CRACK GROWTH IN THE CREEP REGION CRITERIA BASED ON MATERIAL FORCES

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ABSTRACT

The meaning of the C^* concept is discussed. It is shown that this concept is only a global approach of the material force rate concept. The field of the material force rate density is more representative of creep crack growth than the C^* integral. As examples, corrected expressions of C^* are proposed for non isothermal cases and strain hardening creep.

KEYWORDS

Creep Crack growth, Material forces, C^* integral, Path Integral, Fracture Mechanics.

INTRODUCTION

Below the creep region, only the onset of crack propagation was considered at first. Then stable crack propagation was taken into account. There are similar features in the creep region. At first the criterion employed for elevated temperature design has been that the time for crack initiation should exceed the design life. As crack free structures do not exist in practice, more attention is now paid to predicting the crack growth-behaviour under creep conditions. Unfortunately, such a prediction is not easy for the designer, because several parameters have been proposed for characterizing the creep growth rate behaviour of metals (Ellison and Harper, 1978).

To-day, the parameter C^* (Landes and Begley, 1976) seems one of the most popular for correlations of experimental data (Saxona, 1980), but its field of validity is not well defined. Important studies have been made about this point (Riedel and Rice, 1980; Riedel and Wagner, 1981; Ainsworth, 1982). Most of them are related to the strain rate and stress fields near the crack tip and consider very simple formulations for creep behaviour (Like Norton's law). Therefore it seems helpful to give a more general interpretation of the C^* concept, in the same way as J concept (Roche, 1977, 1981, 1983).

VIRTUAL WORK AND CREEP BEHAVIOUR

Principle of Virtual Work

One of the most powerful tool in Mechanics is the principle of virtual work. Equilibrium is obtained if stress working variation is equal to external forces virtual work. But this principle is true only if continuity of the material is satisfied (displacement and strain variations fulfill equations of compatibility). This is not the case of fracture mechanics and a generalized form of the principle of virtual work must be used (Roche, 1977). A simple writting can be obtained in using "generalized forces and displacements" X_a and u_α such that $X_\alpha\ \delta u_\alpha$ is the work variation of external forces

$$dV = \int_V \delta W\ dv = X_\alpha\ \delta u_\alpha - J_\beta\ \delta a_\beta \qquad (1)$$

a_β being parameters describing material properties perturbation related to crack advance (a_β can be called generalized material displacement, and J_β generalized material force). If $\delta W = \sigma_{ij}\ \delta\varepsilon_{ij}$ is the stress working density, $J_\beta\ \delta a_\beta$ is the work due to the perturbation characterized by a_β.

Such a formulation of the principle of virtual work give an exact definition of the J concept. Unfortunately there is a great number of a_β and J_β. Therefore the current practice is to make the following assumption : the perturbation created by crack advance is (fairly) characterized by the value of only one parameter : the crack extension. This leads to the practical, but rough approximation $\delta U = X\ \delta u - J\ B\ \delta a$. This give for the crack growth rate

$$\dot{a} = (X\ \dot{u} - \dot{V})/JB \qquad (2)$$

Quasi Static Problems (Washizu, 1975)

When the time rate of change of external forces and displacement is so gradual that inertial forces can be neglected, it is obvious that the principle of virtual work can be formulated in the conventional manner, except that the time now appears as a parameter. The problem shall be expressed in terms of rate. Therefore, will be considered displacement rate $\dot{u}_i$ and strain rates $\dot{\varepsilon}_{ij}$ and W^* such $\delta W^* = \sigma_{ij}\ \delta\dot{\varepsilon}_{ij}$ is substitued to the strain working density

$$\delta V^* = \delta U^* \quad \text{where} \quad \delta V^* = \int_V \delta W^*\ dv \qquad \delta U^* = \int_S X_i\ \dot{u}_i\ ds \qquad (3)$$

This is only true if the variations of strain rate and displacement rate satisfy conditions of compatibility. Such conditions being not satisfied in case of crack growth, the quasi static formulation must be generalized.

Meaning of W^* and U^*

In many publications W is called strain energy and W^* strain energy rate. Such a practice can lead to misunderstanding. *W^* is not the rate of W* ($\dot{W} = \sigma_{ij}\ \dot{\varepsilon}_{ij} \neq W^*$). The same remark can be made about U^* which is sometimes called "power" "energy rate" or "creep energy dissipation rate". It must be pointed that *U^* is not the power of external forces*. This power is $\dot{U} = X\dot{u}$ which is different from U^* (Harper and Ellison, 1977).

When Norton's law is considered for creep ($\dot{\varepsilon} = B\sigma^n$) it is easy to see that W^* is a state function equal to $n\dot{W}/(n+1)$ and that $U^* = n\dot{U}/(n+1)$. It is the author's opinion that these differences between W^* and $\dot{W}$, U^* and $\dot{U}$ are very important and explain that C^* is not the rate of J.

MATERIAL DISPLACEMENT CONCEPT - MATERIAL FORCE RATE

Spatial and Material Displacements

A particle of the body is identified by its cartesian coordinates x_i in the initial state. This is a Lagrangian formulation, x_i coordinates are only identifying a particle of material. Due to the action of external force X_i, this point is displaced, reaching $x_i + u_i$ in an external referential, and exhibits a state of strain rate ε_{ij} and a state of stress σ_{ij}. It must be pointed out that u_i is a geometrical or spatial displacement (a conventional one), and x_i and $x_i + u_i$ are initial and present coordinates related to an external referential linked for instance to the walls of the laboratory (spatial coordinates). On the contrary, the coordinates of this point are yet x_i if related to the continuum itself (material of referential coordinates).

Now, perturbations are considered as possible in the continuum itself. It is to say that the initial organisation can be changed and that the material coordinates of one particle can change. From the initial material coordinate x_i, the particle goes to a point of material coordinates $x_i + a_i$ (a_i can be called material displacement). A trivial example of such a situation can be given by pastry making. If dried fruits are included in cake dough, these dried fruits can present some displacement before baking. Their displacement is a spatial (or conventional displacement) if it is related to the cake mold, it is a material displacement, if it is related to the initial state of the dough. In other words, material displacements are corresponding to a flow of material properties through the body (including holes and cracks).

Material Force Rate

Due to a virtual material displacement δa_k, there is some change in the distribution of ε_{ij} and W^* in the body. The change in the strain rate is obviously equal to $\dot{\varepsilon}_{ij,k}\ \delta a_k$ leading to a change of W^* equal $\sigma_{ij}\ \dot{\varepsilon}_{ij,k}\ \delta a_k$. Part of this change is due to δa_k acting as a spatial displacement, i.e. $W^*_{,k}\ \delta a_k$. Finally the variation of W^* caused by material variation only (and not taken in account by conventional virtual work) is given by[1]

$$\delta W^* = -\ c^*_k\ \delta a_k \quad \text{where} \quad c^*_k = W^*_{,k} - \sigma_{ij}\ \dot{\varepsilon}_{ij,k} \qquad (4)$$

Regarding material displacement a_k, c^*_k is acting as a force rate density and hence can be called *material force rate density*.

c^*_k is volume density of material force rate. If surface discontinuities exist in the body, surface density $\bar{c}^*_k$ can appear. When the material displacement δa_k crosses a surface discontinuity, the variation of displacement is finite

[1] A comma followed by suffixes will denote differentiation with respect to x, for instance $\dot{\varepsilon}_{ij,k} = \dfrac{\delta\dot{\varepsilon}_{ij}}{\delta x_k}$.

$\Delta u_{i,k}\,\delta a_k$ in this direction and $\delta\Delta W^* = T_i\,\Delta u_{i,k}\,\delta a_k$ where $T_i = \sigma_{ij}\,n_j$. Part of this variation is only caused by spatial displacement $\Delta W^*\,n_k\,\delta a_k$. Therefore the material variation is

$$\delta W^* = -\,\bar{c}_k^*\,\delta a_k \quad \text{where} \quad c_k^* = \Delta W^*\,n_k - T_i\,\Delta\dot{u}_{i,k} \qquad (5)$$

It is useful to pay attention to the exact definition of stress and strain. If u_i, $\dot{u}_i$, ε_{ij} and $\dot{\varepsilon}_{ij}$ are small there is no problem and conventional definitions can be used. If they are not small (Hill, 1959), displacement gradient must be chosen as strain $\varepsilon_{ij} = u_{i,j}$; and stress is the Boussinesq nominal stress tensor (so that $\delta W = \sigma_{ij}\,\delta u_{i,j}$ and $\delta W^* = \sigma_{ij}\,\delta\dot{u}_{i,j}$).

GENERALIZED FORMULATION OF VIRTUAL WORK PRINCIPLE FOR QUASI STATIC PROBLEMS - INTRODUCTION OF C^*

General Formulation

In the previous section, it has been shown that material disorders like holes and cracks evolutions, etc. can be identified by the material displacement concept. The resulting variation of W^* has been computed, leading to the definition of material force rate c_i^* (volume density) and $\bar{c}_i^*$ (surface density). It is now possible to write the quasi static principle of virtual work when compatibility conditions are not satisfied.

$$\delta V^* = \delta U^* - \int_V c_i^*\,\delta a_i\,dv - \int_\Sigma \bar{c}_i^*\,\delta a_i\,ds \qquad (6)$$

where Σ is discontinuity surface (like cracks or material slip lines).

Generalized Material Force Rate

Knowledge of the field of material displacement is needed for using equation (6). Practically this field can be expressed as a function of a finite number of parameters a_β (called generalized material displacement), and equation (6) can be written like eq. (1)

$$\delta V^* = \delta U^* - C_\beta^*\,\delta a_\beta \qquad (7)$$

where C_β^* are generalized material force rates. This show that creep crack growth would be described by several parameters C_β^*, but the current practice is using only one.

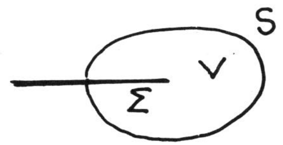
Tridimensional Expression of C^*

It is necessary to define the virtual material displacement field corresponding to the type of perturbation considered. The simplest way is a translation of material properties indicated by δa_i having the same value δa_i in each point. Applying eq. (6) leads to

$$\delta V^* = \delta U^* - C_i^*\,\delta a_i \qquad (8)$$

with

$$C_i^* = \int_V \bar{c}_i^*\,dv + \int_\Sigma \bar{c}_i^*\,ds = \int_S (W^*\,n_i - T_j\,u_{j,i})\,ds \qquad (9)$$



The integral C_i^* relating to a surface S is the resultant of all the material forces rate (volume density and density on discontinuity surfaces Σ) included in the volume V surrounded by S.

Below the creep region it is possible to introduce the energy momentum tensor (Eshelby, 1970). In the creep region it is possible to introduce what correspond to a stress rate tensor (energy momentum rate tensor)

$$P_{kj}^* = W^*\,\delta_{kj} - \sigma_{ij}\,\dot{u}_{i,k} \qquad (10)$$

Conventional C^* (Plane Case)

Current practice is to apply the C^* concept to plane cases with straight crack along x axis. Therefore crack growth is assumed to be a translation δa of material displacement field in x direction (consequently δa is the virtual crack growth). In writting $C^* = C_1^*/B$ where B is the thickness

$$\delta V^* = \delta U^* - C^*\,B\,\delta a \qquad (11)$$

$$C^* = \int_\Gamma W^*\,dy - \vec{T}\,\frac{\partial\vec{\dot{u}}}{\partial x}\,ds$$

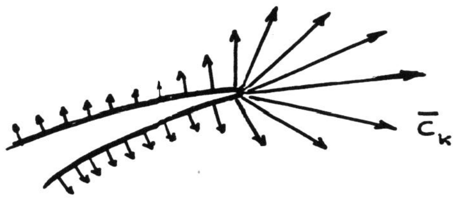
It must be pointed out that such a definition of C^* is only the consequence of a rough approximation (the material perturbation is fully described by crack advance).

Equation (11) can be written like eq. (2) to give an expression of the crack growth rate. When a Norton's law is considered, mixing of eq. (2) and (11) shows that C^* is $n/(n+1)$ times the rate of J.

PATH-INDEPENDENCE OF C^* INTEGRAL

Condition of Path-Independence

The preceding results show that C^* concept is only a consequence of the distribution of material force density c_i^* in the material. Obviously the field of c_i^* in the vicinity of the crack tip is a better indication of the process than C^*. It must be pointed out that on the free surface of crack, there is a field of surface density $\bar{c}_i^*$ which is normal to the surface, the intensity being higher near the crack tip.



As done for the J integral it is useful to consider the conditions of path independence of C^*. Such a property seems to lead to a characterization of stress and strain rate fields near the crack tip. On applying of the preceding equations the condition of path independence is the resultant of material force rate c_i^* equal to zero for all volumes except those including the crack edges. This leads to the condition

$$c_k^* = W^*_{,k} - \sigma_{ij}\ \dot{\varepsilon}_{ij,k} = 0 \qquad (12)$$

in every point of the body (except a small process zone in the vicinity of the crack tip).

Conventional Viscosity

When W^* is a state function, the stress is given by $\sigma_{ij} = \partial W^*/\partial\dot{\varepsilon}_{ij}$. If W^* is only function of the strain rate and of the time (time hardening) $\sigma_{ij}\ \dot{\varepsilon}_{ij,k} = W^*_{,k}$ and the condition (12) is met.

If the material is purely viscous, C^* is not path dependent.

This is the case when the material creeps following a Norton's law $\dot{\varepsilon} = B(t)\ \sigma^n$. Then the expression of W^* is

$$W^* = \dot{\varepsilon}^m_{eq}/m\ B^{1/n} \quad \text{where} \quad \dot{\varepsilon}_{eq} = \sqrt{(2\ \dot{\varepsilon}_{ij}\ \dot{\varepsilon}_{ij}/3)} \quad \text{and} \quad m = (n+1)/n.$$

If creep damage ω is considered (Katchanov, 1960), Norton's law can be written $\dot{\varepsilon} = B(t)\ \sigma^n(1-\omega)^{-n}$, density of material force rate is equal to $\omega_{,k}\ \partial W^*/\partial\omega = \omega_{,k}\ W^*/(1-\omega)$. Therefore C^* is path dependent in the area where the damage ω is strongly variable as a function of x (more exactly where Log $(1-\omega)$ is strongly variable).

Strain Hardening Effect

If strain hardening is considered, W^* is not only depending of the strain rate, but also of the strain. Therefore $W^*_{,k} - \sigma_{ij}\ \dot{\varepsilon}_{ij,k}$ is equal to $\varepsilon_{ij,k}\ \partial W^*/\partial\varepsilon_{ij}$. Therefore C^* is path dependent, but this effect is not significant for creep crack growth. In order to appraise the effect at the crack tip, C^* must be corrected in the following way

$$C_k^*(\text{cor}) = C_k^* - \int_V \frac{\partial W^*}{\partial\varepsilon_{ij}}\ \varepsilon_{ij,k}\ dv \qquad (13)$$

In plane cases, with Norton's law, the corrected value of C^* is written

$$C^*_{\text{corrected}} = C^* - \int_\Gamma W^* \frac{\partial B}{B\partial\varepsilon_{eq}}\ \frac{\partial\varepsilon_{eq}}{\partial x}\ ds \qquad (13')$$

When strain hardening is considered, C^* is path dependent and using a corrected expression of C^* is advised.

Non isothermal bodies

It is well known that B (in Norton's law) is temperature dependent and so must W^* be. Therefore if temperature θ is not uniform in the body, $W^*_{,k} - \sigma_{ij}\ \dot{\varepsilon}_{ij,k} = \theta_{,k}\ \partial W^*/\partial\theta$ and C^* is path dependent. This effect is spurious and does not concern conditions of crack growth. It can be eliminated in using a corrected value of C_k^*

$$C_k^*(\text{corr}) = C_k^* - \int_V \frac{\partial W^*}{\partial\theta}\ \theta_{,k}\ dv \qquad (14)$$

In usual plane configuration, using Norton's law, the corrected value of C^* is given by

$$C^*(\text{corr}) = C^* + \int_\Gamma \frac{W^*}{n}\ \frac{\partial B}{B\partial\theta}\ \frac{\partial\theta}{\partial x}\ ds \qquad (14')$$

If temperature is not uniform in the body, C^* is path dependent. In order to assess crack tip state a corrected value can be used.

Creep and Elasticity

Elastic behaviour of material cannot always be neglected and it is sometimes necessary to consider elastic strain rate $\dot{e}$ in addition[2] of creep strain rate $\dot{v}$. If $\phi(e)$ is the elastic energy density and $\psi(\dot{v})$ a scalar function characterising creep behaviour[3], stress is given by $\sigma_{ij} = \partial\phi/\partial e_{ij} = \partial\psi/\partial\dot{v}_{ij}$. In such a case W^* is not longer a state function and is depending of the real history

$$W^* = \psi + \int_0^t \sigma\ddot{e}dt = \psi + \dot{\phi} - \int_0^t \dot{\sigma}\ \dot{e}\ dt \qquad (16)$$

Therefore the expression of C_k^* is given by

$$C_k^* = \int(\sigma_{,k}\ \ddot{e} + \sigma\ddot{e}_{,k})\ dt - \sigma\dot{e}_{,k} = \dot{\sigma}e_{,k} - \int(\dot{\sigma}_{,k}\ \dot{e} + \dot{\sigma}\ \dot{e}_{,k})dt \qquad (17)$$

and C_k^* is rarely path independent. If σ (and e) are non time dependent (stationnary creep) C_k^* is not path dependent.

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[2] For the sake of simplicity, subscript will be eliminated, e, $\dot{v}$, σ are tensor notations, $\sigma.e$ the contracted product of σ and e $(\sigma_{ij}\ e_{ij})$ and $e_{,k}$ the partial derivative related to x_k.

[3] If Norton's law is considered, the expression of ψ is $\psi = \dot{v}^m_{eq}/m\ B^{1/n}$ where $m = (n+1)/n$ and $\dot{v}_{eq} = \sqrt{(2\ \dot{v}_{ij}\ \dot{v}_{ij}/3)}$.

CONCLUSIONS

In the creep region, quasi static formulation of the principle of virtual work leads to the introduction of functions like $W^* = \int \sigma \, d\dot{\varepsilon}$ which are not equal to the rate of straining work $W = \int \sigma \, d\varepsilon$. This seems the reason why C^* is not the rate of J.

As continuity conditions are not met in crack growth a generalized formulation of this principle must be used. This formulation leads to the introduction a material displacement field a_k, characteristing the evolution of material properties. This displacement is the cause of an additional work equal to its product by material force density c_k^*.

If the material displacement field can be quoted by a finite number of parameters a_β (generalized displacement), the additional work is equal to $C_\beta^* a_\beta$ where C_β^* are generalized material forces characterising the crack growth.

When it is assumed that the crack tip advance is suffisant to characterise the material displacement field, there is only one parameter which is C^*.

It is shown that C^* is the resultant of all the material forces rate included in the region surrounded by integral contour.

This allows to study the path dependence of C^* and to make a relation between conventional C^* concept and the material force rate in the vicinity of the crack tip (process zone). As a consequence corrected expressions of C^* are given, especially when strain hardening is considered and for non isothermal bodies.

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