CORRELATION OF TWO CONSTANTS IN THE PARIS EQUATION FOR FATIGUE CRACK PROPAGATION RATE IN REGION II

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ABSTRACT

The fatigue crack growth rates of 2024-T4 and 2024-T6 aluminum alloys and of commercial purity aluminum cold worked by 40% and 60% have been measured at stress ratio, R, being 0, 0.3 and 0.6. The data in region II followed the Paris relation $dc/dN = A(\Delta K)^m$. When the crack length and stress intensity factor in the Paris equation were measured in meter and $MN/m^{3/2}$, the values of A and m of the alloys followed the relation of log A = a + bm where a = -6.74, b = -1.04 for R = 0 and a = -7.09, b = -0.85 for R = 0.3 indifferent to different compositions and thermomechanical treatments of the alloys. Antolovich's model for the fatigue crack growth rate could account for the behavior reasonably well.

KEYWORDS

Fatigue crack growth rate; effect of stress ratio; the Paris equation; Antolovich's model; Aluminum alloys.

INTRODUCTION

The fatigue crack growth rate in region II can be well described by the Paris equation(Paris, Erdogan, 1963)

$$\frac{dc}{dN} = A(\Delta K)^m \qquad (1)$$

where

$\frac{dc}{dN}$ = fatigue crack growth rate

ΔK = stress intensity factor range ($K_{max} - K_{min}$)

A and m = constants that depend on material, environment, frequency, temperature, and stress ratio.

Recent experimental results showed that there exist a relation between A and m (Kitagawa, Misumi, 1971, 1982; Hickerson, Hertzberg, 1982; Tanaka, Matsuoka 1977; Ichikawa, Takura, Tanaka, 1980; Tanaka, Ichikawa, Akita, 1981; Ishii, Yukawa, 1979; Niccolls, 1976; Kim, Lee, 1981). The relation may be given by

$$\log A = a + bm \tag{2}$$

where a and b have the same values for the same kind of alloys, i.e., ferrous alloys, copper alloys or aluminum alloys, independent of their thermomechanical treatments. There exist substantial experimental data supporting Eq. (2) for ferrous alloys. However, we feel need of more experimental tests of the equation for non-ferrous alloys, and there has been no theoretical explanation of Eq. (2). The purpose of this work is to search a model which can account for the equation and is in more experimental tests of the equation for aluminum alloys.

EXPERIMENTAL METHOD

2mm thick sheets of 2024-T4 and T6 aluminum alloys and commercial pure aluminum cold rolled by 40% and 60% were tensile tested and fatigue tested at the stress ratios of 0, 0.3 and 0.6 using center crack specimens. The crack length was measured by a travelling microscope. All the fatigue tests were performed at the frequency of 25Hz in air. The datailed experimental set up and data processing can be found in a reference (Kim, Lee, 1981)

RESULTS AND DISCUSSION

The tensile properties of the materials are summerized in Table 1. Least

TABLE 1 Tensile Properties of Aluminum Alloy Specimens

Alloys	σ_o	So	σ_u	Su	ε_f	Elongation(%)	n	Hv
Commercial pure Al(C.W.40%)	89	89	94	93	1.084	12.4	0.029	31.6
Commercial pure Al(C.W.60%)	105	105	109	107	0.944	11.3	0.021	33.1
2024-T4 Al	250	249	492	426	0.503	17.3	0.182	130
2024-T6 Al	333	332	462	432	0.414	8.1	0.113	136

* σ_o = true yield stress (MPa)
σ_u = true ultimate tensile strength (MPa)
S_o = engineering yield stress (MPa)
S_u = engineering tensile strength (MPa)
ε_f = true fracture strain
n = strain hardening exponent
Hv = Vickers hardness number (load : 5kg)

squares straight lines of the fatigue crack growth rate data are shown in Fig. 1. The slope of the lines which is equivalent to m in Eq. (1) increased with increasing stress ratio R for a given material, confirming earlier results (Forman and others, 1976; Hudson, 1969). The value of m of 2024-T6 Al alloy was larger than that of 2024-T4 Al alloy for a given R value. Especially, for R = 0, the fatigue crack growth rate of 2024-T6 Al alloy was substantially higher than that of 2024-T4 Al alloy due to the higher toughness (Su x elongation) of the latter alloy. The relations of constants m and A obtained from

data in Fig. 1 are shown in Figs. 2 and 3 for R being 0 and 0.3 in accordance with Eq. (2). The m - A relations may be expressed as

$$\log A = -6.74 - 1.04m \text{ for } R = 0 \tag{3a}$$

$$\log A = -7.09 - 0.85m \text{ for } R = 0.3 \tag{3b}$$

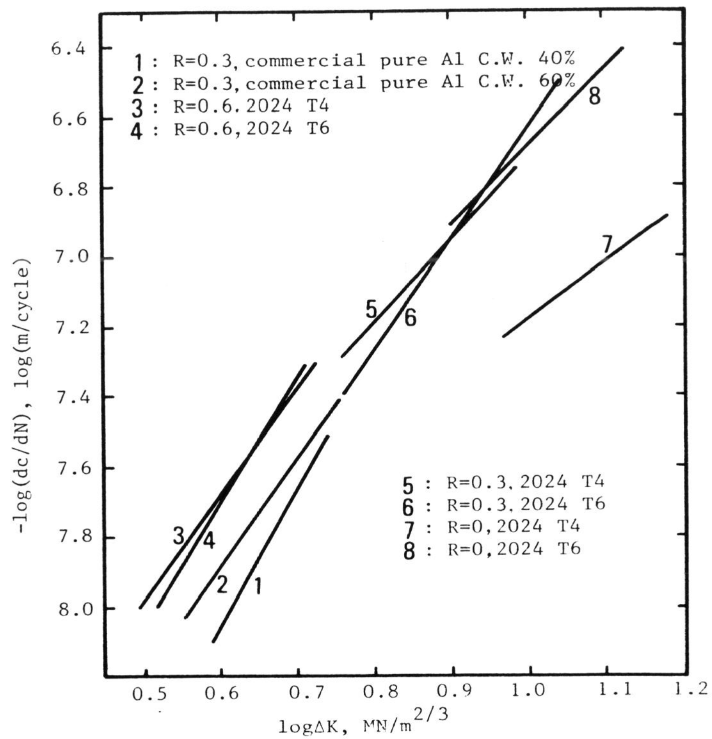

Fig. 1. Fatigue crack growth rates in Al alloys

It is noted that Eqs. (3) satisfy the fatigue crack growth data of aluminum alloys independent of chemical components and thermomechanical treatments. The values of a and b in Eq. (3a) agree very well with Tanaka's results (Tanaka, Matsuoka, 1977). Antolovich's model (Antolovich, 1975; Saxena, Antolovich, 1975) for the fatigue crack growth may be used to explain the relation (3). Antolovich et.al. derived the following equation based on a modified **Manson**-Coffin equation (Chanani, Antolovich, Gerberich, 1972).

$$\frac{dc}{dN} = 4\left(\frac{0.7\alpha}{E\sigma_o^{1+s}\varepsilon_f}\right)^{1/\beta}\left(\frac{1}{\ell^{1/\beta-1}}\right)\Delta K^{(2+s)/\beta} \tag{4}$$

where

E = Young's modulus
σ_o = yield stress
ϵ_f = true strain at fracture
β = exponent in the Coffin-Manson equation
ℓ = fatigue process zone size

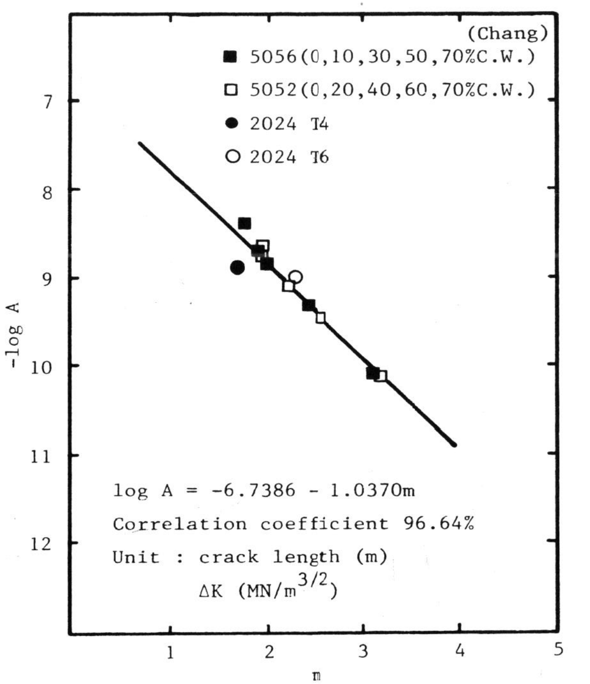

Fig. 2. A-m correlation for Al alloys at R=0. C.W.stands for cold working.

S and α are defined by the following equation :

$$R_p^f = \alpha\left(\frac{\Delta K}{\sigma_o}\right)^{2+s} \qquad (5)$$

where R_p^f is the fatigue plastic zone size and s is small constant indicating a variation in real materials from the theoretically predicted second power dependence of R_p^f on K. Figure 4 shows the various zones involved. Comparing Eq. (4) with Eq. (1) we obtain the following relations,

$$A = 4\left(\frac{0.7\alpha}{E\sigma_o^{1+s}\ \epsilon_f}\right)^{1/\beta}\left(\frac{1}{\ell^{(1/\beta)-1}}\right) \qquad (6)$$

$$m = \frac{2+s}{\beta}$$

Combination of Eqs. (5) and (6) leads to the following equation.

$$\log A = \log 4\ell + \frac{m}{2+s}\cdot\log\left(\frac{0.7\alpha}{E\sigma_o^{1+s}\ \epsilon_f\ \ell}\right) \qquad (7)$$

Comparison between Eq. (2) and Eq. (7) gives us expressions for a and b

$$a = \log 4\ell \qquad (8)$$

$$b = \frac{1}{2+s}\ \log\left(\frac{0.7\alpha}{E\sigma_o^{1+s}\ \epsilon_f\ \ell}\right) \qquad (9)$$

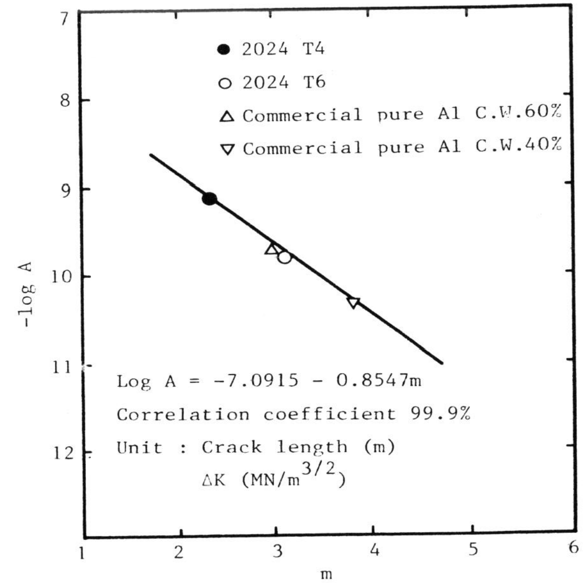

Fig. 3. A-m correlation for Al alloys at R=0.3.

Process zone size ℓ appears to be related to the structural parameters determining strength of materials such as average distance between precipitates in precipitation hardened alloys, cell size or subgain size in high stacking fault energy materials, or grain size in low stacking fault energy materials (Antolovich and others, 1975; Saxena, Antolovich, 1975; Chanani and others, 1972). Thus, the values of ℓ is on the order of μm. The value of a will not be sensitive to ℓ because of the logarithmic variation with ℓ and is on the order of -6 independent of materials. For a given alloy, the value of b given in Eq. (9) is expected to remain approximately constant because E is a structure insensitive property and $\sigma_o^{1+s}\,\varepsilon_f$ will not be sensitive to thermomechanical treatments due to increasing σ_o leading to decrasing ε_f even though σ_o and ε_f are structure sensitive properties, and because b is the logarithmic function of these properties. Setting $E = 10^5$MPa, $\varepsilon_f = 0.1$, $\ell = 10^{-6}$m, $\sigma_o = 10^3$MPa $\alpha = 1/(24\pi)$, $S = 0$ in Eq. (9), we obtain $b = -1.5$

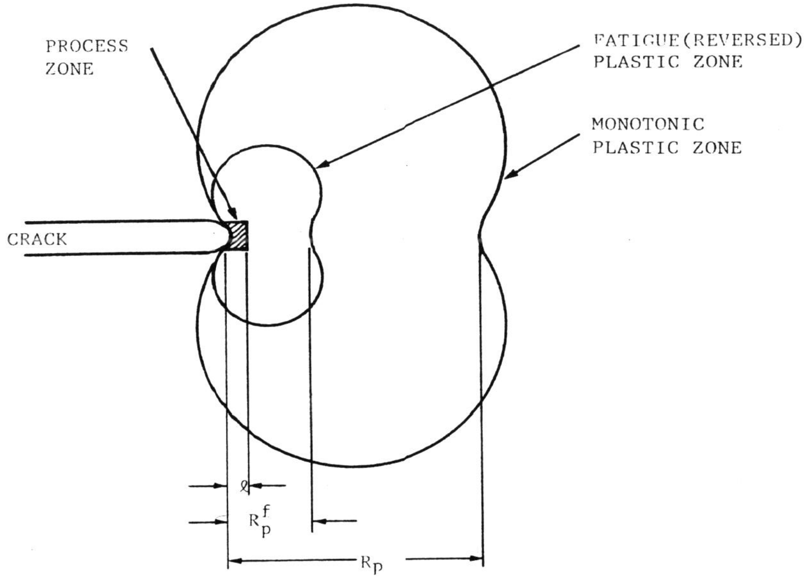

Fig. 4. Schematic illustration of monotonic, fatigue, and process zones ahead of propagating fatigue carack.

The roughly estimated values of a and b compare reasonably well with the measured values in Table 2.

TABLE 2 Measured Values of a and b at R = 0

	Steels(*)	Cu Alloys(**)	Al Alloys
a	-6.3	-6.56	-6.74
b	-1.74	-1.27	-1.04

* Kitagawa, Misumi, 1971 ** Ishii, Yukawa, 1979

Differences in b's depending on alloys may be attributed mainly to Young's modulus. Young's modulus of iron, copper and aluminum are 213, 123 and 70.5GPa, respectively. Experimental data show that increasing stress ratio, R, gives rise to a decrease in the absolute magnitude of b. The effect of R may be accounted for by increasing s in Eq. (9) in light of the fact that m increased with increasing R.

CONCLUSIONS

1. When fatigue crack growth rate is given by the Paris equation, $dc/dN = A(\Delta K)^m$, the a-m correlation may be expressed as $\log A = a + bm$ where $a = -6.74$, $b = -1.04$ at $R = 0$ and $a = -7.09$, $b = -0.85$ at $R = 0.3$ for aluminum alloys.

2. The fact that a and b in $\log A = a + bm$ have constant values at a given stress ratio, R, could be derived from the Antolovich model for the fatigue crack growth rate.

3. The value of m of 2024-T6 Al alloy was larger than that of 2024-T4 Al alloy.

4. For stress ratio of zero, the fafigue crack growth rate of 2024-T6 Al alloy was substantially higher than that of 2024-T4 Al alloy.

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