FATIGUE CRACK PROPAGATION UNDER VARIOUS TYPES OF LOADING

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ABSTRACT

A model of fatigue crack propagation was developed with consideration of the behaviour of crack opening and closing during the part of cycles and the limiting stress at which no cumulative cyclic plastic energy has occurred, namely "consuming stress". The effects of stress ratio and over- and/or under-spike load on fatigue crack propagation were analyzed by using the model. An example of \( \Delta K_{th} \) value under zero-to-tension stress condition was also calculated by using the model. The results show the good agreement with expriments.

KEYWORDS

Fatigue crack propagation; crack opening and closing; "Consuming stress"; stress ratio; over- and/or under-spike loading; Dugdale model; threshold stress intensity factor.

INTRODUCTION

Fatigue cracks remain closed during part of the load cycle under fatigue loading. Newman (Newman, 1984) introduced the calculation model of the crack opening and closing phenomenon which was based on the Dugdale model (Dugdale, 1960), but was modified to leave plastically-deformed materials along the crack surface as the crack advances.

In his model, he neglected the stress-redistribution on the crack line in the calculation of Elber's crack opening load (Elber, 1971). Moreover he also neglected the elastic deformation for the materials along the crack surface. In this paper, Newman's calculation model is modified to take those neglected problems into consideration. Moreover we considered that a fatigue crack does not propagate if a local part in the vicinity of a crack tip remains elastic, i.e., a plastic strain does not accumulate at a crack tip. This limiting stress is named "consuming stress \( S_{CS} \)" which is replacing and corresponding with a crack opening stress \( S_{OP} \) proposed by Elber.

Fatigue crack propagation tests using center notched tension test specimens were carried out under constant amplitude loading with various kinds of stress ratios and over- and/or under-spike loading conditions, for the purpose of confirming the effectiveness of our model.

CALCULATION MODEL OF CRACK OPENING AND CLOSING

The closure model is schematically shown in Fig. 1 which is similar to Newman's model\(^3\). For simplicity, a crack on an infinity wide plate subjected to uniform tension is analyzed.
The crack-opening displacement $V(x)$ for the configuration shown in Fig. 2 is given by

$$V(x) = \frac{2(1-\nu^2)S}{E} \sqrt{d^2-x^2} = S \cdot f(x)$$

where $\nu$ is the Poisson's ratio for plane strain and $\nu=0$ for plane stress. The crack-opening displacement $\delta$ in the configuration shown in Fig. 3 is given by

$$V(x) = \frac{2(1-\nu^2)h}{\pi E} \left[ \frac{1}{b} \cdot \text{cosh}^{-1} \left( \frac{1}{d} \sqrt{d^2-x^2} \right) - \frac{b}{d} \cdot \arctan \left( \frac{d}{b} \right) \right]$$

where $h$ and $d$ are the dimensions of the plate.

The equations which govern the response of the complete system are obtained by requiring that compatibility be met between the elastic plate and all of the bar elements which are exact elastic-plastic bodies along the crack surface and plastic-zone boundaries. The displacements are given by

$$\delta = \frac{V(x)}{S} = \frac{1}{S} \cdot \sigma \cdot g(b_i, x)$$

from Eq.(3), the displacements at the center of the j-th bar element is

$$\delta_j = \frac{V(x_j)}{S} = \frac{1}{S} \cdot \sigma_j \cdot g(b_i, x_j)$$

where $n$ is the total number of bar elements and $x_j = (b_j + b_{j+1})/2$. The value of $d$ is the fictitious crack length, that is, a half crack length plus tensile plastic zone, which is given as follows.

$$d = d_m + \frac{\pi S_{\text{max}}}{\sigma_y}$$

where $d_m$ is the maximum fictitious half crack length at a certain time and $S_{\text{max}}$ is the maximum stress at a certain time.

The gage length $L_j$ of the bar elements at $x_j$ remains constant given by

$$L_j = V(x_j) \left( 1 - \frac{\sigma_y}{E} \right)$$

The region of contact zone within the fictitious crack, $L_j$ corresponds to $V(x_j)$. From Eq.(4) and (8), following equations are obtained.

$$L_j = \frac{1}{S} \cdot \sigma_j \cdot g(b_i, x_i) \quad (j=1, ..., n)$$

where $\sigma_j$ becomes zero in the crack opening region. From Eq.(9), the following iterative equations are obtained:

$$\begin{align*}
[\sigma_j]_{k+1} &= \left[ \begin{array}{c}
[S] \cdot f(x_j) - \frac{1}{S} \cdot \sigma_j \cdot g(b_i, x_i) \\
\vdots \\
[S] \cdot f(x_n) - \frac{1}{S} \cdot \sigma_n \cdot g(b_i, x_n)
\end{array} \right] \\
&= \frac{1}{S} \cdot \left[ \sum_{j=1}^{n} \frac{1}{S} \cdot \sigma_j \cdot g(b_i, x_i) - L_j \right] / \left[ \sum_{j=1}^{n} \frac{1}{S} \cdot \sigma_j \cdot g(b_i, x_i) + L_j \right]
\end{align*}$$

where $(k+1)$ is the current iteration number. Equations (10) are solved with the following constraints:

For elements in the plastic-zone $(x_i > C)$, if $\sigma_j > \sigma_Y$, set $\sigma_j = \sigma_Y$.

For elements along the crack surface $(x_i \leq C)$, if $\sigma_j > 0$, set $\sigma_j = 0$.

These constraints are due to elements separation and compressive yielding. In solving Eq.(10), $V(x_j)$ in the plastic zone at the maximum stress has to be first obtained by using Dugdale model and then $L_j$ is set by Eq.(6). After that, crack growth is simulated by extending the crack by small incremental value. Making initial guess for $[\sigma_j]_{1}$, iterative calculations are made using equation (10).

At every iteration, the stresses $[\sigma_j]$ are checked against the constraint conditions (11) through (13) and are updated if necessary. This is repeated until $[\sigma_j]_{k+1}$ become $[\sigma_j]_{k+1}$, COD $V(x)$ is obtained by Eq.(3).

A fatigue crack may not propagate, if a local part of a crack tip, i.e., the bar element at the crack tip remains elastic, because a plastic strain does not accumulate. Let this limiting stress be called "consuming stress $S_{\text{CS}}$" in this paper. The numeral identifying the bar element at the crack tip is $m$ and the length of the bar element at $S_{\text{CS}}$ is

$$S_{\text{CS}} = L_m (1 + \frac{\sigma_y}{E})$$

Then from Eq.(9), the following iterative equations are obtained.

For $j \neq m$,

$$\begin{align*}
[\sigma_j]_{k+1} &= \left[ \begin{array}{c}
[S] \cdot f(x_j) - \sigma_m \cdot g(b_i, x_i) \\
\vdots \\
[S] \cdot f(x_n) - \sigma_n \cdot g(b_i, x_n)
\end{array} \right] \\
&= \frac{1}{S} \cdot \left[ \sum_{j=1}^{n} \frac{1}{S} \cdot \sigma_j \cdot g(b_i, x_i) - L_j \right] / \left[ \sum_{j=1}^{n} \frac{1}{S} \cdot \sigma_j \cdot g(b_i, x_i) + L_j \right]
\end{align*}$$

For $j = m$,

$$S_{\text{CS}} = \left( L_m (1 + \frac{\sigma_y}{E}) + \sigma_m \cdot g(b_m, x_m) \right) + \frac{1}{S} \cdot \left[ \sum_{j=1}^{n} \frac{1}{S} \cdot \sigma_j \cdot g(b_i, x_i) \right] / f(x_m)$$

The $S_{\text{CS}}$ value is calculated in the similar way of solving Eq.(10).

In the present computation, the plastic zone and the wake zone were arbitrarily divided into 25 graduated bar elements except the bar at the tip. The smallest element was located at the crack tip and had the width $(b_m - b_{m-1})$ of 0.0035mm which was sufficiently small to calculate $S_{\text{CS}}$ accurately.

Now let us consider the phenomenon of fatigue crack propagation for under- and over-spoke loading. Fig. 4 shows schematically crack opening and closing phenomenon during fatigue crack propagation under constant amplitude loading including an under-spoke loading. A local part ahead and in the vicinity of a crack tip is stretched in the transverse direction to a crack line due to tensile yielding at a maximum stress. When small amount of a crack grows at a maximum stress, elastic stress is partially released at the location of the newly generated crack surfaces and then, the new crack opens a little due to the released elastic stress at the maximum stress. Therefore the new crack may close at a little lower stress than the maximum stress under unloading process. Then the crack may close over the certain area in the vicinity of the crack tip at a minimum stress as shown in condition (3) in Fig. 4. Condition (2) corresponds to the state with the stress of $S_{\text{CS}}$ at which tensile plastic zone starts to be generated at the crack tip under loading process and then the effective stress range is given by the stress at condition (1) minus the stress at condition (2).
Fig. 4 Crack opening and closing during fatigue crack propagation due to under-spike loading

The crack opening leads to growing and stretching a compressive yielding zone in the contact zone at minimum stress for under-spike loading (see condition 5 in Fig. 4). The crack opens in the manner of having dents in the vicinity of the crack tip at the maximum stress due to the compressive yielding zone for the under-spike loading (see condition 6). Due to dents, the crack may still pen at the minimum stress (see condition 7). Under loading stress, then, a tensile plastic zone acts to be generated at a little higher than the minimum stress and $S_{Ct}$ level may be lower than that under stable crack propagation stage (see condition 8, cf. condition 2). The effective stress range, $max - S_{Ct}$, in this stage becomes larger than that in the stable crack propagation stage. The crack is then expected to be accelerated.

When the crack propagates further, the shape of the crack opening at the maximum stress may be of a cross-sectional shape of the top part of “Gothic tower”, as shown in condition (9). Unloading leads to crack closure, as shown in condition (10) and the crack might occasionally touch at a region part from the crack tip in addition to the vicinity of the crack tip. Just after reloading, stress may concentrate at A point in Fig. 4-Condition (10), which is inside the crack. Therefore $S_{Ct}$ may be larger than that at the stable crack propagation stage, as shown in stage (d) (see condition 11). Hence $max - S_{Ct}$, the effective stress range, at stage (d) is less than that at stage (a), and the delayed retardation of a fatigue crack propagation may occur.

Fig. 5 shows a schematic view of crack opening and closing phenomenon during fatigue crack propagation under constant amplitude loading including over-spike loading. Similar phenomenon to under-spike loading occurs in this case. But the effective zone due to over-spike loading is greater than that due to under-spike loading. As a result, the effect of overspike loading on fatigue crack propagation may be larger than that of under-spike loading.

**RESULTS AND ANALYSIS OF FATIGUE CRACK PROPAGATION TEST**

Figure 6 shows the specimen configurations and the material used is mild steel of which chemical compositions and mechanical properties are shown in Table 1. Table 2 shows the testing conditions including the conditions of constant amplitude loading with various stress ratio and with various spike loading.

The test results of constant amplitude loading with various stress ratios are shown in Fig. 7. It shows that fatigue crack propagation rates increase as stress ratios increase. Figure 8 shows the calculated $S_{Ct}$ for various stress ratios using above model. The results show that $S_{Ct}$ value increases as a crack advances until it reaches a certain length, and then increase in $S_{Ct}$.

### Table 1 Chemical composition and mechanical properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Composition (%)</th>
<th>Mechanical properties</th>
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<tbody>
<tr>
<td></td>
<td>C</td>
<td>Si</td>
</tr>
<tr>
<td>CCT-1</td>
<td>0.1</td>
<td>0.68</td>
</tr>
<tr>
<td>CCT-2</td>
<td>0.1</td>
<td>0.55</td>
</tr>
<tr>
<td>CCT-3</td>
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</tr>
<tr>
<td>CCT-4</td>
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<td>0.55</td>
</tr>
<tr>
<td>CCT-5</td>
<td>0.1</td>
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### Table 2 Test condition

<table>
<thead>
<tr>
<th>Test Condition</th>
<th>Stress (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCT-1</td>
<td>0.68 - 6.8</td>
</tr>
<tr>
<td>CCT-2</td>
<td>3.0 - 10.1</td>
</tr>
<tr>
<td>CCT-3</td>
<td>3.0 - 10.1</td>
</tr>
<tr>
<td>CCT-4</td>
<td>3.0 - 10.1</td>
</tr>
<tr>
<td>CCT-5</td>
<td>3.0 - 10.1</td>
</tr>
</tbody>
</table>

Fig. 6 Test specimen
becomes very gradual and finally $S_{cr}$ maintains almost constant as the crack advances. The sharp increase in $S_{cr}$ in early stage of crack propagation may lead to no existance of compressive plastic deformation at the first time in the wake zone. Figure 9 shows the relation between stress intensity factor range $\Delta K$ divided by yield stress and $U$ value which is the ratio of the effective stress range $\Delta S_{eff}$ to the applied stress range, for various kinds of materials with different yield stresses and Young's moduli. Figure 10 shows the re-arranged results of the fatigue crack propagation rate in the basis of the effective stress intensity factor range $\Delta K_{eff}$ ($\Delta S_{max} - S_{cr}/\sqrt{\pi a}$) (at, a): correction factor) by using Fig. 9. This results show that fatigue crack propagation rates are mainly given as a function of $\Delta K_{eff}$ and Fig. 9 is available to predict the effect of fatigue crack propagation rates on stress ratios.

Figure 11 shows the experimental fatigue crack growth rate of specimen CCT-2 in which a single over-spike loading is applied. Figure 12 is the calculated result of $S_{cr}$ for specimen CCT-2. From the calculated result of $S_{cr}$, $\Delta K_{eff}$ is obtained and the prediction of the fatigue crack propagation rate is given as shown by the solid line in Fig. 11 from Fig. 9 and Fig. 10. The predicted fatigue growth rate is in good agreement with the experimental fatigue growth rate except in the region of accelerated fatigue crack propagation. In this region, growth rate could not be measured because of the measurement interval of crack length of 1 mm in the test. Experimental results as compared with the predicted propagation life curve are shown in Fig. 13. The predicted propagation life is fairly in good agreement with experimental one.

Figure 14 shows the fatigue crack growth rate data and the predicted propagation rate for specimen CCT-3 in which a single under-spike loading is applied as shown in Table 2. It also shows that the predicted growth rate on the basis of $S_{cr}$ concept is in good agreement with the experimental one. It is clear by comparison between Fig. 11 and Fig. 14 that delayed retardation zone due to an under-spike loading is smaller than that due to an over-spike loading.

Figure 16 shows the consuming stress and opening stress changes with crack growth on an overloading or an overloading after an underloading.
Figure 15 shows the fatigue crack growth rate for specimen CCT-4 in which a single over-spike loading is applied and then an overload is applied just after an underload is applied.

The calculated result of $S_{cr}$ value for specimen CCT-4 is shown in Fig. 16. Calculated result of crack opening stress $S_{open}$, which corresponds to an applied stress at which stress acting the bar element at a crack tip in the model come to zero during re-loading process, is also shown in the figure. The general trend of the change of $S_{open}$ with crack growth due to spike-loadings seems to be similar to that of $S_{cr}$. Moreover the value of $S_{open}$ is almost the same as the $S_{cr}$, but is little less than $S_{cr}$. The prediction of the crack growth rate for specimen CCT-4 using the $S_{cr}$ value in Fig. 16 is shown by a solid line in Fig. 15 which is in good agreement with the experimental value.

Figure 17 shows the crack propagation life for specimen CCT-4. The experimental life agrees better with the predicted life due to $S_{cr}$ than that due to $S_{open}$.

Figure 18 shows an example of the calculated $S_{cr}$ value in the case of a decreasing step loading with the stress ratio of zero, simulating a $\Delta K_{th}$ test. As shown in Fig. 18, $S_{cr}$ value finally becomes equal to $S_{max}$ and then fatigue crack is supposed not to propagate any more. From the result, the $\Delta K_{th}$ value was obtained to be 22 kgl-$\cdot$mm$^{\frac{3}{2}}$ (6.8MPa $\sqrt{m}$). It seems that the calculated $\Delta K_{th}$ values are in good agreement with previous experimental ones.

From above results, our model is available to predict fatigue crack propagation life under various types of loading.

CONCLUSION

An analytical fatigue crack-closure model is developed and used in a crack-growth analysis program by proposing the concept which a fatigue crack does not propagate if a local part in the vicinity of a crack tip remains elastic and the limit stress is named "consumine stress $S_{cr}$" in place of Sopen concept by Elber to predict crack growth under various kinds of loading. The model was introduced to modify the analytical crack growth model by Newman. The proposed model needs only one fatigue crack growth data under constant amplitude loading for a material to predict fatigue growth life under other general loading. On the contrary, Newman’s model needs at least data from four loadings.

The present model correlated the constant amplitude data over a wide range of stress ratio quite well. The model also predicted well the effects of load interaction, such as retardation and acceleration. Moreover, $\Delta K_{th}$ value can be expected to be calculated by the model.

REFERENCES