SHORT CRACK DEVELOPMENT IN MECHANICAL STRUCTURES

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ABSTRACT

Theoretical models of crack development based on linear fracture mechanics don´t adequately describe the real conditions under which the cohesion defects are originating and developing : such conditions are characterized by short crack systems as well as by interacting stresses in close critical zones. The paper aims at presenting some ideas of solving reliability problems in mechanical structures involving short cracks.

KEYWORDS

Mechanical systems; machine parts; reliability problems; fatigue mechanism; short crack development; life to safe crack estimation.

PHYSICAL BACKGROUND OF SHORT CRACK DEVELOPMENT

Relating to above mentioned theoretical models, the following main differences can be emphasized :

1. The fatigue crack initiation takes place within material macro-defects of various nature, the state of the material being defined by its structure, purity and the local damage intensity including residual- and backstresses in exposed zones. Should the fatigue crack start on body´s surface without considerable macro-scopic stress concentrations, the surface layers material properties would differ substantially from the material´s behaviour in the internal volume of body. As a rule, it has a different Young modulus, different chemical compositions of material and a different state of internal stresses by technology. As a consequence, fatigue cracks are starting sooner and propagating faster in respect to theoretical assumptions based on cyclic hardening and deformational energy deposition in standardly conceived material.

2. A substantial part of the lifetime of real bodies consists in connecting and developing short cracks of order values between 10^{-3} and 10^{-1} cm, whereby the damage cumulation processes in this stage involve some specific phases as follows :

a) Original forming of barriers against short crack propagation, the decrease of the velocity of the grown up cracks being due to cyclic strain hardening of initially weak material. This process is very inhomogeneously divided in micro-volumes depending on the material structure around the initial macro-defect in a body.

b) Short crack decelerating when leaving the higher stress field around the initiation zone, the extent of which approaches the size of macro-defects or sharp notches as well as the change in character of a deformation cycle in the root of a developing crack causing the shape shift of the deformation loops towards the pulsating one both when alternatively loading the body. This is the effect of closing the narrow fatigue crack faces by compression in the half of the loading cycle.

The theory of short crack development is not yet worked trough so far; generally, it is not possible to find a physical basis for it in linear fracture mechanics. The plastic deformation extent within the short cracks substantially exceeds the crack size itself, whereby one cannot assume a homogeneous deformation density. One cannot accept a model of isolated plastic zones in tips of small cracks and many interactions of small cracks and micro-defects of material structure have to be taken into account. There are no barriers formed by compression backstresses around plastic zones in front oß cracks as it is the case in grown up macro-cracks. Hence, short cracks grow below the value of the treshold stress intensity factor of stopping of macro-cracks.

The mentioned differences among simplified models and real structures are so sharply expressed that they hardly can be overlooked. The applicability of both linear fracture mechanics and theoretical fracture dynamics is therefore limited to developed macrocracks and hence, to a small part of the lifetime of real engineering structures, only, see Fig. 1. for reference.

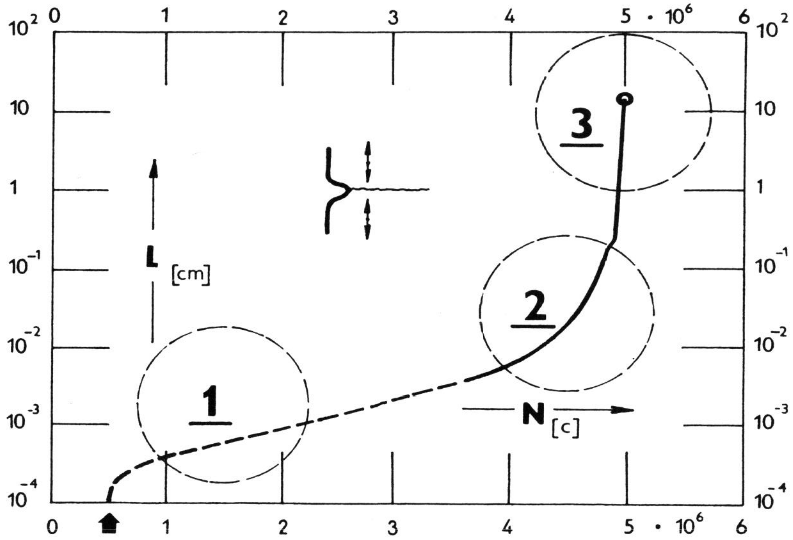

Fig. 1. A simplified crack development curve in high-cycle fatigue. **1** - micro-crack zone (grains) , **2** - short crack zone (surface layers), **3** - macro-cracks zone (cross section of bodies), crack initiation ⬆, **N** - cycle number.

This fact is shown in Fig. 2., too; this figure summarizes results of an analysis of fatigue crack development in driving axles of locomotives and in other bodies. Significant deviation from theoretical assumptions may be found in macro-crack propagation problem field, too, for example within thin-walled structures when exceeding deformation stability limit in region of developed macro-cracks or by impulse loading.

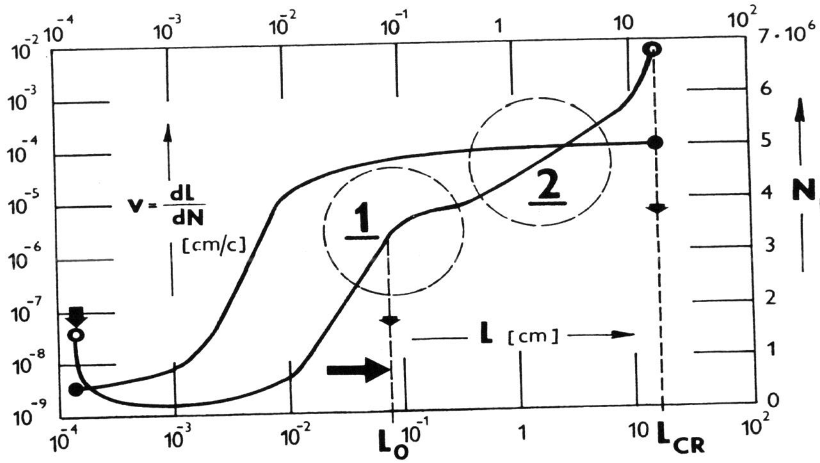

Fig. 2. An example of the fatigue crack grow in real structures : ⬇ crack initiation, **1** - notch effect zone , **2** - linear fracture zone, L_O- surface layer crack size, L_{CR}- critical crack length, **O** fracture.

It results from the preceeding discussion that a considerable lifetime percentage is covered by short crack propagation. A feasible formula for a crack length estimation may be derived as based on real case analysis giving the relation between the crack length L and loading cycle number N as follows

$$L = const.\exp\left\{k\cdot\left(\frac{N-N_0}{N_{CR}}\right)^{1/n}\right\} \qquad (1)$$

Involving the critical crack length varying by material ageing one gets

$$c\cdot\left(\frac{L_{CR}}{L}\right) = f\left(\frac{\sigma_a}{\sigma_{kt}}\right)\cdot\exp\left\{k\cdot\sqrt[n]{\frac{N_L}{N_{CR}}}\right\} \qquad (2)$$

whereby $8 \leq n \leq 12$, $N_L = N - N_0$.

A basic importance for practical application is to be attributed to a residual life map covering the body limit state defined by total fracture, see Fig. 3. Should such a map be known for every engineering structure, there would be possible to estimate its residual life on basis of known instantaneous crack length and supposed crack critical value. Assuming quasibrittle fracturing mode at the end of the damaging process, we obtain

$$\left(\frac{N^*}{N} - 1\right).100\,\% = const\left\{\ln\left(\frac{K_{Ic}}{K_a}\right)\right\}^n \qquad (3)$$

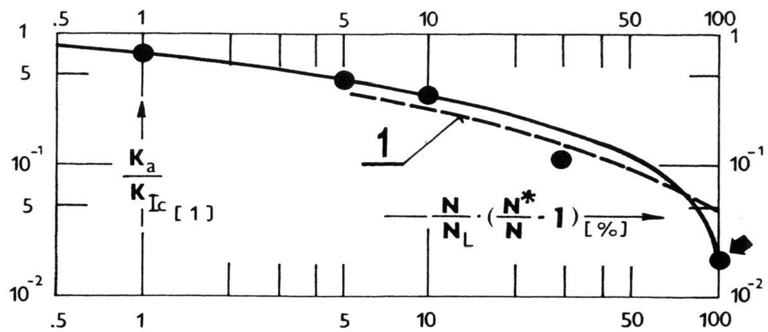

Fig. 3. Dimensionless presentation of equ.(3) relating residual life ($N^*/N - 1$) to acceptable crack length L and to stress intensity level K_a/K_{Ic} . 1 - sharp notch, ➡ crack initiation.

SHORT CRACK SYSTEMS IN AIRPLANE TURBINE DISKS

In several machine parts and mechanical structures an important interaction with decohesion processes is to be mentioned due to dynamic phenomena. Mass-, dissipative and stiffness properties of machines and structures under discussion participate on evoking couples in forces flows and hence, in stressing states of critical zones, see Fig. 4 presenting typical gas turbine disk. In the last figure right on the bottom , an interaction effect is suspected in crack occurences on the front and back sides of the fir-tree attachments (FTA) of the disk under investigation due to stiffness changes of the attachments when damaged by growing cracks.

In general, when solving operational reliability problems of machine parts and structures, three questions of basic importance are arrising as follows :
i) Which one may be taken as an adequate state description of a machine part or structure involving short cracks ?
ii) Which one is the adequate way of referring short crack characteristics to a structure reliability parameter ?
iii) What is the way of estimating the fatigue life of a structure involving a short crack system ?

It results from the preceeding paragraphe that there is no universal method satisfying all three questions under all possible variety of corresponding operational conditions and stressing states. Obviously, an acceptable solution of practical problems is to be searched for a specified machine type and a corresponding sample of operational characteristics, only.

In this connection, the experience as having been gained while investigating low-cycle fatigue resistance of airplane gas turbine disks yields some information answering the above questions.

Referring to questions i) and ii) : an adequate desription of a disk structure state involving short cracks in fir-tree attachments is to be related to the crack system as a whole, see the mentioned Fig. 4.; in this respect, the crack system is to be presented by a family of crack propagation curves. Statistical analysis of the problem

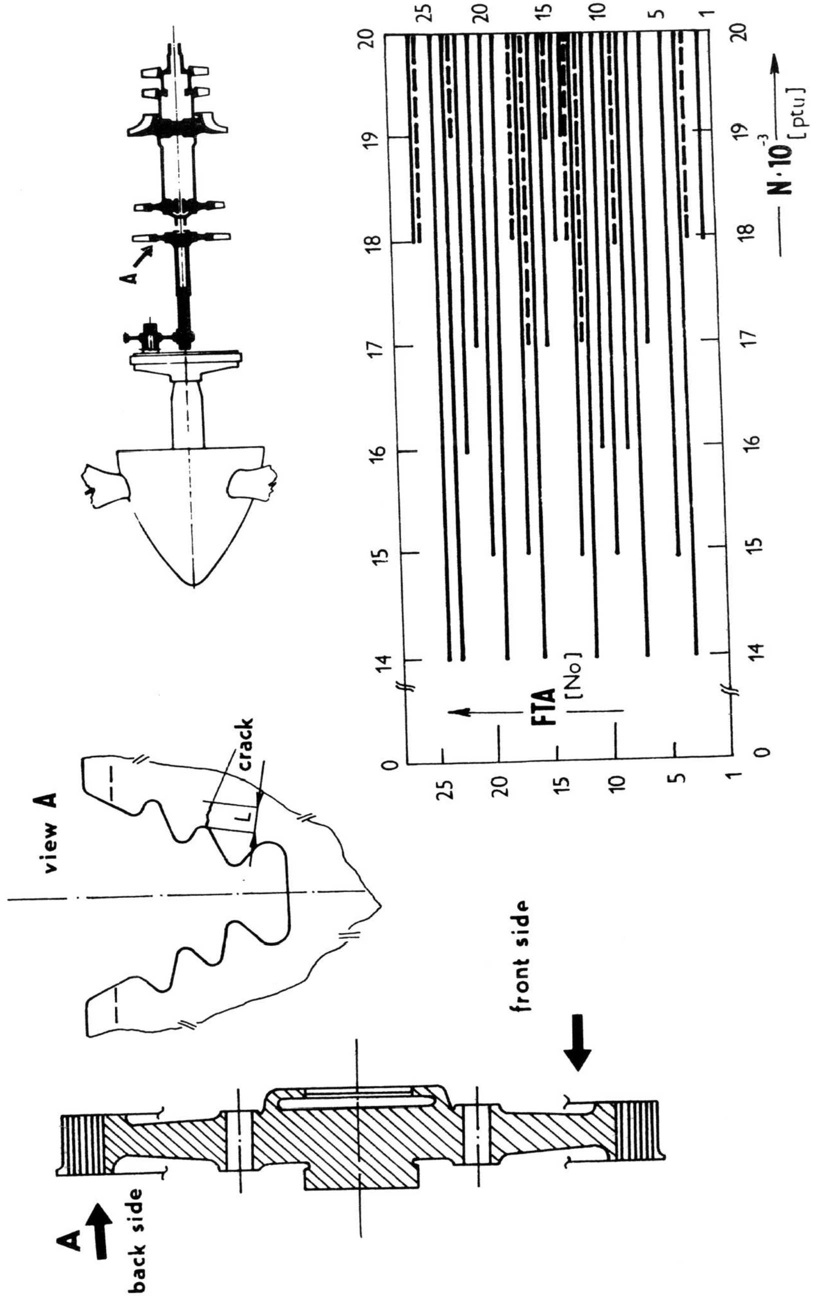

Fig. 4. Cracking story of the fir-tree attachment system (FTA) of a gas turbine disk, see right above for reference. Bottom right : FTA front (- - - -) and back side cracks (———) reveal interaction occurences due to dynamic effects of the turbine blades and to FTA stiffnesses changing with growing cracks.

of finding a representative state description of a machine part or structure, see the references to this paper, has prooved the adequacy of taking the following two of disk state parameters into consideration : the quantile crack length L_Q for $Q = 0.05$ and the total crack number on the disk n_c. Furthermore, the above analysis has shown the relating of L_Q to n_c as argument to be of good use from detection point of view , namely

$$L_Q = a\left[(1 - \exp\{b.n_c\}) + \frac{A}{a} \cdot \exp\{B.n_c\}\right] \quad (4)$$

where a , b , A , B are parameters describing the short crack propagation characteristics; the total number of cracks n_c is in function of the test programme unit number N as follows

$$n_{cj} = \mathrm{INT}\left[(m+1) \cdot \left(1 - \exp\left\{-\left(\frac{N_j - N_0}{\theta}\right)^{\beta}\right\}\right)\right] \quad (5)$$

m being the total number of fir-tree blade attachments to the disk, parameters θ and β presenting their technological quality level.

Referring to question iii) : technologically identical fir-tree attachments to the disk become - on the basis of equations (4) and (5) - through the n_c number a good measure of the residual life of the disk under question : i.e. of the lifetime up to the so called safe crack length grown to the value L_{FSC}, which may be expressed usefully through a L_Q length percentage. Using the idea of taking the real operation of the disk for an experiment being run simultaneously on all the fir-tree attachments of the blades to the disk, one can get the following relations for occurence probability of the safe crack number n_{FSC} related through equations (4) and (5) to the safe crack length L_{FSC} and to the safe number of programme test units

$$\mathrm{Prob}[n_{FSC} \cdot n_F ; m] = \sum_{j=1}^{n_{FSC}} c_j^m \cdot p_a^j \cdot (1 - p_a)^{m-j} \quad (6)$$

where crack occurence probability on an arbitrary disk fir-tree attachment p_a is estimated by

$$p_a = \frac{n_F}{m+1} \quad (7)$$

By analogy, we find the occurence probability of the total crack number n_F corresponding to the achievement of the catastrophical crack length L_F and to the N_F number of testing units to catastrophical damage to the disk. The disk operational life expressed in safe number N_{FSC} of programme test units may be estimated in the following way being in accordance with airworthiness requirements : for the last one of the testing and/or operational unit it is required for the probability of a catastrophical damage being brought to the gas turbine disk not to exceed the value $\mathrm{Prob}[N < N_{FSC}]$, i.e.

$$\mathrm{Prob}[N < N_{FSC}] = 1 - \exp\{-\lambda . N_{FSC}\} \geq \mathrm{Prob}[n_{FSC} / n_F ; m]$$

where $\lambda = 1.10^{-8}$, the conditional probability on the right side being given, using equations (6) and (7), by the expression

$$\mathrm{Prob}[n_{FSC} / n_F ; m] = \frac{\mathrm{Prob}[n_{FSC} \cdot n_F ; m]}{\mathrm{Prob}[n_F ; m]}$$

Hence, the acceptable life estimation meeting the airworthiness requirements for an airplane structure involving a short crack system follows from the formula

$$N_{FSC} \geq \lambda^{-1} \cdot \{-\ln(1 - \mathrm{Prob}[n_{FSC} / n_F ; m])\}$$

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