EFFECT OF VARIATION OF STRESS FREQUENCY ON FATIGUE CRACK PROPAGATION IN TITANIUM

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ABSTRACT

The effect of variation of stress frequency on fatigue crack propagation in 99.5 % pure titanium, which has remarkable strain rate dependence in the plastic region, was studied. Fatigue crack propagation tests were carried out under four stress frequencies : two constant stress frequencies (20Hz, 0.02Hz) and two stress frequencies changed step-wise (20→0.02Hz, 0.02→20Hz). An elasto/visco-plastic analysis of fatigue crack propagation was performed by the finite element method (FEM), and the comparison between the change of crack propagation rate due to variation of stress frequencies and the visco-plastic strain behavior at the crack tip calculated by the analysis was made. The results obtained in this study are summerized as follows :

(1) It was found from the experiments that the crack propagation rate was characteristically changed depending on the variation of stress frequency.

(2) A parameter closely related to the fatigue crack growth rate is the visco-plastic strain range at the crack tip.

(3) The effect of variation of stress frequency on the fatigue crack propagation rate may be explained by variation of the visco-plastic strain range at the crack tip based on the strain rate dependence of the material.

KEYWORDS

Fatigue crack propagation; variation of stress frequency; titanium;elasto/visco-plastic overlay model; FEM; visco-plastic strain range.

INTRODUCTION

As for studies on fatigue crack propagation, the effect of stress frequency on the crack propagation rate has been almost all ignored except for the case of consideration of atmospheric or environmental effect and the results of the crack propagation test under a constant stress frequency have been applied to practical structures in spite of the fact that various loading rates act on them. There is a trend toward the disregard of stress frequency effects, considering them in general to be small in the usual testing rate range. Some materials, however, have remarkable strain rate dependence even at room temperature, and it is very important to make clear the effct of frequency on the crack propagation rate for such materials from not only a scientific viewpoint but also a practical standpoint.

In the previous paper (Takezono and Satoh, 1982), the authors gave attention to the viscosity of the material in plastic region as a cause of an observed frequency effect on the fatigue crack propagation rate, in 99.5 percent pure titanium, a material which has remarkable strain rate dependence in plastic region. In addition a FEM elasto/visco-plastic analysis of fatigue crack propagation was performed. As a result it was found that the crack propagation rate, depended on loading frequency and was approximately in inverse proportion to f^n ($n > 0$), where is f is the frequency. It was also found that if the experimental fatigue crack propagation rate is plotted against visco-plastic strain range obtained by elasto/visco-plastic analysis, the relation can be expressed by a straight line in logarithmic coordinates for any loading frequency and stress amplitude.

In the present paper, the change of crack propagation rate for step-like variation in stress frequencies was observed in 99.5 percent pure titanium, and FEM elasto/visco-plastic analysis of fatigue crack propagation was performed. In this analysis, the elasto/visco-plastic overlay model (Takezono, Tao and Kanezaki, 1980) was used, which is able to express the Bauschinger effect and the strain rate dependence in plastic region.

FATIGUE CRACK PROPAGATION TESTS

Material and Specimen. The material used in this investigation was 99.5 percent pure titanium in plate form. The chemical composition is given in Table 1. Uni-axial tension test specimens and crack growth specimens were cut out from the titanium plate in such a manner that the loading direction coincided with the direction transverse to the rolling direction. Uni-axial tension tests under various strain rates ($\dot{\varepsilon}$ =1.14×10^{-5}~1.56×10^{-1}1/s) were performed. The stress-strain relations were measured by strain gauges pasted on the surface of the specimens. The stress-strain relations obtained are shown in Fig.1. It is found that the stress level increased with higher strain rate in the plastic region. (The broken lines in Fig.1 will be explained later.)

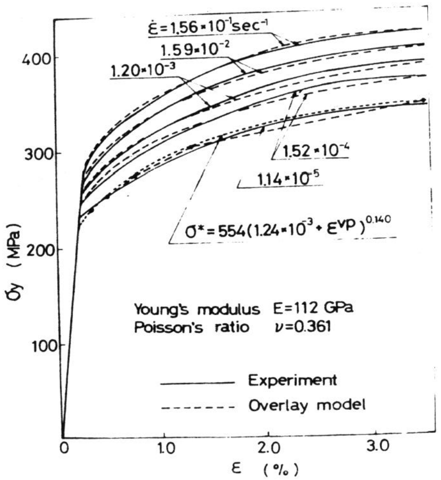


Fig. 1. Tensile stress-strain curves.

Table 1. Chemical Composition (Wt %).

Fe	N	O	H
0.038	<0.007	0.073	0.0026

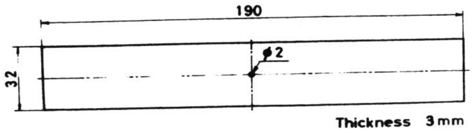


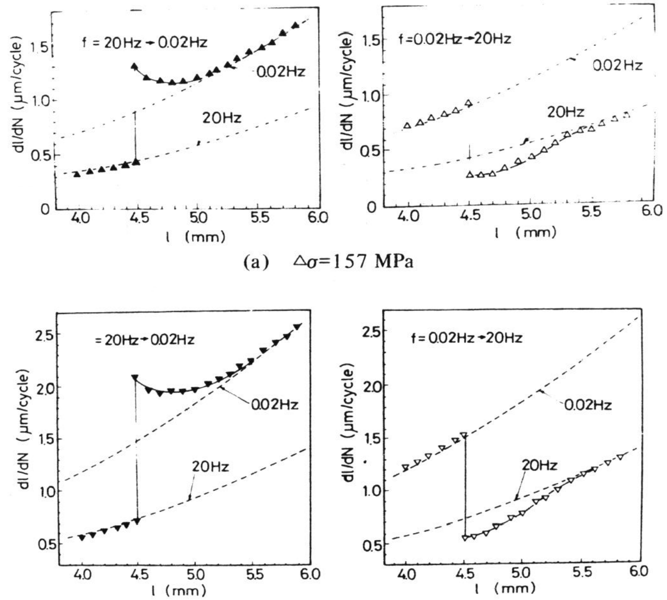
Fig. 2. Geometry and dimensions of the crack-growth specimen (in mm).

The geometry and dimensions of the crack-growth specimen are shown in Fig.2. All specimens were annealed under argon atmosphere at 540°C for one hour after machining. A circular hole of 2 mm in diameter was made at the center of the specimen to serve as a crack starter. The surfaces of the specimens were polished with No. 1500 Emery paper and alumina polishing suspension for microscopic observation.

Experimental Method. Fatigue tests were performed in an electro-hydraulic servo-type fatigue testing machine. In this study, pulsating loads in tension with four loading frequencies, which were two constant loading frequencies (20Hz, 0.02Hz) and two loading frequencies changed step-

wise (20→0.02Hz, 0.02→20Hz), were chosen to observe the effect of variation of frequency on fatigue crack propagation. The tests were conducted with the stress amplitude, $\Delta\sigma = \Delta\sigma_{max}$ = constant ($\Delta\sigma_{max}$: maximum nominal stress of specimens without a crack). The crack was observed and its length on a surface of the specimen was measured with a travelling microscope. In the case of 20Hz, crack length was measured by synchronizing stroboscope.

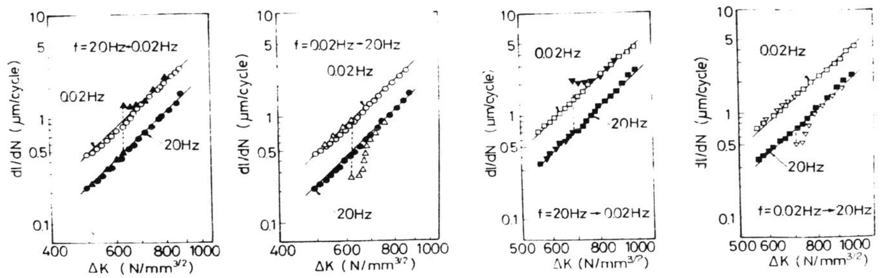
Experimental Results. Figures 3 (a), (b) show the relations between the crack propagation rate, dl/dN obtained from crack propagation curves (l - N curves) and crack length. Figures 4 (a), (b) plot the relations between dl/dN and the stress intensity factor range, ΔK. The value of ΔK was calculated from the following equation for the case of a specimen with a center crack (Tada, 1971).



(a) Δσ=157 MPa

(b) Δσ=176 MPa

Fig. 3. Relation between crack propagation rate (dl/dN) and l.



(a) Δσ=157 MPa (b) Δσ=176 MPa

Fig. 4. Relation between crack propagation rate (dl/dN) and ΔK.

$$\Delta K = \Delta\sigma\sqrt{\pi l}(1-0.025\lambda^2+0.06\lambda^4)[\sec(\pi\lambda/2)]^{1/2}, \qquad \lambda = 2l/W \tag{1}$$

where W is the width of the specimen. For each value of $\Delta\sigma$, it was observed that the crack propagation rate depended on the frequency, i.e., for a low value of f, a high crack propagation rate was obtained for the same ΔK value.

In the case when the stress frequency was changed step-wise from 20Hz to 0.02Hz, it was seen that the propagation rate became higher than natural one of 0.02Hz immediately after changing and approached it with increase in ΔK or l. Inversely, changed from 0.02Hz to 20Hz, it was seen that the propagation rate became lower than natural one of 20Hz and approached it with increase in ΔK or l.

ELASTO/VISCO-PLASTIC ANALYSIS FOR FATIGUE CRACK PROPAGATION

In order to introduce strain rate dependence and the Bauschinger effect in plastic region, we employ the elasto/visco-plastic overlay model (Takezono, Tao and Kanezaki, 1980), which applies the Perzyna equation (Perzyna, 1966) for the constitutive relation of each layer of the overlay model proposed by Zienkiewicz, Nayak and Owen (1972) as shown in Fig.5. In the plane stress state assumed in this analysis the constitutive relations of each layer may be expressed as follows:

$$\begin{Bmatrix} \dot\varepsilon_x \\ \dot\varepsilon_y \\ \dot\gamma_{xy} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix}\begin{bmatrix} \dot\sigma_{xk} \\ \dot\sigma_{yk} \\ \dot\tau_{xyk} \end{bmatrix} + \begin{Bmatrix} \dot\varepsilon_{xk}^{vp} \\ \dot\varepsilon_{yk}^{vp} \\ \dot\gamma_{xyk}^{vp} \end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix} \dot\varepsilon_{xk}^{vp} \\ \dot\varepsilon_{yk}^{vp} \\ \dot\gamma_{xyk}^{vp} \end{Bmatrix} = \gamma_k\langle\Phi(F_k)\rangle\frac{3}{2\bar\sigma_k}\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & 0 \\ -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{Bmatrix} \sigma_{xk} \\ \sigma_{yk} \\ \tau_{xyk} \end{Bmatrix} \tag{3}$$

and

$$\dot\varepsilon_{zk}^{vp} = \gamma_k\langle\Phi(F_k)\rangle\frac{3}{2\bar\sigma_k}\left(-\frac{1}{3}\sigma_{xk} - \frac{1}{3}\sigma_{yk}\right) \tag{4}$$

where E is Young's modulus, ν is Poisson's ratio. The dot denotes partial differentiation with respect to time, Δt is time increment and the index k means the kth layer component of overlay model. The symbol $\langle\Phi(F_k)\rangle$ is defined as follows :

$$\langle\Phi(F_k)\rangle = 0;\ \text{when } F_k \leqq 0, \quad \langle\Phi(F_k)\rangle = \Phi(F_k);\ \text{when } F_k > 0 \tag{5}$$

where function F_k is :

$$F_k = (\bar\sigma_k - Y_k)/Y_k, \qquad \bar\sigma_k = (\sigma_{xk}^2+\sigma_{yk}^2-\sigma_{xk}\sigma_{yk}+3\tau_{xyk}^2)^{1/2} \tag{6}$$

and $F_k = 0$ denotes the von Mises yield surface, $\bar\sigma_k$ is the equivalent stress and Y_k is the statical yield stress. Multiplying the stress $\{\sigma_{xk}, \sigma_{yk}, \tau_{xyk}\}$ by the thickness t_k of the kth layer and summing up these values in each layer, the stresses are obtained as follows :

$$[\sigma_x, \sigma_y, \tau_{xy}] = \sum_{k=1}^{n}[\sigma_{xk}, \sigma_{yk}, \tau_{xyk}]t_k, \qquad \sum_{k=1}^{n} t_k = 1 \tag{7}$$

Suppose that the visco-plastic strain rate, $\{\dot\varepsilon_k^{vp}\}$ is constant in the time increment Δt, then the increment of visco-plastic strain, $\{\Delta\varepsilon_k^{vp}\}$, is calculated from the next equation

$$\{\Delta\varepsilon_k^{vp}\} = \{\dot\varepsilon_k^{vp}\}\Delta t \tag{8}$$

where Δt should be selected by taking into consideration the stability of the solution. In this analysis, Δt was selected by referring to the next calculation stability limit by Zienkiewicz and Cormeau (1974) :

$$\Delta t \leqq \frac{4(1+\nu)}{E}\,\frac{\gamma_k}{(d\Phi/dF_k)} \tag{9}$$

The model which consists of 6 layers was employed in this analysis. First, the quasi-static stress σ^* was expressed in consideration of experimental data for the case of the smallest strain rate ($\dot\varepsilon$ = 1.14×10^{-5}1/s) shown in Fig.1 as follows :

$$\sigma^* = 554(1.24\times10^{-3} + \varepsilon^{vp})^{0.140} \tag{10}$$

Next, the yield stress Y_k and the thickness t_k of each layer was calculated from this equation and viscosity constant γ_k and the function $\Phi(F_k)$ in equation (3) were determined as follows on the basis of the experimental data :

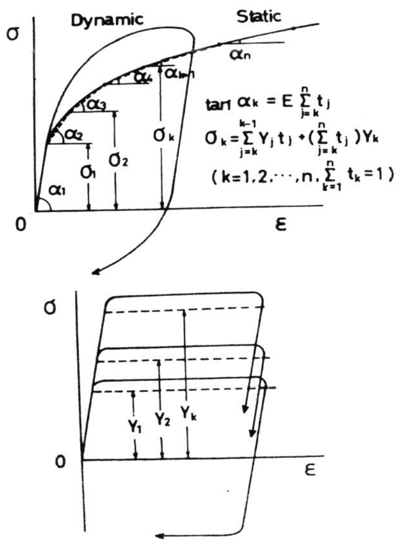


Fig. 5. Elasto/visco-plastic overlay model.

$$\left.\begin{aligned} Y_k &= Y_1 \sim Y_6 = 222,\ 395,\ 633,\ 1032,\ 1977,\ 7476 \quad \text{MPa} \\ t_k &= t_1 \sim t_6 = 0.887,\ 0.038,\ 0.026,\ 0.021,\ 0.018,\ 0.010 \\ \gamma_k &= \gamma_1 \sim \gamma_6 = 500,\ 400,\ 300,\ 200,\ 100,\ 50 \\ \Phi(F_k) &= \left(\frac{\bar\sigma_k - Y_k}{Y_k}\right)^{7.5} \end{aligned}\right\} \tag{11}$$

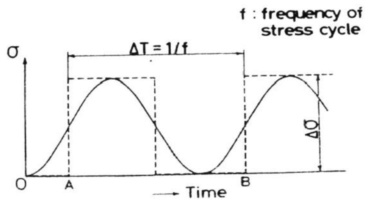


Fig. 6. Stress wave in experiment and calculation.

The stress-strain curves calculated from the overlay model are shown by broken lines in Fig.1. It is seen from Fig.1 that the strain rate dependence of the material is well approximately by equations (11).

In the case of an incremental analysis for elasto/visco-plastic calculation by the finite element method, the relation between the apparent nodal force increment, $\{\Delta f^*\}$ and nodal displacement increment, $\{\Delta\delta\}$ is obtained by the use of $\{\Delta\varepsilon^{vp}\}$ in equation (8) as follows :

$$\{\Delta f^*\} = [K]\{\Delta\delta\}, \qquad \{\Delta f^*\} = \{\Delta f\} + Ah[B][D^e]\{\Delta\varepsilon^{vp}\} \tag{12}$$

where $\{\Delta f\}$ is the increment of actual nodal force, $[K]$ is the stiffness matrix of an element, A is the area of triangular element, h is the thickness of element, $[B]$ is the strain-displacement matrix, and $[D^e]$ is the elastic stress-strain matrix.

Although the stress wave in the fatigue crack propagation tests was sinusoidal as shown in Fig.6, a rectangular stress wave as shown by the broken line was employed for simplification of the calculation. In this figure, point A is the start of calculation and 1 cycle is closed at point B. In this analysis it was assumed that a crack was extended in each loading cycle by a prescribed length equal to the side length of a crack tip element. The assumption is similar to that of other studies not taking account the viscoplasticity (Ogura and Ohji, 1977).

ANALYTICAL RESULTS AND DISCUSSION

Figure 7 shows stress-strain hysteresis loops under cyclic strain obtained by experiment and calculated from isotropic hardening model and overlay model. The experimental loops are stable ones after dozens cycles. It is seen from Fig.7 that overlay model used in this analysis is more actual than isotropic hardening model. Although there is a little difference between experimental loops and loops from the overlay model in the vicinity of reloading point, because of lowering of yield point of the material due to cyclic loading, it seems that the parameters of the overlay model obtained by uni-axial tension test (Fig.1) are appropritiate in general also in the cyclic loading.

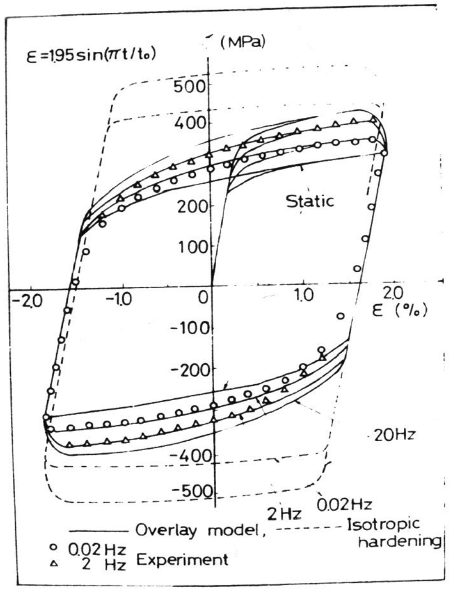


Fig. 7. Stresses under cyclic strain.

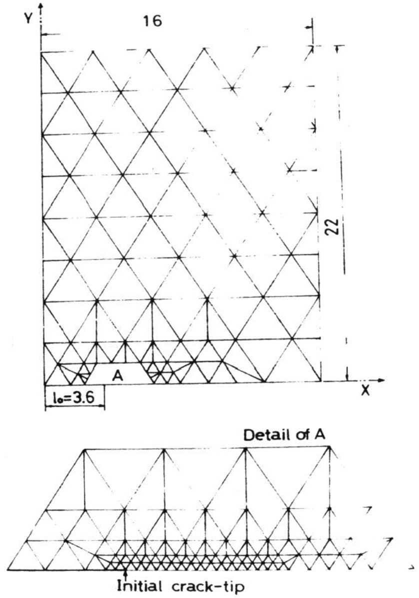
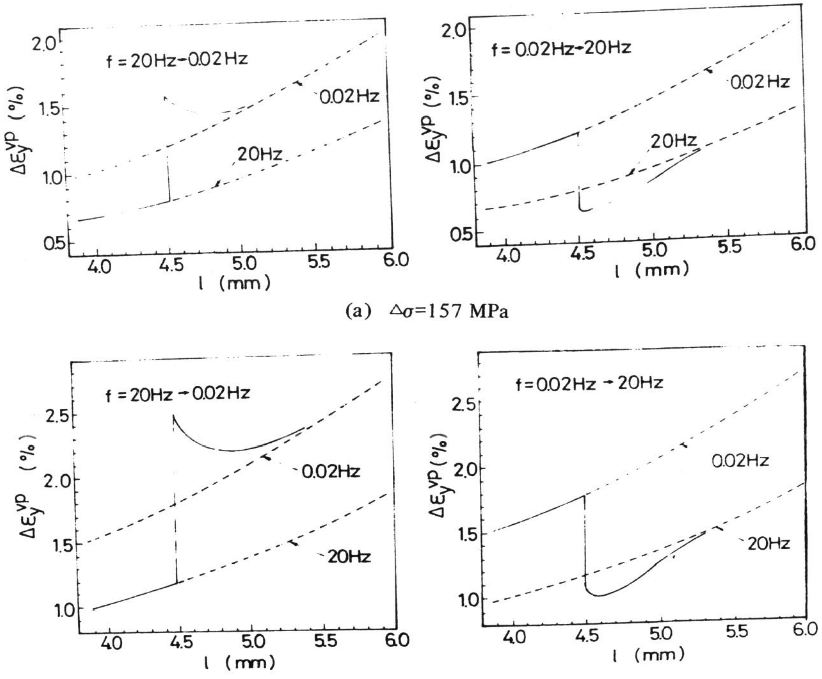


Fig. 8. Example of finite element subdivision.

Since it is difficult in terms of computing time to calculate the crack propagation continuously from the start, the calculations for the case of several initial crack lengths, l_0, have been carried out. As an example, the finite element subdivision in the case of l_0 =3.6 mm is shown in Fig.8. The size of the elements around the crack tip is 0.1 mm in side length.

Figuers 9 (a), (b) show relation between crack length, l and the crack tip visco-plastic range, $\Delta\varepsilon_y^{vp}$ obtained by the elasto/visco-plastic analysis. When the loading frequency is changed step-wise from 20Hz to 0.02Hz, $\Delta\varepsilon_y^{vp}$ is higher than natural one of 0.02Hz immediately after changing and approach natural one with increase in l. Inversely, changed from 0.02Hz to 20Hz, $\Delta\varepsilon_y^{vp}$ is lower than natural one of 20Hz and also approaches it with increase in l.



(a) $\Delta\sigma$=157 MPa

(b) $\Delta\sigma$=176 MPa

Fig. 9. Relation between $\Delta\varepsilon_y{}^{vp}$ and l.

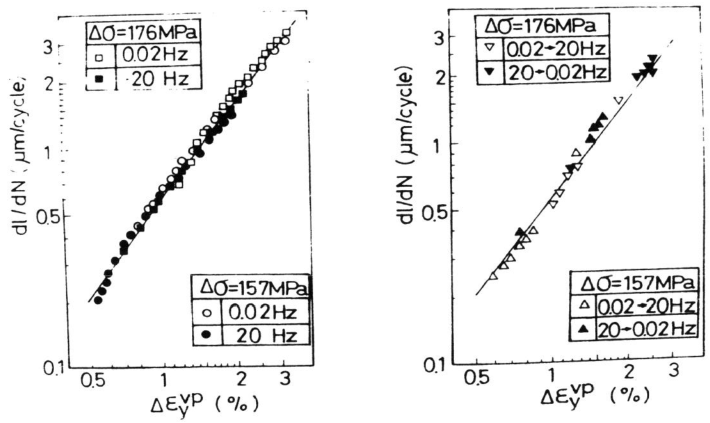


Fig. 10. Relation between dl/dN and $\Delta\varepsilon_y{}^{vp}$.

These relations correspond well with the trend of the dl/dN curves in Figs.4 (a), (b). The facts mentioned above suggest that there is a probability of expressing methodically the fatigue crack propagation rate in terms of $\Delta\varepsilon_y^{vp}$.

Figures 10 (a), (b) show the relation between dl/dN and $\Delta\varepsilon_y^{vp}$ in the case of constant stress frequencies and stress frequencies changed step-wise, respectively. The solid lines in Figs. 10 (a), (b) coincide. If dl/dN is plotted against $\Delta\varepsilon_y^{vp}$, the relation follows a straight line in logarithmic coordinates and the data fall in narrow range regardless of loading frequency, its variation and stress amplitude.

From Figs. 10 (a), (b) we conclude that the experimental fatigue crack propagation rate is closely related to the crack tip visco-plastic strain range, $\Delta\varepsilon_y^{vp}$. Consequently the dependence of the fatigue crack propagation rate on variations of loading frequency, where there is no emvironmental effect, is well explained by variations of $\Delta\varepsilon_y^{vp}$ at the crack tip based on visco-plasticity of the material.

CONCLUSIONS

Fatigue crack propagation tests in 99.5 percent pure titanium, a material having a strong strain rate dependence, were carried out for four loading frequencies (20Hz, 0.02Hz, 20→0.02Hz, 0.02→20Hz). In addition, FEM elasto/visco-plastic analysis of fatigue crack propagation was performed and the effect of variations of loading frequency on the fatigue crack propagation rate was investigated. Some of the conclusions drawn from the present study are as follows :

(1) As a result of fatigue crack propagation tests in 99.5 percent pure titanium plate, it was found that the crack propagation rate was characteristically changed depending on the variation of loading frequency.

(2) The experimental fatigue crack propagation rate, dl/dN , is closely related to $\Delta\varepsilon_y^{vp}$ obtained by elasto/visco-plastic analysis. If dl/dN is plotted against $\Delta\varepsilon_y^{vp}$, the relation can be expressed by a straight line in logarithmic coordinates for any loading frequency, its variation and stress amplitude.

(3) The effect of step-wise variation of stress frequencies on fatigue crack propagation rate, may be explained by variation of the visco-plastic strain range, $\Delta\varepsilon_y^{vp}$ at the crack tip based on material viscoplasticity.

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