STATISTICAL CHARACTERISTICS OF LOW-CYCLE FATIGUE CRACKS PROPAGATION FROM STRESS CONCENTRATION ZONES

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ABSTRACT

The method of estimating some crack characteristics, i.e. initiation, propagation and COD has been developed. This method is based on the statistical analysis of the initiation and propagation of cracks in specimens with central hole under low--cycle fatigue and also on the statistical analysis of main mechanical properties of the material and its distribution functions. The data obtained are used for study of probabilistic approach to the calculation of low-cycle strength at stress concentration zones in terms of the strain criterion of failure. The peculiarities of short cracks growth in terms limit of crack resistance are supposed to be taken into account.

KEYWORDS

Low-cycle fatigue; crack initiation, propagation; concentration zones; statistical analysis; dustribution function; strain criterion of failure.

PREFACE

The evaluation of low-cycle strength and life of most elements of modern highly loaded machine structures, which are characterized by cyclic elasto-plastic strains arising in stress concentration zones, must include the estimation of life according to the following two stages: the moment of crack formation and the moment of crack development with defining the statistical characteristics of crack propagation. This way of getting up the problem is quite justified because, for the zones of stress concentration, the duration of these two stages is usually comparable. The available data on crack formation are quite plentiful both in theoretical and experimental field. However there are practically no statistical data on the initial growth of low-cycle fatigue cracks from concentration zones.

EXPERIMENTAL PART

The investigations are based on the statistical analysis of the regularities of crack initiation and propagation in flat specimens with central hole of 18-8 steel (α_σ = 2,4), based on the statistical analysis of main mechanical properties, on the establishing and evaluation of the parameters of experimental distribution functions. Tensile tests and cyclic tests under tension-compression at symmetric loading on 3 levels of stress (18-20 specimens for each level) in cycle numbers $3 \cdot 10^2$, 10^3 and 10^4 at room temperature were carried out. The specimens for tensile tests were cut out from the non-deformed part of the main sample.

Measuring of crack length ℓ and its opening displacement ϑ during the experiment was carried out by an optical method by means of a microscope with magnification up to 120. The methods of testing and data processing are described by Mackhutov, Zatsarinny and Novikov (1983). As an example in Fig. 1 is shown the field of the lengths of all cracks which appeared on the surface of 18 specimens (cycle numbers - 10^3). The crack length was measured only on one side of the specimen. Such fields were also obtained for the COD ϑ and crack growth rate $d\ell/dN$ for all the three statistical series of tests.

The experimental distribution functions of cycle numbers according to the moment of crack formation N_0 and the moment of failure N_c as well as the distribution functions of cycle numbers for certain crack length $N(\ell)$ (Fig. 2), size of COD $N(\vartheta)$ and crack growth rate $N(d\ell/dN)$ were checked for homogeneity (according to the criteria of Grubbs and Irvin) and for correspondence to the chosen type of distribution function (according to the criteria ω^2 and Shapiro-Wilk W).

The results of graphical analysis and calculation analysis permitted the establishment of a satisfactory correspondence of the distributions to the normal and logarithmically normal laws. The normal distribution law corresponded better to the initial stage of crack growth; before failure, as a rule, better correspondence was observed for the logarithmically normal distribution law. The dispersion of characteristics of main mechanical properties ($\sigma_{Y0,02}$, $\sigma_{Y0,2}$, σ_u, ψ_c, S_c) was sufficiently well described by the normal distribution law.

With the aim of obtaining a calculated estimate of cycle numbers dispersion for the moment of crack initiation N_0, the calculation was done according to the strain criterion of failure including the probabilistic characteristics of the main mechanical properties of the material. The number of cycles before crack initiation was estimated by means of the equation (1) of Langer type for the fatigue curve from Norms of design (1973) for constant amplitude of strain mode; this equation reflects the influence of main mechanical properties and the asymmetry of local elasto-plastic strains in the concentration zone:

$$\sigma_a^* = \frac{E}{4N_0^{0,5} + \frac{1+r^*}{1-r^*}} \ln \frac{1}{1-\psi_c} + \frac{\sigma_{-1}}{1 + \frac{\sigma_{-1}}{\sigma_u} \cdot \frac{1+r^*}{1-r^*}} ; \qquad (1)$$

where: E - elastic modulus, MPa; ψ_c - relative reduction of the specimen cross-section area at static tension; σ_{-1} - fatigue limit of a smooth specimen at cyclic tension-compression, MPa; σ_u - ultimate tensile strength of a smooth specimen, MPa; r^* - coefficient of skew of local strains; σ_a^* - conventional elastic stress, which is equal to the product of strain (elastic or elasto-plactic) and elastic modulus ($\sigma_a^* = e \cdot E$).

The conventional elastic stress σ_a^* and the coefficient of skew r^* have been determined according to the method (Mackhutov, 1981):

$$\sigma_a^* = \frac{\bar{\varepsilon}_{max}^{(1)}}{2} \cdot E ; \qquad r^* = \frac{\bar{e}_{max}^{(0)} - \bar{\varepsilon}_{max}^{(1)}}{\bar{e}_{max}^{(0)}} ;$$

$$\bar{e}_{max}^{(0)} = \frac{e_n^{(0)}}{e_{Y0,02}} \cdot K_e^{(0)} ; \qquad \bar{\varepsilon}_{max}^{(1)} = \frac{\varepsilon_n^{(1)}}{e_{Y0,02}} \cdot K_\varepsilon^{(1)} .$$

where $\bar{e}_n^{(0)}$, $\bar{\varepsilon}_n^{(1)}$, $\bar{e}_{max}^{(0)}$, $\bar{\varepsilon}_{max}^{(1)}$ are nominal and maximum local strains in the zero and first semicycles of deformation, $K_e^{(0)}$ and $K_\varepsilon^{(1)}$ are strains concentration factors.

It is assumed that the strain value in the first semicycle is determined for the cyclic strain diagram stabilized in the first and successive semicycles. In the power-mode approximation of strain diagram, the value of the strain concentration factor is determined (Mackhutov, 1981) from the following expression:

$$K_{e,\varepsilon} = \frac{\alpha_\sigma^{\frac{2}{1+m}}}{\left(\alpha_\sigma \cdot \frac{\sigma_n}{\sigma_{Y0,02}}\right)^{0,5(1-m)\left[1-\left(\frac{\sigma_n}{\sigma_{Y0,02}} - \frac{1}{\alpha_\sigma}\right)\right]/(1+m)}} ; \quad \frac{\sigma_n}{\sigma_{Y0,02}} \geq 1 \qquad (2)$$

where m is the characteristic of hardening at power-mode approximation of strain diagram (it is determined for the zero or K-th semicycle of loading).

The hardening factor for steels at E = 200 GPa in the zero semicycle may be defined through the main mechanical properties (Mackhutov, 1981):

$$m = 0,75 \cdot \frac{\lg\left[\frac{\sigma_u}{\sigma_{Y0,2}} \cdot (0,8 + 2,06\,\psi_c)\right]}{\lg\left[\left(10^5 \ln \frac{1}{1-\psi_c}\right) / \left(200 + 0,5\,\sigma_{Y0,2}\right)\right]} ; \qquad (3)$$

Thus, after the statistical processing of the results of tensile tests and knowing the empirical distribution functions of the main mechanical properties, it is possible, to estimate by equation (1) the number of cycles before crack initiation N_0 depending on the preset probability level P.

The results of processing of experimental data obtained in tensile tests (the number of tested specimens was about 60) did not permit the establishment of functional connections between such characteristics as σ_u and ψ_c; σ_u and $\sigma_{Y0,2}$; σ_u and $\sigma_{Y0,02}$. Which is why, when probabilistic estimates were being done, the values of the static characteristics introduced into the equations (1÷3) were chozen from the experimental functions of their distributions for the preset probability levels P .

For checking the correspondence of estimated and experimental results, four variants of estimation have been made. The obtained coefficients of variation for estimation distributions are comparable with experimental coefficients (Fig. 3). This fact proves the possibility of estimation description of cyclic characteristics dispersion(of cycle numbers N_0) through the dispersion of main machanical properties.

The estimation variants 1, 2 and 3 differ by the method of obtaining the values $\sigma_{Y0,02}$ and S_C (estimated or experimental). For the variant 1 the value of the yield limit was found from the experiment with the tolerance 0,02% for plastic deformation. For the variants 2 and 3 the value $\sigma_{Y0,02}$ was taken as estimated and it was calculated on the basis of equation:

$$\sigma_{Y0,02} = \left[\frac{\sigma_{Y0,2}}{(E \cdot 0,2 \cdot 10^{-2} + \sigma_{Y0,2})^m}\right]^{\frac{1}{1-m}} ;$$

For the variant 2 the puer ultimate tensile strength S_C was estimated according to the relation:

$$\frac{S_C}{\sigma_u} = 0,8 + 2,06\psi_C$$

For the variant 1 and 3 the value S_C was taken from the experimental data. For the variant 4 the method of obtaining the values $\sigma_{Y0,02}$ and S_C is the same as for variant 1, but, unlike the latter, here were used the values of static characteristics which were obtained not from empirical distribution functions (for equal probabilities P), but from the experimental data for a given specimen.

The analysis of Fig. 3 shows that in the field of limited life there is a satisfactory accordance of the estimate with the mean value of the experimental life distribution curves. When cyclic stresses decrease and life increases these estimates give results without the safety factor. In most important field of small probabilities of failure P , these errors exhaust to 55% of the presently assumed safety factor n = 10. The coefficient of life variation ϑ , on appearing of the crack N_0, varies in the range from 2,5 to 7% for estimated data and from 2,5 to 4,5% for experimental data for logariphmically normal distribution law. We can also note the decrease of coefficients of variation with the increase of cycle numbers.

The results of experiment showed the absence of the threshold value of the stress intensity factor range ΔK at the crack growth rates $d\ell / dN > 10^{-4}$ mm/cycle (which corresponds to initial cracks more than 0,05÷0,1 mm long). As an example, Fig. 4 shows the experimental results for a series of 10^3 cycles (10 specimens). For short cracks was observed an increase of spread in results and a rather high value of experimental crack growth rates as compared with estimated values according to Forman's equation (curve 1).

The calculation was made for the conventional values of the

stress intensity factor (on the basis of the equation of the linear fracture mechanics):

$$\Delta K = \Delta\sigma_n \sqrt{\pi\ell} \cdot f_1 \cdot f_2$$

where $\Delta\sigma_n$ is the range of nominal stresses ($\Delta\sigma_n = \sigma_a$ is accepted according to the assuption of a fluctuating stress cycle at the crack tip, R = 0); f_1 and f_2 are dimensionless correction functions which take into consideration the crack length, the dimension of the hole and the width of the specimen (Broek, 1974).

The values of crack growth rates have been estimated on the basis of Forman's equation:

$$\frac{d\ell}{dN} = \frac{A \cdot \Delta K^B}{(1-R)K_C - \Delta K} ;$$

where: A and B - parameters; R = 0 - coefficient of skew; K_C - critical stress intensity factor (its value was conventionally assumed to be equal to 50 MPa $m^{1/2}$).

Introducing into the estimation the limit of crack resistance I_C (Morozov, 1975) instead of K_C , considered as a material constant, imroves the correspondence of the estimation to the experiment in the field of short cracks. The limit of crack resistance I_C is connected with the critical stress intensity factor K_C according to the relation $I_C = K_C \cdot \varphi(\ell)$ or $I_C = \varphi(\frac{\sigma_C}{\sigma_u})$ where σ_C is the ultimate stress which is usually defined experimentally. The limit of crack resistance I_C is calculated according to the equation:

$$I_C = K_C \sqrt{1 - \left(\frac{\sigma_C}{\sigma_u}\right)^{n'}} ; \qquad (4)$$

where n = 2÷4. The values of the ultimate stress σ_C were calculated according to the method described by Markochev (1980). The obtained estimated crack growth rates correspond to different exponents n in equation (4): the curve 2 - n = 4; the curve 3 - n = 2.

The represented data don't contain statistical characteristics of the cyclic fracture mechanics in evident form. But it may be supposed that the dispersion of crack growth rates in the initial and successive parts of their development may be determined by statistical characteristics of the critical stress intensity factor K_C and of the parameters A and B , which, in turn, are connected with the main mechanical properties.

Due to the high plastic deformations in the crack zones, the analysis of the regularities of their development must be made not in the values of the stress intensity factors, but in the values of the strain intensity factors. The particularities of kinetic curves shown in Fig. 4 (in the field of short cracks) should be connected with the analysis of damage accumulation in concentration zones at the moment of crack formation.

CONCLUSIONS

The experimental distributions of cracks according to different parameters (lengths, COD, growth rates) satisfactorily correspond to the normal and logarithmically normal distribution laws.

It is possible to estimate distribution functions according to the moment of crack formation on the basis of strain criterion of failure, introducing into it the dispersion of main mechanical properties.

When considering the particularities of short cracks development, it is expedient to use the limit of crack resistance I_c. However, even in this case, the influence of plastic deformations in the crack zone is essential and must be taken into account when calculating the values ΔK and I_c.

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Нормы расчета на прочность элементов реакторов, парогенераторов, сосудов и трубопроводов атомных электростанций, опытных и исследовательских ядерных реакторов и установок. (1973). (Norms of design for strength parts of reactors, steam generators, vessels and pipes of nuclear power stations, pilot and research nuclear reactors and installations). Металлургия, Москва.

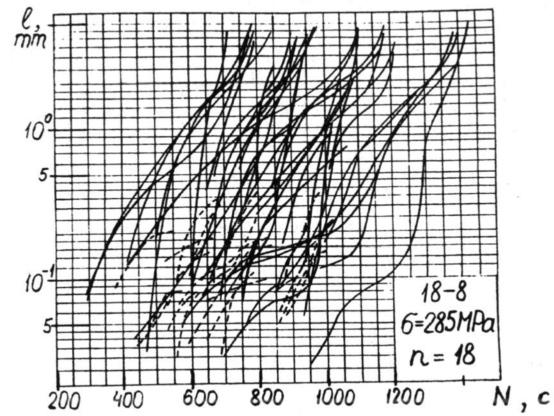

Fig. 1. The experimental field of crack lengths for the test series with the stress amplitude σ_a = 285 MPa.

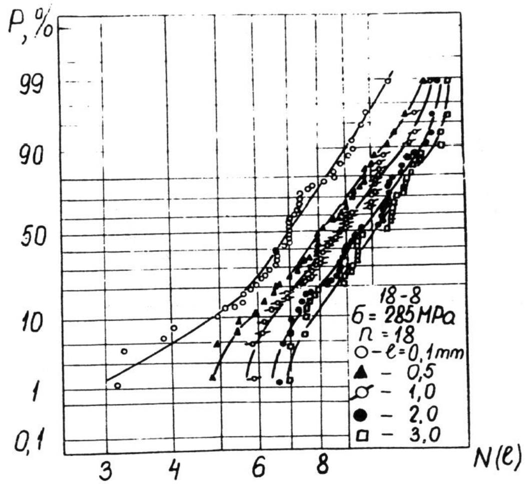

Fig. 2. Experimental distribution functions of cycle numbers according to the parameter of crack length $N(\ell)$ (σ_a = 285 MPa).

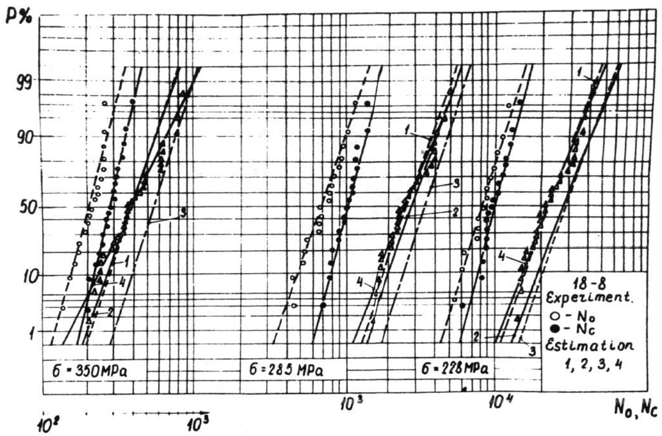

Fig. 3. Experimental and estimated distribution functions of cycle numbers before crack initiation N_0 for different estimation variants (1, 2, 3, 4):

○ – experimental values of cycle numbers before the initiation of a crack 0,3 mm long N_0 ;

● – experimental values of cycle numbers of failure N_c ;

Δ – estimated values of cycle numbers before crack initiation for variant 4.

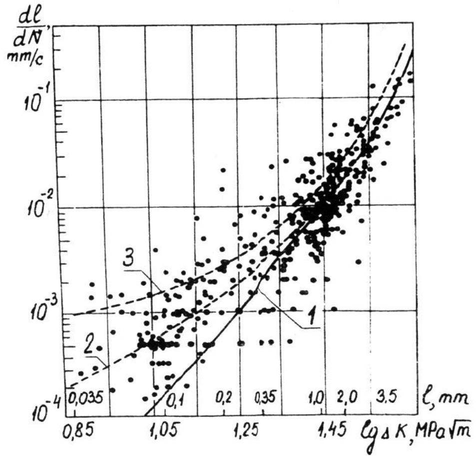

Fig. 4. Experimental (points) and estimated (lines 1,2,3) crack growth rates.