DETERMINATION OF COMBINED MODE STRESS INTENSITY FACTORS USING MIXED ELEMENTS

D. P. Mazumdar* and P. N. Murthy**

**Department of Applied Mechanics, Regional Engineering College, Durgapur 713209, India*
***Tata Consultancy Services, Hyderabad 500004, India*

ABSTRACT

The paper presents an application of mixed finite elements based on Reissner's variational principle to the determination of combined mode stress intensity factors for general crack problems. The crack tip elements do not contain any built-in stress singularities. Relatively coarse meshes with a compatible mixture of mostly linear elements and a few quadratic elements near the crack tips have been employed. Several numerical examples that illustrate the accuracy and efficiency of the analysis are presented.

KEYWORDS

Combined mode stress intensity factors; mixed elements.

INTRODUCTION

The finite element method has been used extensively to evaluate Mode I as well as some combined modes I and II stress intensity factors. The major difficulty in the application of the finite element technique to crack problems is due to the singular nature of the stress field near the crack tip. Kobayashi and others (1969) and Chan, Tuba and Wilson (1970) estimated the stress intensity factors by fitting displacement-based element solutions for the near tip crack surface displacements with the analytical expressions. Highly refined meshes were required near the crack tip in order to accurately calculate the near field displacements. Watwood (1970) and Anderson, Ruggles and Stibor (1971) used strain energy release rates computed from displacement-based element analysis. Better accuracy was obtained with relatively coarse meshes, but in combined mode cracks the stress intensity factors for the two modes could not be separated. Pian, Tong and Luk (1971), Atluri, Kobayashi and Nakagaki (1975) and Barsoum (1976) used special crack tip elements with the correct stress singularities. Excellent accuracy was obtained by using substantially smaller number of elements.

Recently, Mazumdar and Murthy (1979) have presented highly accurate results of the application of mixed elements to the determination of Mode I stress intensity factors using relatively coarse meshes without resorting to any singular crack tip elements. Use of mixed elements was motivated by the fact that mixed elements are considerably more accurate than other comparable elements and are highly effective in capturing steep stress and displacement gradients. In the present paper, the results of the application of mixed elements to the determination of Mode I and Mode II stress intensity factors for combined mode crack problems are discussed.

MIXED ELEMENTS

The formulation of the regular mixed elements used in the present study has been described in detail in Mazumdar and Murthy (1979) and will not be repeated here. The elements are based on Reissner's variational principle utilizing both stresses and displacements as the unknown variables. Compatible mixtures of mostly linear elements and a few quadratic elements near the crack tips have been employed. Such mixtures of compatible elements have been conveniently generated by using isoparametric formulations. Same interpolation functions have been used for both the stresses and the displacements within an element. Triangular elements used at the crack tips for grading the meshes have been obtained by appropriately degenerating the quadrilateral elements (Zienkiewicz, 1973).

SEPARATION OF K_I AND K_{II}

The mixed element solutions for the displacement components U and V (Fig. 1) for two nodes i and p on the two crack surfaces at the same radius were first transformed to directions parallel and perpendicular to the crack line by the standard operations $u = U\cos\alpha + V\sin\alpha$ and $v = -U\sin\alpha + V\cos\alpha$. The crack opening displacement (COD) and the crack sliding displacement (CSD) were

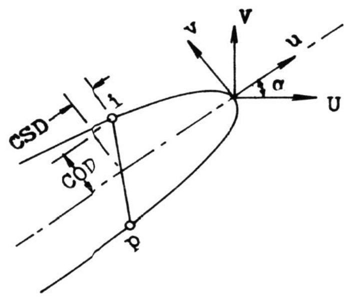

Fig. 1 Separation of Mode I and Mode II displacements

taken as $\frac{1}{2}(v_i - v_p)$ and $\frac{1}{2}(u_i - u_p)$ respectively. The stress intensity factors for the two modes were then calculated by the least square fitting of the two modes of crack surface displacements for nodes close to the crack tip with the analytical distributions. The stress intensity factors were also estimated by fitting the solutions for the stresses on the crack extension line, but were found to be less accurate.

COMPUTATIONS

A FORTRAN IV computer program based on the analysis described above was written. The program is extremely short and simple because of the simplicity of the mixed isoparametric formulation. The program is fully automatic including the computation of Mode I and Mode II stress intensity factors. The element matrices for the mixture of elements are generated through a general interpolation function routine. For the solution of the system equations, an efficient Gauss elimination routine that exploits the prominent 'skyline' feature of the system matrix caused due to the mixing of linear and quadratic elements has been developed.

NUMERICAL RESULTS

The results of the analysis presented above for three combined mode crack problems with known solutions are given below. In each problem only two layers of quadratic elements were used near the crack tip. The size of the elements at the crack tip was taken as one-twentyfifth of the crack length.

Oblique Edge Cracked Tension Plate

The geometry and finite element breakdown of a tension plate with an oblique edge crack is shown in Fig. 2. Because of asymmetry, the entire plate had to be used in the analysis. For $a/b = 0.4$, the mesh composed of 63 elements and 111 nodes. Figure 3 shows a comparison of the computed values of K_I and K_{II} with the modified mapping-collocation solution by Bowie (1973). An excellent agreement between the two solutions for K_I is noted for short cracks, but slightly higher values of the stress intensity factor are predicted for deep cracks by the present analysis. Attempts were not made at the refinement of finite element grids for the deep cracks to study the convergence of the solution. The agreement of the two solutions for K_{II} is excellent for short as well as deep cracks. For $a/b = 0.4$, an error of +1.74 percent in K_I, and an error of +1.79 percent in K_{II} were noticed in the present solution.

Central Curved Cracked Tension Plate

Figure 4 shows the geometry and finite element representation of a plate with a central quarter-circle crack. Due to symmetry, only one half of the plate was needed in the analysis. A total

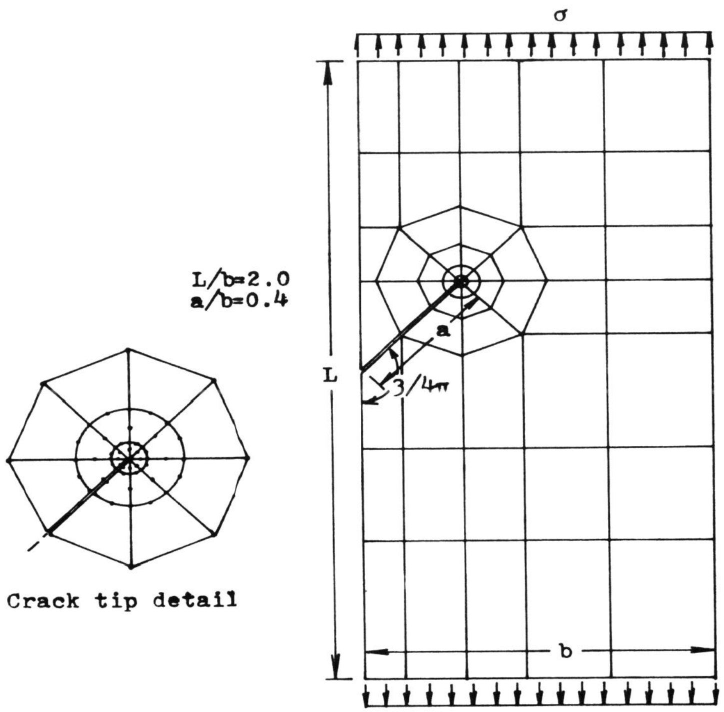

Fig. 2 Element breakdown of a tension plate with an oblique edge crack

of 76 elements with 132 nodes were employed. Elements with curved boundary were used for the curved crack. The same plate was used for biaxial and uniaxial tension. A comparison of the computed values of K_I and K_{II} with the complex variable stress-function solution by Sih, Paris and Erdogan (1962) is given in Table 1. The correct values of the stress intensity factors for uniaxial tension re-derived in Atluri, Kobayashi and Nakagaki (1975) have been used in the comparison. Slight discrepancies are found in the two solutions for K_I, but the computed values of K_{II} are not as accurate. The discrepancy between the two solutions is probably to some extent due to finite width effect in the computed stress intensity factors.

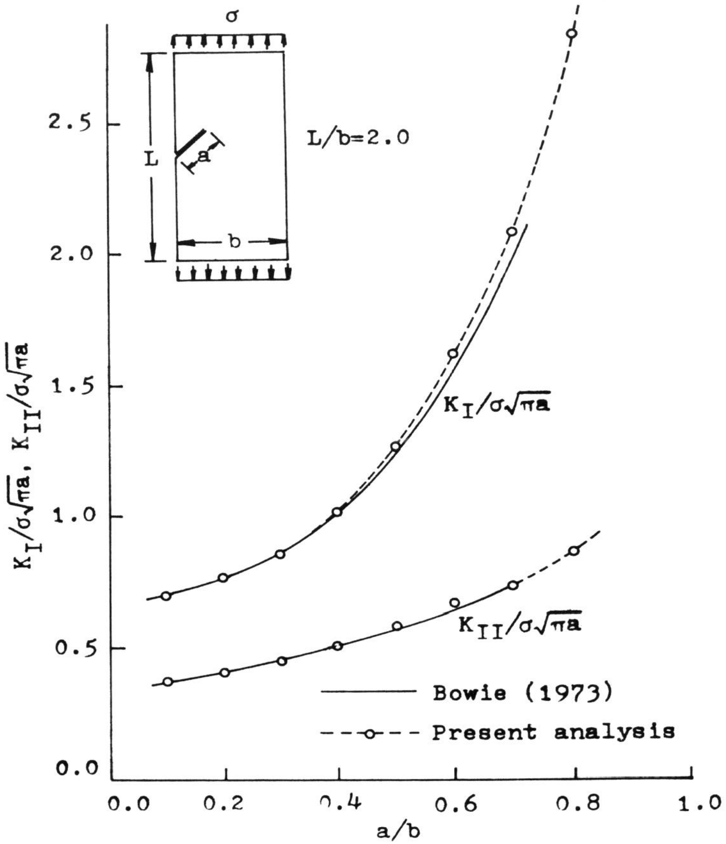

Fig. 3 Stress intensity factors for an oblique edge crack in a tension plate

TABLE 1 Stress Intensity Factors for a Central Quarter-Circle Crack in a Tension Plate

	Present analysis		Sih, Paris and Erdogan (1962)[a]		Percent difference	
	K_I	K_{II}	K_I	K_{II}	K_I	K_{II}
Biaxial tension	1.227	-0.526	1.201	-0.498	+2.16%	+5.62%
Uniaxial tension	0.813	-0.944	0.811	-0.906	+0.24%	+4.19%

[a]Infinite plate solution

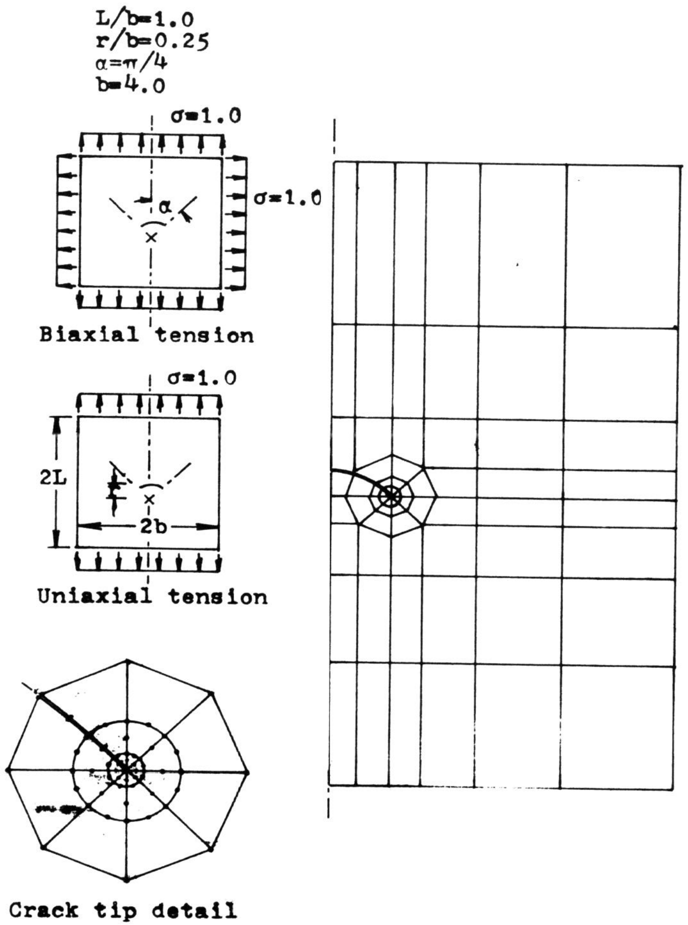

Fig. 4 Element breakdown of a plate with a central quarter-circle crack (biaxial and uniaxial tension)

Centre Cracked Tension Plate with Adjacent Edge Cracks

The geometry and finite element model of a tension plate with a centre crack and four adjacent edge cracks is shown in Fig. 5. Exploiting the double symmetry, one quarter of the plate was subdivided into 67 elements and 132 nodes. Table 2 shows a comparison of the computed values of K_I and K_{II} for the edge cracks with the crack tip hybrid singular element solution of Pian, Tong and Luk (1971). The agreement between the two finite element solutions for the edge cracks is good. The value of K_I for the centre crack is not reported in Pian, Tong and Luk (1971).

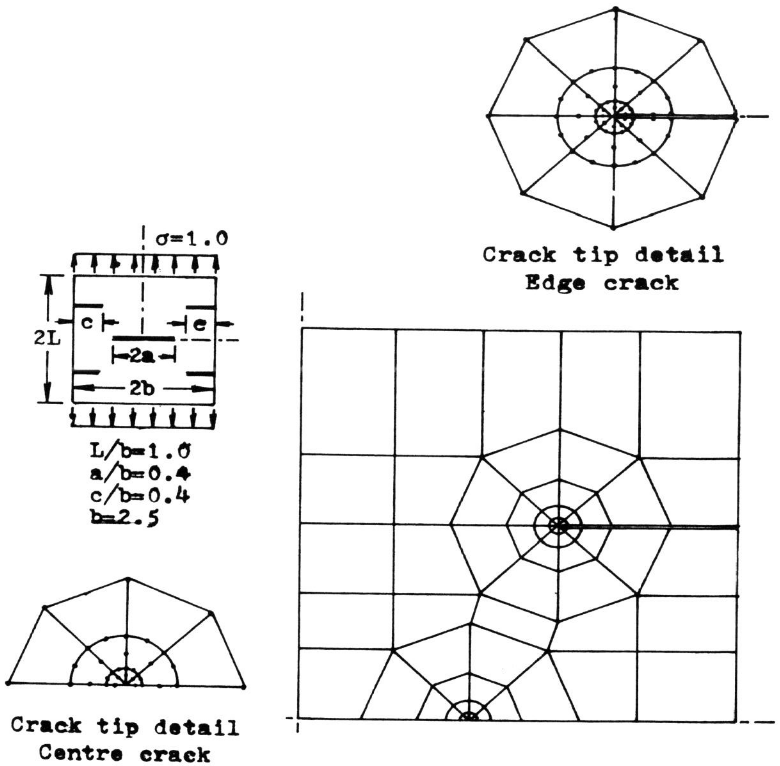

Fig. 5 Element breakdown of a tension plate with a centre crack and four adjacent edge cracks

TABLE 2 Stress Intensity Factors for a Centre Crack and adjacent edge cracks in a tension plate

	Present analysis		Pian, Tong and Luk (1971)[a]	
	K_I	K_{II}	K_I	K_{II}
Centre crack	4.723	–	–	–
Edge cracks	4.804	-0.920	4.70	-0.94

[a]Finite element solution

CONCLUSION

A mixed finite element procedure for the determination of combined mode stress intensity factors for general crack problems has been presented. The numerical results demonstrate that fairly accurate estimation of combined mode stress intensity factors can be obtained by using relatively coarse meshes of mixed elements without resorting to any crack tip singular elements. Due to the generality and accuracy, the mixed elements can be usefully employed in fundamental studies on fracture mechanics.

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