FATIGUE CRACK PROPAGATION FROM AN INCLINED CRACK UNDER COMBINED MODE LOADING

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ABSTRACT

The fatigue crack propagation under combined mode loading has been experimentally investigated in an Al-alloy and a steel. A centre crack panel geometry loaded under uniaxial cyclic tension has been used with variation in crack angle from 30^{o}-90^{o}. The effects of crack size and stress amplitude on the direction of crack propagation and crack growth rate have been studied. The results have been compared with a theoretical prediction based on Sih's strain energy density factor approach. A fair agreement is found in the Al-alloy between the experimental and the theoretical results.

KEYWORDS

Fatigue crack growth, mixed mode loading, crack growth profile, crack angle, strain energy density factor.

INTRODUCTION

In a number of practical situations, the cracks or defects present in a component do not encounter loading on a pure mode. For example, a damaging defect or crack present in a structure at an angle with respect to the tensile loading axis meets mode I and II combined. The cracks may initiate on micro-structural inhomogeneties, inclusions, welding or casting defects and also under a hostile environment. At the tip of such cracks a mixed mode (combined mode I and II) loading prevails under an uniaxial tensile loading. Under a cyclic tensile loading situation, the Paris equation can no more be used to predict the rate of fatigue crack propagation from an inclined crack.

Few investigations are reported on combined mode (Mode I and II) fatigue crack growth in literature, (Roberts and Kilbur, 1971; Toor, Mori and Miyata, 1975; Tanaka, 1974; Hills and Ashelby, 1980; IIda and Kobayashi. 1969; Sih & Barthelemy, 1980, Badaliance, 1980, Sih, 1973, Patel and Pandey, 1981). The majority of the investigations however are computational in nature. The experimental data available are insufficient to substantiate the predicted path of fatigue crack propagation. The present investigation has been conducted with inclined crack panels in sheet materials to study the effect of crack angle, the crack size and the stress amplitude on the fatigue crack propagation rate, cyclic life and the crack growth profile. The results have been used to verify a crack propagation law (Patel and Pandey, 1981) based on the 'strain energy density factor (s) approach.

EXPERIMENTAL

Materials and Test Specimens

The materials used are an Al-alloy and a steel which are employed in the sheet form as aircraft structural materials. The strength properties of the alloys are given below:

L-72 Al-Alloy : 0.2 per cent Proof stress (σ_{ys}) 247 MPa,
Tensile strength - 386 MPa

4130 Steel : 0.2 per cent Proof stress (σ_{ys}) 520 MPa,
Tensile strength - 636 MPa

The preparation of final specimen from the initial one is shown in Fig. 1. Precracking was done by fatigue cycling using $\Delta\sigma/\sigma_{ys}$ = 0.155-0.191. The crack angle β has values 30,45 and 60^o and crack length $2a_o$ = 16,20 and 24 mm.

Fatigue Testing and Evaluation of Paris Constants

The specimens were tested in a high frequency vibrophore fatigue testing machine at a frequency of 90 Hz and stress ratio, R = 0. The aluminium specimens were tested at the stress amplitudes, σ/σ_{ys} = 0.156, 0.174, 0.191, and the steel specimens 0.155 and 0.191. The propagation of fatigue crack was monitored with a travelling microscope and number of cycles were noted for each 0.50 mm of crack growth. Testing was continued for a crack extension of around 9 mm from each crack tip and the crack growth trajectories were traced. Mode I crack growth rates were found to be as follows:

$$da/dN = 5.107 \times 10^{-10}(\Delta K)^{2.07} \quad \text{- Al-alloy} \qquad (1)$$
$$da/dN = 1.641 \times 10^{-13}(\Delta K)^{3.97} \quad \text{- Steel}$$

RESULTS

Fatigue Crack Growth Profile and Crack Propagation Direction

The average direction of crack propagation, θ_o was measured and its variation as a function of crack angle β is shown in Fig. 2. Following observations can be made:
(1) No distinct effect of crack length is noticed over the crack length investigated in θ_o value is noticed. (2) The stress range $\Delta\sigma$ does not seem to affect the θ_o significantly in the range of $\Delta\sigma/\sigma_{ys}$ = 0.156 - 1.191. (3)The^o θ_o is somewhat larger in case of steel as compared to the Al-alloy, especially for smaller β values. (4) As β increases, the value θ_o decreases. Almost in all the cases, the fatigue crack had a^o tendency to deviate from the original crack plane. In no case, the crack was found to grow in self-similar manner even during the initial stages contrary to the observation made in an earlier investigation (Tanaka,1974).

Determination of Crack Growth Rate

The experimental crack growth rate was determined in the following steps:- I. determination of 'effective crack length' a_{eff} and instantaneous crack growth θ_i at each increment of crack growth. II. obtaining a_{eff} vs N (Number of fatigue cycles) diagram. III. determination of da/dN for different values of a_{eff}. The growth of fatigue crack was taken as a number of discrete incremental steps of about 0.5 mm length. For each increment of crack growth, one has to consider the instantaneous crack length a_i and instantaneous crack angle β_i. Values of a_i and β_i were determined using the vectorial method (Patel and Pandey 1981) and corresponding θ_i values were also determined graphically. The a_i is taken as 'the effective crack length'. The da/dN as a function of a_{eff} (Fig. 3) was obtained.

Fatigue Crack Growth Rate as a Function of Various Parameters

Effect of crack length - Fig. 4 shows the variation of da/dN with angle β_o for three different crack lengths. Mode I crack propagation rate (β = 90^o) was also computed and shown in the diagram. It may be seen from the figure that the crack growth rate increases with β. Higher value of da/dN is observed in case of smaller crack as compared to the larger one. The da/dN increases by one order over a crack extension range of 0.5 to 5 mm in the steel. The increase in da/dN for the Al-alloy is less significant.

Effect of stress level - Fig. 5 shows the effect of stress amplitude on da/dN vs β diagrams. As expected, the da/dN is maximum for $\Delta\sigma/\sigma_{ys}$ = 0.191 and minimum for $\Delta\sigma/\sigma_{ys}$ = 0.156 (Al-alloy) and 0.155 (steel). Also, the crack growth rate in the Al-alloy is higher as compared to the steel for a given $\Delta\sigma/\sigma_{ys}$ value, during initial stages of crack propagation but in the later stages the reverse trend is observed.

Effect of crack angle and stress level on the cyclic life: The number of fatigue cycle decreases with increasing crack angle (Fig. 6). Also the number of cycles required for steel are larger than the Al-alloy for a given value of β and $\Delta\sigma/\sigma_{ys}$ ratio. Fig. 7 exhibits the cyclic life ratio (CLR) as a function of crack angle in both the alloys. The CLR is defined as the ratio of number of cycles for a specified amount of crack propagation (i.e. Δa = 5 mm) under combined mode loading to the number of cycles in the opening mode. With increasing β, the CLR decreases and approaches unity for β = 90^o, the CLR decreases and approaches unity for β = 90^o. Also, the CLR decreases with increasing stress amplitude. For very small values of β, the CLR on extrapolation, becomes extremely large. The CLR for the Al-alloy appears to be greater than the steel for $\Delta\sigma/\sigma_{ys}$ = 0.191.

DISCUSSION

Crack Propagation Angle

In literature, some attempts have been made to predict the direction of crack growth, while loaded monotonically (Sih, 1973; Erdogan and Sih, 1963; Williams and Ewing, 1972). Sih (1973) showed that the crack will grow in a direction along which the strain energy density is a minimum. The applicability of Sih's theory in fatigue crack propagation is yet to be examined. Using Sih's theory, the direction of crack extension for mixed mode loading (mode I + II) is obtained as,

$$2(1-2\nu)\sin(\theta_o-2\beta)-(2\sin 2(\theta_o-\beta))-\sin 2\theta_o = 0, \ \beta \neq 0 \qquad (2)$$

where ν is the Poisson's ratio. The values of crack propagation direction, θ_o obtained by solving equation (2) are reported in Table 1 and compared with experimental θ_o values.

TABLE 1. Comparison of Experimental θ_o with the Theoretical θ_o values (Sih's theory)

	Al-Alloy		Steel	
β	Experimental θ_o	Theoretical θ_o	Experimental θ_o	Theoretical θ_o
30°	58-72°	63.5°	62-77°	63.5°
45°	44-57.5°	53°	54-64°	53°
60°	33.5-40.5°	41.5°	32.5-42°	41.5°

From Table 1, it may be seen for the Al-alloy that angle, θ_o as computed from S approach agrees fairly well with the experimental values for small crack angles (β=30°, 45°). For larger crack angles, the predicted value of θ_o is greater than the experimental values. For the steel, the Sih's theory appears to underestimate the crack growth angle, θ_o in the regime of smaller β values (β = 30°, 45°). However, for larger β values, (β = 60°), the S approach predicts a higher value of θ_o than the one obtained experimentally.

Crack Growth Profile

The fatigue crack growth profile can be obtained theoretically using the S approach by taking the values of a_i and β_i, using the vectorial method (Fig. 8). The crack growth profile as predicted by S approach is also shown in Fig. 8. The crack propagates by varying angle θ and it has a tendency to approach the opening mode situation. Nevertheless, over a great deal of length, the crack propagates in mixed mode state. The experimental crack has however, an innate tendency to approach the opening mode without undergoing significant amount of growth in a mixed mode manner.

Experimental vs Theoretical Crack Growth Rate

Recently, a criterion for mixed mode crack growth under cyclic loading has been proposed (Patel and Pandey, 1981) by combining, strain energy density factor for static loading and the Paris equation. The da/dN may be expressed as,

$$\frac{da}{dN} = c\left(\frac{4\pi\mu}{1-2\nu}\cdot\Delta S\right)^{n/2} \qquad (3)$$

where c and n are the Paris constants, μ is the shear modulus and ΔS is the range of strain energy density. The ΔS value was determined by substituting the appropriate expressions for K_I and K_{II} for the angled crack (Brown and Sraweley, 1966) and corresponding f(g) value as,

$$f(g) = 1.0.1(2a_{eff}/W_{eff})+(2a_{eff}/W_{eff})^2 \qquad (4)$$

where a_{eff} and W_{eff} are the effective values of crack length and width. W_{eff} is the width of the specimen along the a_{eff} direction. The expression for ΔS was obtained as,

$$\Delta S=f(g)\Delta\sigma^2 \cdot a_{eff}(a_{11}\sin^2\beta+2a_{12}\sin\beta\cos\beta+a_{22}\cos^2\beta)\sin^2\beta \qquad (5)$$

Since the da/dN (experimental) at a given a_{eff} is known, the same is plotted against corresponding ΔS value in Fig. 9. From the above figures, it may be noticed that the experimental points are fairly scattered around the theoretical line in case of the Al-alloy. Thus, there is a fair agreement between the experimental data and the theoretical predicted values. However, in case of steel, the predicted crack growth rate is significantly larger than the experimental one. Therefore, the S approach does not appear to predict the fatigue crack growth rate in steel satisfactorily.

CONCLUSIONS

1. The average direction of fatigue crack growth is independent of crack size, stress amplitude and has a tendency to move towards mode I situation after some growth.
2. The θ_o can be predicted in the Al-alloy using the S approach for smaller crack angles ($\beta \leq 45°$). However, the same does not seem to be applicable in the steel for any value of β.
3. The da/dN increases with β, $\Delta\sigma$, and the crack extension, Δa. The Al-alloy shows higher values of da/dN than the steel for a given crack angle and stress amplitude.
4. A criterion for mixed mode fatigue crack growth based on S approach may be used to predict the da/dN in the Al-alloy within reasonable limits. This however overestimates the crack growth rate in the steel.
5. The CLR for a specified amount of crack growth increases with $\Delta\sigma$ and decreases with crack angle β, and $\Delta\sigma$ in both the alloys.

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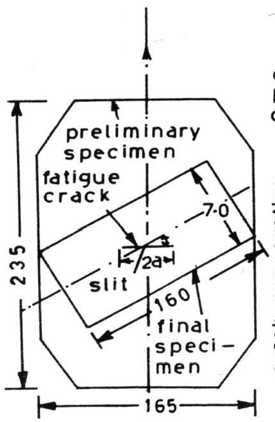

Fig-1 Geometry of test specimen

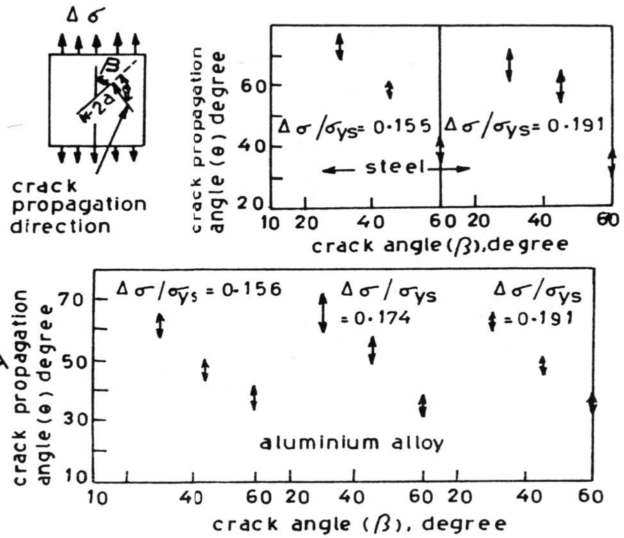

Fig. 2 Crack propagation angle (θ) as a function of β [$2a_0$ = 16-24 mm, R=0]

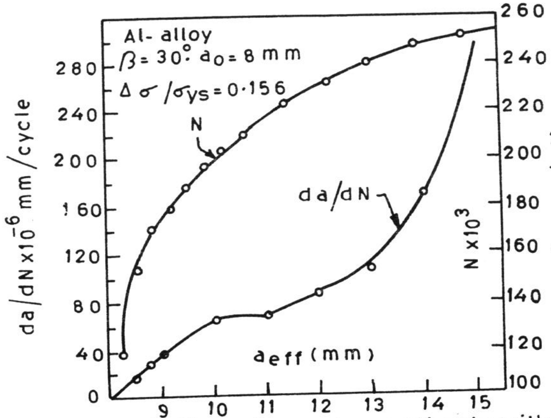

Fig.3 Variation of crack growth rate with effective crack length

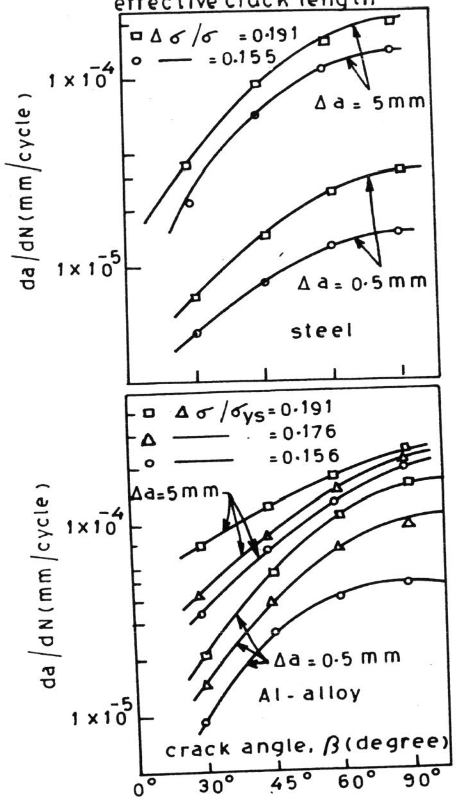

Fig 5 Effect of stress amplitude on rate of crack propagation

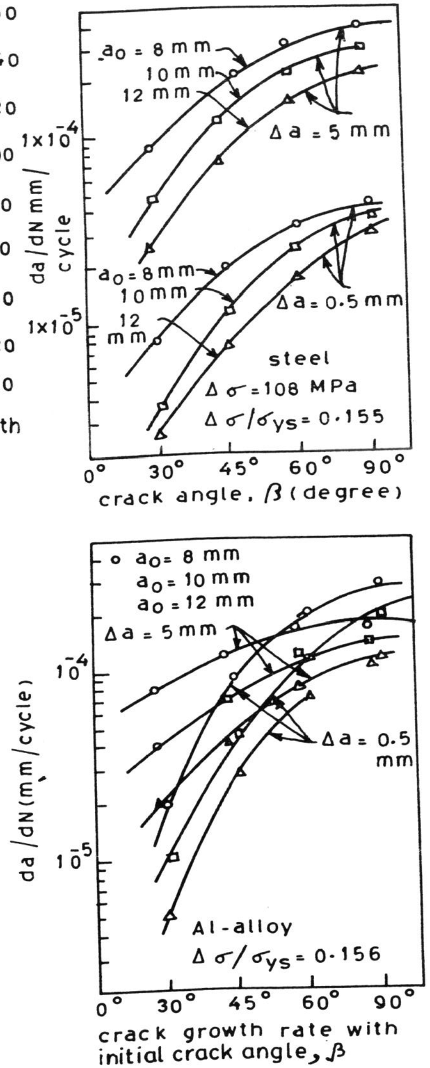

Fig. 4 Variation of crack growth rate with crack angle, β

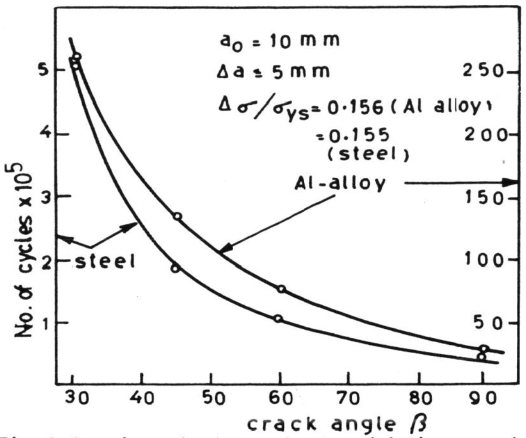

Fig.6 Crack angle dependence of fatique cycle

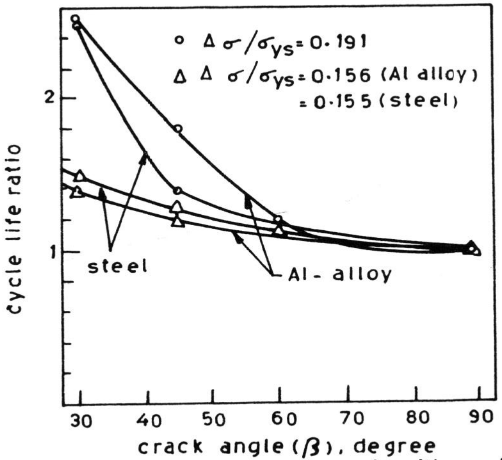

Fig. 7 Variation of cycle life ratio with crack angle (β) $a_0 = 8$ mm, $\Delta a = 5$ mm

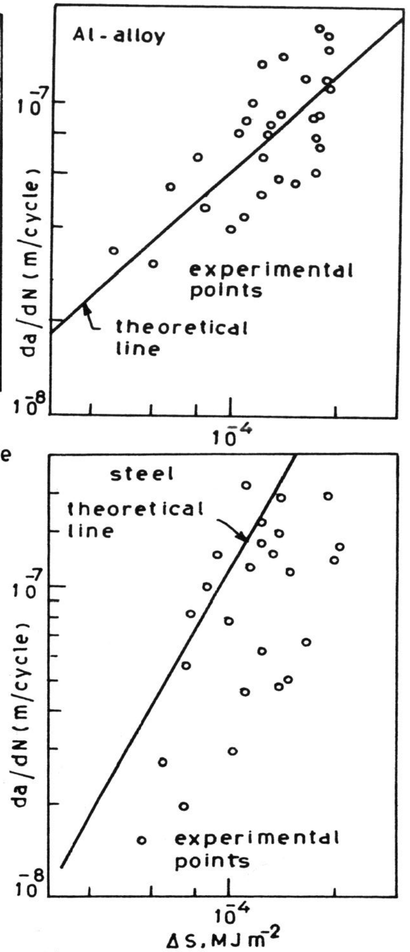

Fig.9 Crack growth rate as a function of strain energy density range.

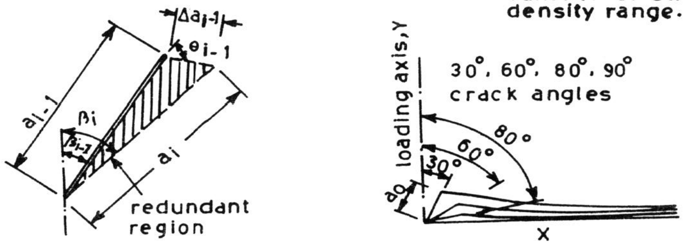

Fig.8 Determination of crack growth profile