INTERACTION OF A MACROCRACK WITH MICRODAMAGES

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ABSTRACT

The problem of crack propagation through an elastic material, containing many arbitrary located microcracks, is examined. Mathematically, the problem is reduced to the system of singular integral equations and its solution is obtained by using the method of a small parameter. The change of stress intensity factor and the initial propagation direction of a macrocrack is obtained for different orientations of microcracks.

KEYWORDS

Stress intensity factor; microcracks; singular integral equations; small parameter method.

The problem of interaction of a number of cracks in a solid body is one of the topical problems of fracture mechanics. When materials, especially heterogeneous ones, undergo disperse or volume fracture, the stage of accumulation of small defects is prevalent in the lifetime of a specimen or structure (Kuksenko, Tamužs, 1981). Stochastic models of defect accumulation (Bolotin, 1981; Tamužs, 1982) are constructed so that the interaction of damages is disregarded. It is apparent that interaction of cracks affects, at least, the final stage of fracture, calling forth coalescence of small cracks and facilitating (or hindering) propagation of a macrocrack through the field of microdefects. The account of this phenomenon is of prime importance for correct evaluation of crack resistance of materials having small cracks (for instance, ceramics) and it can also be applied to geophysical problems.

In the plane case, one of the most effective methods of solving the problem of a multitude of cracks is a method of singular integral equations (Muskhelishvili, 1962). The method is based on construction of a complex potential by means of superposition. If the complex potential of an elastic plane problem provides continuity of displacement everywhere, except the crack line, then naturally superposition of similar potentials will define the field of displacements with discontinuities (unknown for the moment) in places of cracks. All the discontinuities of displacements, however, are interrelated and can be found by solving a system of singular integral equations, obtained by satisfying boundary conditions for all crack lines. Such a system of singular integral equations is given by Panasyuk, Savruk, Datsishin (1976). Yet, an approximate solution of the system was obtained with sufficient efficiency only for cracks placed greatly apart from one another.

The present paper examines a case when one crack is much greater than the others. This means that here we shall consider the above formulated physical problem of macrocrack propagation through a damaged material.

We shall examine a plane problem of deformation of an infinite body having a crack of length $2\ell_0$ and arbitrary distributed N microcracks. We shall assume that all the microcracks are of length $2\ell_\kappa << 2\ell_0$, $\kappa = 1 \dots N$ (Fig. I). The x and y -coordinate axes are chosen relative to the macrocrack direction, while the position of microcracks is defined by the coordinates of their centres z_κ and the slope angle α_κ to the x -axis. For convenience we shall also use a local system of coordinates x_κ and y_κ (Fig. I) for each microcrack.

The problem is resolved by using the method of singular integral equations.

If a self-balanced load is applied to crack surfaces, that is, when boundary conditions take the form

$$\sigma_\kappa^+ + i\tau_\kappa^+ = \sigma_\kappa^- - i\tau_\kappa^- = p_\kappa(x_\kappa); \quad |x_\kappa| \leqslant \ell_\kappa, \quad \kappa = 0, 1, \dots N \qquad (I)$$

(a macrocrack is not yet singled out), then the system of integral singular equations takes the form:

$$\int_{-\ell_n}^{\ell_n}\frac{g_n'(t)dt}{t-x_n} + \sum_{\substack{\kappa=0\\ \kappa\neq n}}\int_{-\ell_\kappa}^{\ell_\kappa}\left[g_\kappa'(t)K_{n\kappa}(t,x)+\overline{g_\kappa'(t)}L_{n\kappa}(t,x)\right]dx = \pi p_n(x) \qquad (2)$$

Here $g_\kappa'(t)$ are the derivatives of displacement discontinuities for the κ -th crack (according to the formula

$$g'(x_\kappa) = \frac{2\mu}{i(\varkappa+1)}\frac{\partial}{\partial x_\kappa}\left([u_\kappa]+i[v_\kappa]\right);$$

μ is the shear modulus, $\varkappa = 3-4\nu$ for plane deformation, $[u_\kappa]$ and $[v_\kappa]$ are the jumps of normal and tangential displacements; and $K_{n\kappa}$, $L_{n\kappa}$ are regular nuclei whose expressions are given by Panasyuk, Savruk, Datsishin (1976) (the bar denotes a complex conjugate).

After regularization, using the conversion formulas for Cauchy-type integrals, the system (2) takes the form

$$g_0'(x) = \frac{1}{\pi\sqrt{\ell_0^2-x^2}}\left\{-\int_{-\ell_0}^{\ell_0}\frac{\sqrt{\ell_0^2-t^2}\,p_0(t)dt}{t-x} + \sum_{\kappa\neq0}\int_{-\ell_\kappa}^{\ell_\kappa}\left[g_\kappa'(t)M_{0\kappa}(t,x) + \overline{g_\kappa'(t)}N_{0\kappa}(t,x)\right]dt\right\} \quad |x|<\ell_0 \qquad (3)$$

$$g_n'(x) = \frac{1}{\pi\sqrt{\ell_n^2-x^2}}\left\{-\int_{-\ell_n}^{\ell_n}\frac{\sqrt{\ell_n^2-t^2}\,p_n(t)dt}{t-x} + \sum_{\kappa\neq0}\int_{-\ell_\kappa}^{\ell_\kappa}\left[g_\kappa'(t)M_{0\kappa}(t,x) + \overline{g_\kappa'(t)}N_{0\kappa}(t,x)\right]dt\right\} \quad |x|<\ell_n, \quad n=1,\dots N$$

Here the indices of variables are omitted and the following designations are introduced:

$$M_{n\kappa}(t,x) = -\frac{e^{i\alpha_{\kappa n}}}{2\pi}\int_{-\ell_n}^{\ell_n}\frac{\sqrt{\ell_n^2-\xi^2}}{\xi-x}\left(\frac{1}{\xi-T_{n\kappa}} + \frac{1}{\xi-\overline{T}_{n\kappa}}\right)d\xi; \qquad (4)$$

$$N_{n\kappa}(t,x) = \frac{e^{i\alpha_{n\kappa}}}{2\pi}\int_{-\ell_n}^{\ell_n}\frac{\sqrt{\ell_n^2-\xi^2}}{\xi-x}\left[-\frac{1}{\xi-\overline{T}_{n\kappa}} + \frac{\xi-T_{n\kappa}}{(\xi-\overline{T}_{n\kappa})^2}\right]d\xi$$

$$T_{n\kappa} = e^{i\alpha_{n\kappa}}\left[t - d_{n\kappa}e^{i(\alpha_\kappa-\beta_{n\kappa})}\right];$$

where $d_{n\kappa}$ is a distance of the n -th and κ -th crack centres, $\beta_{n\kappa}$ defines their relative location and $\alpha_{n\kappa} = \alpha_n - \alpha_\kappa$ defines their relative orientation.

Let us substitute the variables $t = \ell_\kappa\tau$ in the integrals and let $x = \ell_\kappa\chi$. Then Eq. (3) takes the form

$$g_0'(\chi\ell_0) = \frac{1}{\pi\sqrt{1-\chi^2}\,\ell_0}\left\{-\ell_0\int_{-1}^{1}\frac{\sqrt{1-\tau^2}\,p_0(\tau\ell_0)d\tau}{\tau-\chi} + \sum_{\kappa\neq0}\ell_\kappa\int_{-1}^{1}\left[g_\kappa'(\tau\ell_\kappa)M_{0\kappa}(\tau\ell_\kappa,\chi\ell_0) + \overline{g_\kappa'(\tau\ell_\kappa)}N_{0\kappa}(\tau\ell_\kappa,\chi\ell_0)\right]d\tau\right\} \quad |\chi|\leqslant1 \qquad (5)$$

$$g_n'(\chi\ell_n) = \frac{1}{\pi\sqrt{1-\chi^2}\,\ell_n}\left\{-\ell_n\int_{-1}^{1}\frac{\sqrt{1-\tau^2}\,p_n(\tau\ell_n)d\tau}{\tau-\chi} + \ell_0\int_{-1}^{1}\left[g_0'(\tau\ell_0)M_{n0}(\tau\ell_0,\chi\ell_n) + \overline{g_0'(\tau\ell_0)}N_{n0}(\tau\ell_0,\chi\ell_n)\right]d\tau + \sum_{\substack{\kappa=1\\ \kappa\neq n}}^{N}\ell_\kappa\int_{-1}^{1}\left[g_\kappa'(\tau\ell_\kappa)M_{n\kappa}(\tau\ell_\kappa,\chi\ell_n) + \overline{g_\kappa'(\tau\ell_\kappa)}N_{n\kappa}(\tau\ell_\kappa,\chi\ell_n)\right]d\tau\right\}$$

Subsequently we shall assume that $l_k = l$; k =I... N and seek a solution of the system (5) in the form of a series by using a small parameter $\lambda = l/l_0$

$$g_0'(\chi l_0)=\sum_{p=0}^{\infty} g_{0p}'(\chi l_0)\lambda^p \quad ; \quad g_n'(\chi l)=\sum_{p=0}^{\infty} g_{np}'(\chi l_n)\lambda^p \tag{6}$$

For this purpose, we shall expand all the functions $M_{0k}, N_{0k}, M_{k0}, N_{k0}, M_{nk}, N_{nk}$ in terms of powers λ :

$$M_{0k}(\tau,\chi)=-\frac{e^{i\alpha_k}}{2\pi}\left\{\sum_{u=1}^{\infty}(\tau\lambda)^{u-1}\left[e^{-i\alpha_k(u-1)}\int_{-1}^{1}\frac{\sqrt{1-\xi_1^2}\,d\xi_1}{(\xi_1-\chi)\left(\xi_1-\frac{Z_k}{l_0}\right)^u}+\right.\right.$$

$$\left.\left.+e^{i\alpha_k(u-1)}\int_{-1}^{1}\frac{\sqrt{1-\xi_1^2}\,d\xi_1}{(\xi_1-\chi)\left(\xi_1-\frac{\bar Z_k}{l_0}\right)^u}\right]\right\}=\sum_{u=0}^{\infty} m_{0ku}\lambda^u$$

$$N_{0k}(\tau,\chi)=-\frac{ie^{-i\alpha_k}y_k}{\pi l_0}\sum_{u=1}^{\infty}(\tau\lambda)^{u-1}\left\{\int_{-1}^{1}\frac{\sqrt{1-\xi_1^2}}{\xi_1-\chi}\left[u\frac{e^{-i\alpha_k(u-1)}}{\left(\xi_1-\frac{\bar Z_k}{l_0}\right)^{u+1}}+\right.\right.$$

$$\left.\left.+\frac{l_0\sin\alpha_k e^{-i\alpha_k(u-2)}(u-1)}{y_k\left(\xi_1-\frac{Z_k}{l_0}\right)^u}\right]d\xi_1\right\}=\sum_{u=0}^{\infty}\lambda^u n_{0ku}$$

$$M_{k0}(\tau,\chi)=-\frac{e^{i\alpha_k}}{2}\sum_{u=1}^{\infty}\lambda^u\left[\frac{1}{(T_{k0}/l_0)^u}+\frac{1}{(\bar T_{k0}/l_0)^u}\right]\left[\chi^u-\sum_{2\tau=2}^{u}\frac{|2\tau-3|!!}{(2\tau)!!}\chi^{u-2\tau}\right]=$$

$$=\sum\lambda^u m_{k0u};$$

$$N_{k0}(\tau,\chi)=-\frac{e^{-i\alpha_k}}{2}\sum_{u=1}^{\infty}\lambda^u\left[\frac{T_{k0}/l_0-\bar T_{k0}/l_0}{(T_{k0}/l_0)^u}\right]\left[\chi^u-\sum_{2\tau=2}^{u}\frac{|2\tau-3|!!}{(2\tau)!!}\chi^{u-2\tau}\right]= \tag{7}$$

$$=\sum\lambda^u n_{k0u};$$

$$M_{nk}(\tau,\chi)=2\,\mathrm{Re}\left\{\frac{e^{i(\alpha_{kn}+\alpha_k)}}{\pi\left(\frac{Z_k}{l_0}-\frac{Z_n}{l_0}\right)}\sum_{u=1}^{\infty}\lambda^u\int_{-1}^{1}\frac{\sqrt{1-\xi_1^2}}{\xi_1-\chi}\left[\frac{(\xi_1-\tau e^{i(\alpha_n-\alpha_k)(u-1)})e^{\alpha_k(u-1)}}{}\right]d\xi_1\right\}=$$

$$=\sum\lambda^u m_{nku}$$

$$N_{nk}(\tau,\chi)=\frac{ie^{i\alpha_{nk}}}{\pi}\sum_{u=1}^{\infty}\lambda^u\frac{e^{-i\alpha_n u}}{\left(\frac{\bar Z_n}{l_0}-\frac{\bar Z_k}{l_0}\right)^u}\int_{-1}^{1}\frac{\sqrt{1-\xi_1^2}(\xi_1-\tau e^{i(\alpha_n-\alpha_k)(u-2)})}{\xi_1-\chi}\left\{\frac{u(\tau e^{i(\alpha_n-\alpha_k)}-\xi_1)}{e^{i\alpha_n}\left(\frac{\bar Z_n}{l_0}-\frac{\bar Z_k}{l_0}\right)}\times\right.$$

$$\left.\times\,\mathrm{Im}\left[e^{i\alpha_n}\left(\frac{\bar Z_n}{l_0}-\frac{\bar Z_k}{l_0}\right)\right]+(u-1)\tau\sin(\alpha_n-\alpha_k)\right\}d\xi_1$$

In expansion of expressions M_{k0} and N_{k0} in terms of λ , the values of the Cauchy-type integrals were used

$$\int_{-1}^{1}\left[\left(\sqrt{1-\xi^2}\,\xi^{u-1}\right)/(\xi-\chi)\right]d\xi=\pi\left[\chi^u+\sum_{2\tau=2}^{u}(|2\tau-3|!!\,\chi^{u-2\tau})/(2\tau)!!\right] \quad |\chi|<1$$

It should be noted that the expansion (7) is valid in the case when the microcracks have no mutual intersections and they do not intersect the macrocrack: $Z_k \notin \Omega$ (Fig. I). Substitution of (6) into jump expressions (5), taking into account (7) and equating the indices at λ^n give a recurrent sequence of formulas for calculation of g_{ij}' :

$$g_{00}'=-\frac{1}{\pi\sqrt{1-\chi^2}}\int_{-1}^{1}\frac{p_0\sqrt{1-\tau^2}\,d\tau}{\tau-\chi};$$

$$g_{0p}'(\chi)=\frac{1}{\pi\sqrt{1-\chi^2}}\left\{\sum_{k\neq 0}\int_{-1}^{1}\left[\sum_{q=0}^{p-1}g_{kq}'(\tau)m_{0k(p-1-q)}(\tau,\chi)+\sum_{q=0}^{p-1}\overline{g_{kq}'(\tau)}n_{0k(p-1-q)}(\tau,\chi)\right]d\tau\right\} \tag{8}$$

$$g_{n0}'(\chi)=-\frac{1}{\pi\sqrt{1-\chi^2}}\left\{\int_{-1}^{1}\frac{\sqrt{1-\tau^2}\,p_n(\tau)\,d\tau}{\tau-\chi}+\int_{-1}^{1}\left[g_{00}'(\tau)m_{n01}(\tau,\chi)+\overline{g_{00}'}\,n_{n01}(\tau,\chi)\right]d\tau\right\}$$

$$g_{np}'(\chi)=\frac{1}{\pi\sqrt{1-\chi^2}}\left\{\int_{-1}^{1}\left[\sum_{q=0}^{p}g_{0q}'(\tau)m_{n0(p+1-q)}(\tau,\chi)+\sum_{q=0}^{p}\overline{g_{0q}'(\tau)}n_{n0(p+1-q)}(\tau,\chi)\right]d\tau+\right.$$

$$+\sum_{q=0}^{p-1}\int_{-1}^{1}\left[\sum_{\substack{k=1\\k\neq n}}^{\infty}g_{kq}'(\tau)m_{nk(p-q)}(\tau,\chi)+\right.$$

$$\left.\left.+\sum_{q=0}^{p-1}\overline{g_{kq}'(\tau)}\,n_{nk(p-q)}(\tau,\chi)\right]d\tau\right\}$$

Let us assume that a body with cracks is stretched to infinity by forces $\sigma_y^{\infty}=p$. Then $p_k=-\frac{p}{2}(1+e^{-2i\alpha_k})$. We note that factor at λ is g_{01}' =0, since the integrals of the type $\int[g_{k0}'(\tau)m_{0k0}(\chi)+\overline{g_{k0}'(\tau)}n_{0k0}(\chi)]d\tau=0$ are expressed as integrals of an uneven function within the range of (-I, +I). We shall calculate the second approximation of the stress intensity factor at the tip of the main crack. By using Eq. (8), we get

$$g_{02}'(\chi)=\frac{1}{\pi\sqrt{1-\chi^2}}\sum_{k\neq 0}\int_{-1}^{1}\left[g_{k1}'(\tau)m_{0k0}(\chi)+\overline{g_{k1}'(\tau)}n_{0k0}(\chi)+g_{k0}'(\tau)m_{0k1}+\overline{g_{k0}'(\tau)}n_{0k1}\right]d\tau \tag{9}$$

It is easy to prove that $\int[g_{k1}'(\tau)m_{0k0}(\chi)+\overline{g_{k1}'(\tau)}n_{0k0}(\chi)]d\tau=0$ therefore in order to calculate g_{02}' it is enough to know the values $g_{k0}'(\tau)$ and $\overline{g_{k0}'(\tau)}$:

$$g_{k0}'(\chi)=\frac{1}{\pi\sqrt{1-\chi^2}}\left\{\int_{-1}^{1}\left[g_{00}'(\tau)m_{k01}(\tau,\chi)+\overline{g_{00}'(\tau)}n_{k01}(\tau,\chi)\right]d\tau-\frac{p\pi(1+e^{-2i\alpha_k})}{2}\chi\right\} \tag{IO}$$

Calculation of the integrals in Eq. (7) and substitution of all the found expressions into (9) and then into the relations of the stress intensity factor (Panasyuk, Savruk, Datsishin, 1976) give:

$$K^{\pm}_{I02} - iK^{\pm}_{II02} = \mp \lim_{x \to \pm l_0} \left[\sqrt{\frac{l_0^2 - x^2}{l_0}}\, g'_{02}(x) \right] = \mp \lim_{\chi \to \pm 1} \sqrt{l_0(1-\chi^2)}\, g'_{02}(l_0 \chi) \qquad \text{(II)}$$

In such a way, we shall obtain a value of the second approximation of the stress intensity factor for an arbitrary microcrack in the form:

$$K^{\pm}_{I02} - iK^{\pm}_{II02} = -\frac{p\lambda^2\sqrt{l_0}}{4}\left\{\frac{1}{2}\left[1 - e^{-2i\alpha_k} - \frac{u_k}{\sqrt{u_k^2-1}} - \frac{\bar{u}_k}{\sqrt{\bar{u}_k^2-1}} - \frac{(\bar{u}_k - u_k)e^{-2i\alpha_k}}{(\bar{u}_k^2-1)^{3/2}}\right] \times \right.$$

$$\times \left[\frac{1}{(\bar{u}_k \mp 1)\sqrt{\bar{u}_k^2-1}} + \frac{e^{2i\alpha_k}}{(u_k \mp 1)\sqrt{u_k^2-1}}\right] + \left[1 - e^{2i\alpha_k} - \frac{u_k}{\sqrt{u_k^2-1}} - \frac{\bar{u}_k}{\sqrt{\bar{u}_k^2-1}} - \right.$$

$$\left. - \frac{(u_k - \bar{u}_k)e^{2i\alpha_k}}{(u_k^2-1)^{3/2}}\right]\left.\left[\frac{(\bar{u}_k - u_k)(2\bar{u}_k \pm 1)e^{-2i\alpha_k}}{2(\bar{u}_k^2-1)^{3/2}(\bar{u}_k \mp 1)} + \frac{1 - e^{-2i\alpha_k}}{2(\bar{u}_k \mp 1)\sqrt{\bar{u}_k^2-1}}\right]\right\}$$

where the designation $u_k = \frac{z_k}{l_0}$ is used.
From (I2) we have the known particular cases of relative location of two cracks with the assumption that the length of one crack is much longer than that of the other. For instance, if $\alpha_k = 0$ and $y_k = 0$, that is, when a microcrack is located straight ahead of the macrocrack, we shall get an expression which agrees with the first two expansion terms of the exact solution (Panasyuk, Savruk, Datsishin, 1976)

$$K^{\pm}_{I0} = p\sqrt{l_0}\left\{1 + \frac{\lambda^2\, x_k/l_0}{2(x_k/l_0 \mp 1)[(x_k/l_0)^2 - 1]}\right\}$$

Let us note that the second approximation of the stress intensity factor does not comprise expressions which take into account interrelations of the microcracks. The second approximation of the stress intensity factor considers only interaction of the macrocrack with each microcrack. The formula (I2) gives us a chance to calculate the influence of any microcrack on macrocrack propagation.

Let us examine a doubly-periodic system of microcracks, located as shown in Fig. 2. We shall assume that all the microcracks are oriented in one direction at an angle α to the x -axis. The coordinates of the microcrack centres are expressed as $x_n = l_0 n/r$; $y_m = l_0 m/s$, where m , n =I,2..., while r and s are some natural numbers, indicating how many microcracks are located on the section of length l_0 in x - and y -directions, respectively.

Let us determine now the propagation direction θ of the main crack with respect to orientation of the microcracks. As a criterion we shall use, for instance, the criterion of

maximum normal stresses (Erdogan, Sih, 1963)

$$\theta = 2\,\mathrm{arctg}\,\frac{K_I - \sqrt{K_I^2 + 8K_{II}^2}}{4K_{II}}$$

The ultimate load p^* related to the ultimate load p_0 (for the case of a single macrocrack without damages) is determined from the formula

$$p^*/p_0 = \frac{p\sqrt{l_0}}{\cos^3\theta/2\,(K_I - 3K_{II}\,\mathrm{tg}\frac{\theta}{2})}$$

The graphs of the initial angle of crack growth θ^0 and the relative values of the ultimate load p^* as a function of orientation of the microcracks are given in Figs. 3 and 4. At any orientation of the microcrack field the ultimate load for a damaged material is less than its initial strength.

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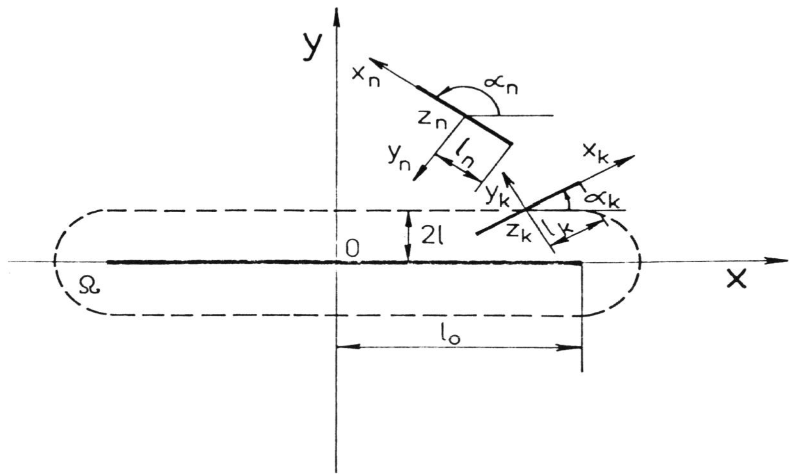

Fig. 1.

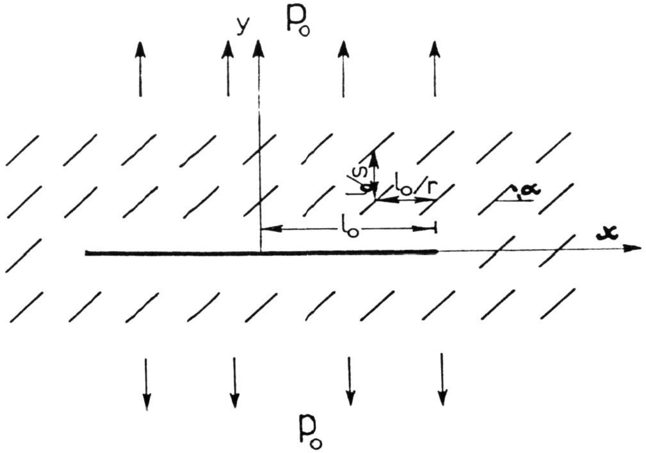

Fig. 2.

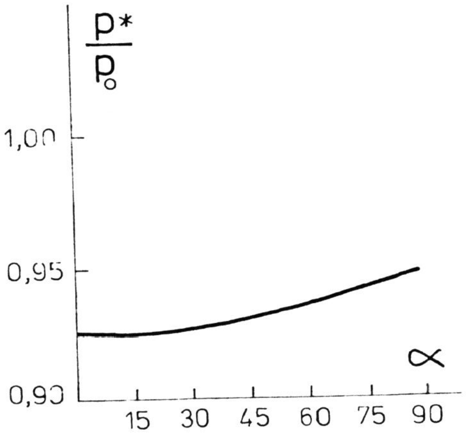

Fig. 3.

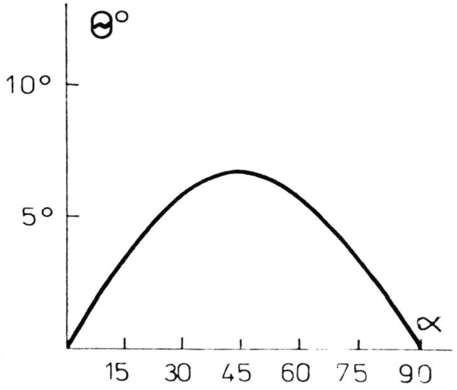

Fig. 4.