The effect of Contact Stress Intensity Factor Kc on fatigue crack propagation under constant amplitude loading has been investigated by Lam and Williams (1982,1984). Using Kc for the modelling of crack closure, the effect of R can be accounted for. The important equations under constant amplitude loading are summarized as follows:-

$$Ka = K + Kc \tag{1}$$

$$Ka_{max} = K_{max} \tag{2}$$

$$Ka_{min} = K_{min} + Kc_{max} \quad (3)$$

$$\Delta K = K_{max} - K_{min} \quad (4)$$

$$Kc_{max} = f(R)\Delta K \quad (5)$$

$$\Delta Ke = Ka_{max} - Ka_{min} - K^*$$

$$= K_{max} - (K_{min} + Kc_{max}) - K^*$$

$$= \Delta K - Kc_{max} - K^* \quad (6)$$

$$da/dn = f\ (\Delta\ Ke) \quad (7)$$

where

- K = Stress Intensity Factor due to the applied loads.
- Ka = Actual Stress Intensity Factor experienced at the crack tip.
- Kc = Contact Stress Intensity Factor due to crack closure.
- ΔKe = The Effective Stress Intensity Factor Range causing fatigue crack growth.
- K* = A material constant related to the minimum energy required to cause fatigue crack growth. (Threshold Stress Intensity Factor)
- da/dn = Crack growth rate.

The subscripts max and min indicate respectively the maximum and minimum values of the appropriate parameter in a loading cycle.

In the existing literature, crack closure is modelled by using the crack opening stress σ_{op} as proposed by Elber (1970, 1971). However, as demonstrated by Sharpe and co-workers (1976), Chanani and Mays (1977), Brown and Weertman (1978) and Gan and Weertman (1981), σ_{op} cannot account fully for crack retardation under variable amplitude loading.

Thus, it is a natural extension to investigate the effect of Kc on fatigue crack propagation under variable amplitude loading. For the present discussion, attention will be focussed on two-step block loading.

It should be pointed out that the concept of Kc is based on the compressive stress developed along the wake of the crack due to crack closure. In these investigations, Kc is deduced experimentally using the Crack Arrest Method. Details of the concept and the experimental procedure employed for the determination of Kc may be obtained from Lam and Williams (1984). Thus, both the concept and measurement of Kc are different from Elber's σ_{op}.

TWO-STEP BLOCK LOADING WITH CONSTANT K_{max}

The first type of two-step block loading to be examined is one with constant K_{max} but different K_{min} as shown in Fig. 1.

Considering that crack closure occurs during the unloading part of the cycle, it is reasonable to assume that Kc is a continuous increasing quantity during unloading. At any point along the unloading path, Kc has a finite but changing value, varying from zero to K_{max} to a maximum (Kc_{max}) at K_{min}. It is further reasonable to suggest that under constant amplitude loading, at any point of the unloading path, the value of Kc is governed by an equation similar to equation 5. That is:-

$$Kc = f(R_p)\Delta K_p \quad (8)$$

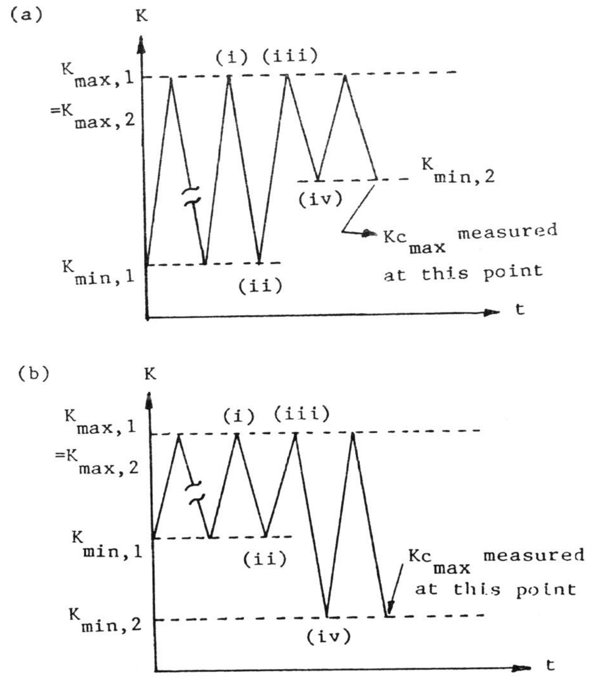

Fig. 1. Two-step block loading with constant K_{max}.

where the subscript p indicates the value of the variable at any point on the unloading path. In subsequent discussion, the subscripts 1 and 2 will be used to denote quantities associated with block one and block two loadings respectively.

Thus, for the case depicted in Fig. 1a, Kc_{max} for unloading from (i) to (ii) will be given by :-

$$Kc_{max,1} = f(R_1)\Delta K_1 \quad (9)$$

which is the value of Kc_{max} typical of constant amplitude loading for block one.

Unloading from (iii) to (iv) is similar to partially unloading from (i) to (ii), that is the value of Kc will be governed by equation 8. However, the value of Kc, R_p and ΔK_p are the values of $Kc_{max,2}$, R_2 and ΔK_2 respectively. That is :-

$$Kc_{max,2} = f(R_2)\Delta K_2 \quad (10)$$

Comparing equation 10 with equation 5, the value of $Kc_{max,2}$ turns out to be the Kc_{max} value for block two as if under constant amplitude loading. This amounts to saying that for the case shown in Fig. 1a where K_{max} remains constant, a sudden change in K_{min} will not produce any transient or load interaction effect since Kc_{max} and thus ΔKe will always acquire the value under constant amplitude loading instantaneously.

A similar argument may be applied to the case as shown in Fig. 1b, provided that Kc will still be governed by equation 8 even when ΔK_1 is less than ΔK_2.

From the above analysis, it may be generalised that as long as K_{max} remains unchanged, there will be no load interaction effect. This is in fact observed experimentally by McMillan and Pelloux (1967), Porter (1972) Wei and co-workers (1973) and Druce and co-workers (1979).

To confirm the above hypothesis that Kc_{max} will adjust itselt instantaneously if K_{max} is kept constant, the value of Kc_{max} was measured using the Crack Arrest Method after two cycles of block two loading were applied. The loading sequences are indicated in Fig.1 and the results are tabulated in Table 1.

TABLE 1

Specimen No.		30	29
Load Sequence		Fig. 1a	Fig. 1b
ΔK_1	$(MNm^{-3/2})$	34.82	17.65
R		0.0	0.5
$Kc_{max,1}$ (cal.)	$(MNm^{-3/2})$	16.03	4.90
ΔK_2	$(MNm^{3/2})$	17.63	34.87
R		0.5	0.0
$Kc_{max,2}$ (cal.)	$(MNm^{-3/2})$	4.89	16.06
$Kc_{max,2}$ (mea.)	$(MNm^{-3/2})$	5.04	15.20

In Table 1, the values of $Kc_{max,1}$ and $Kc_{max,2}$ were calculated using equation 5. $Kc_{max,2}$ was measured experimentally using the Crack Arrest Method. From Table 1, it is apparent that the value of Kc_{max} adjusts instantaneously to the value of block two loading immediately after the transition from block one to block two loading. The difference between the calculated values of Kc_{max} for block two (assuming constant amplitude loading) and the measured values are in agreement, with only 4% and 5% difference between them for the loading sequence as shown in Fig. 1a and 1b.

STEP-DOWN LOADING

In the step-down loading sequence shown in Fig. 2, both crack arrest and retardation will be discussed.

Crack Arrest

On unloading from (i) to (ii), Fig. 2, Kc_{max} is given by the constant amplitude value, that is according to equation 9:-

$$Kc_{max,1} = f(R_1)\,(K_{max,1} - K_{min,1})$$

On loading from (ii) to (iii), ΔKe, according to equation 6, is given by:-

$$\Delta Ke = \Delta K_2 - Kc_{max,1} - K^* \qquad (11)$$

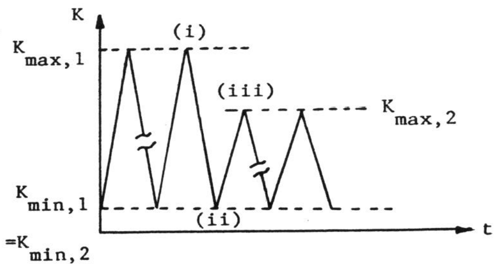

Fig. 2. Step-down loading.

Thus from equation 7, crack arrest occurs when $\Delta Ke = 0$, that is:-

$$\Delta K_2 < f(R_1)(\Delta K_1) + K^* \qquad (12)$$

In the following discussion of crack arrest, the effect of K^* will be ignored. The reason being that if the applied stress intensity factor is much larger than the threshold stress intensity factor, the value of K^* will be small and negligible when compared with K_{max} and K_{min}.

Note that ΔKe cannot be negative since a negative ΔKe is meaningless. This implies that for crack arrest, from equation 6 and ignoring the effect of K^* :-

$$Ka_{min} = Ka_{max} \qquad (13)$$

A closer examination indicates that the second line of equation 6 is obtained by assuming $Ka_{max} = K_{max}$. This means that at maximum load, the crack is completely open. However, with crack arrest, at K_{max} there will still be crack closure. This implies that at Ka_{max} will be the sum of K_{max} and Kc associated with K_{max} at that instant. In fact, if K and Kc are the applied and contact stress intensity factors at any point of the loading or unloading path with K greater that K_{min} and with crack arrest, equation 13 becomes :-

$$K_{min} + Kc_{max} = K + Kc \qquad (14)$$

When crack arrest occurs, according to equation 13, the actual stress intensity factor experienced by the crack tip material is constant since $Ka_{min} = Ka_{max}$ at all times. From equation 14, this is brought about by the interaction of the contact stress intensity factor due to crack closure and the applied stress intensity factor. As the latter increases, the amount of crack closure and thus Kc, decreases.

Therefore with crack arrest, the crack tip material experiences no change in the stress intensity factor. It may be thought of as being "frozen" in the particular condition before the occurrence of crack arrest. The number of loading cycles experienced during that period of crack arrest will not alter the subsequent behaviour. If at any time, the loading level is raised such that crack growth occurs, the behaviour of crack growth will then be dictated by the conditions existing just before crack arrest.

Experimental observations by Wei and co-workers (1973) indicate that the number of cycles during which the crack is arrested has negligible effect on the subsequent retarded crack growth behaviour. This gives support to the above analysis.

Crack Retardation

From the above discussion, it is apparent that the onset of crack arrest is caused by the crack tip conditions existing at Ka_{min}. Since crack arrest may be treated as the most severe case of crack retardation, it is reasonable to consider that crack retardation is governed by the same condition.

As the crack propagates under block two loading, the value of Ka_{min} will change. It is reasonable to assume that the magnitude of deviation of the various quantities from their constant amplitude loading values in block two will in turn affect the way in which they change. Thus the relevant parameters to be considered will be $(Ka_{min} - Ka_{min,c2})$ and $(Ka_{min,c1} - Ka_{min,c2})$ where Ka_{min} is the actual minimum stress intensity factor at any instant. The quantities with subscripts c1 and c2 indicate the values associated with constant amplitude loading for block one and block two respectively.

Further, immediately after step-down loading, the conditions ahead and in the wake of the crack are typical of block one. As the crack propagates under block two loading, the influence of block one will slowly decay. Thus, crack retardation should be related to the distance the crack propagates into the monotonic plastic zone due to $K_{max,1}$. The relevant parameters should then include $(a-a_o)$ and d_1 where a and a_o are respectively the crack lengths at any instance and at the instance of stepping down and d_1 is the monotonic plastic zone due to $K_{max,1}$.

Based on the parameters identified above, it is proposed that:-

$$(Ka_{min} - Ka_{min,c2})/(Ka_{min,c1} - Ka_{min,c2}) = g[(a-a_o)/d_1)] \qquad (15)$$

The boundary conditions to be satisfied are:-

$$a-a_o = 0 \qquad Ka_{min} = Ka_{min,c1} \qquad g = 1.0 \qquad (16)$$

$$a-a_o \rightarrow \infty \qquad Ka_{min} = Ka_{min,c2} \qquad g = 0.0 \qquad (17)$$

If Kc_{max} is known, the values of Ka_{min} may be calculated according to equation 3. In this investigation, as the crack propagated under block two loading, Kc_{max} was measured as a function of crack length using the Crack Arrest Method. Two specimens were used for each set of loading limits and generally three readings were obtained from each specimen.

For the calculation of $Ka_{min,c1}$ and $Ka_{min,c2}$, equation 3 was used. However, the value of Kc_{max} was obtained using equation 5.

To calculate d_1, in order to take into account the effect of the finite specimen size, Rice's (1966) longitudinal shear solution for an elastic perfectly plastic material and the analogy between longitudinal shear and tensile loading (McClintock, 1961; McClintock and Irwin, 1965) was utilized.

Alternatively the value of Ka_{min} may be calculated through the measurement of da/dn. If da/dn is known, ΔKe may be obtained from the log-log plot of da/dn versus ΔKe, see Lam and Williams (1984). Then from equation 6, with the value of K_{max} and K* known, Ka_{min} may be calculated. The value of K* may be obtained from Lam and Williams (1984). For the measurement of da/dn after step-down loading, a different specimen was used for each loading limit. It should be noted that if the values of Ka_{min} obtained independently by either the Crack Arrest Method or the da/dn Method agree, then a rational explanation of retardation by crack closure will be substantiated.

Two loading conditions were investigated with R approximately equal to zero. The results are shown in Fig. 3 with the parameter $(Ka_{min} - Ka_{min,c2})/(Ka_{min,c1} - Ka_{min,c2})$ plotted against $(a-a_o)/d_1$.

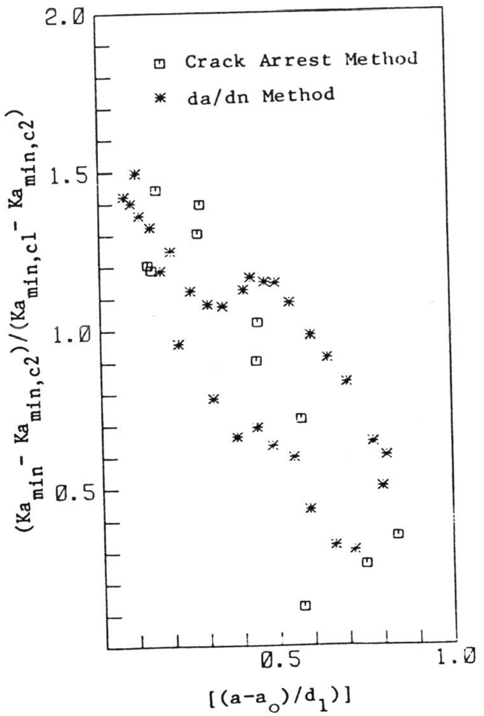

Fig. 3. Showing the variation of Ka_{min} as the crack propagated under block two loading.

From Fig. 3 it can be seen that indeed excellent agreement is obtained between the values of Ka_{min} measured by the Crack Arrest Method and by the da/dn method. This indicates that the phenomenon of crack closure as modelled by Kc can account for crack retardation.

A close examination of Fig. 3 indicates that the boundary condition as stated in equation 16 for the relationship proposed by equation 15 does not seem to be satisfied. At $a-a_o = 0$, the function $g[(a-a_o)/d_1]$ assumes a value approximately equal to 1.5 instead of 1.0 as required. The higher value of the function indicates a higher value of Ka_{min} and Kc_{max} than expected.

A possible explanation of the above discrepancy is due to the effect of crack branching which was commonly observed after step-down loading. The interaction between the two different branched cracks may bring about a higher value of Kc_{max}. It should also be noted that the parameter $(Ka_{min} - Ka_{min,c2})$ is very sensitive. Small changes in the measurements of Ka_{min} may bring large changes in the value of the parameter.

CONCLUSION

Using the concept of crack closure as modelled by the Contact Stress Intensity Factor Kc, it was demonstrated by Lam and Williams (1982, 1984) that the effect of R under constant amplitude loading can be accounted for.

Under two-step block loading, using Kc, it is further predicted and demonstrated that as long as K_{max} is kept constant, there will be no load interaction effect with changing K_{min}. Further, excellent experimental agreement was obtained between the measured value of Kc and its effect on the fatigue crack propagation rate after a reduction of K_{max} where crack growth retardation was observed. To the best knowledge of the authors, it is the first time that good agreement has been obtained between the measured value of crack closure and retarded crack growth. Since Kc is measured by the Crack

Arrest Method, this further implies that using the concept of crack closure, the phenomena of crack arrest and crack retardation under variable amplitude loading and the R effect under constant amplitude loading are intimately related to one another.

Disagreement exists between the suggested relationship as defined by equation 16 for the variation of Ka_{min} during crack retardation and the experimental observations. A higher values of Ka_{min} and Kc were observed. This most probably can be explained by the presence of crack branching after step-down loading. With crack branching, due to the interaction of the branched cracks, a higher crack arrest load and a lower crack growth rate should be expected. This will then give rise to a higher calculated value of crack closure.

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