DETERMINATION OF COMBINED MODE STRESS INTENSITY FACTORS USING MIXED ELEMENTS

D. P. Mazumdar* and P. N. Murthy**

*Department of Applied Mechanics, Regional Engineering College, Durgapur 713209, India
**Tata Consultancy Services, Hyderabad 500004, India

ABSTRACT

The paper presents an application of mixed finite elements based on Reissner's variational principle to the determination of combined mode stress intensity factors for general crack problems. The crack tip elements do not contain any built-in stress singularities. Relatively coarse meshes with a compatible mixture of mostly linear elements and a few quadratic elements near the crack tips have been employed. Several numerical examples that illustrate the accuracy and efficiency of the analysis are presented.

KEYWORDS

Combined mode stress intensity factors; mixed elements.

INTRODUCTION

The finite element method has been used extensively to evaluate Mode I as well as some combined modes I and II stress intensity factors. The major difficulty in the application of the finite element technique to crack problems is due to the singular nature of the stress field near the crack tip. Kobayashi and others (1969) and Chan, Tuba and Wilson (1970) estimated the stress intensity factors by fitting displacement-based element solutions for the near tip crack surface displacements with the analytical expressions. Highly refined meshes were required near the crack tip in order to accurately calculate the near field displacements. Watwood (1970) and Anderson, Rugeles and Stihor (1971) used strain energy release rates computed from displacement-based element analysis. Better accuracy was obtained with relatively coarser meshes, but in combined mode cracks the stress intensity factors for the two modes could not be separated. Plan, Tong and Luk (1971), Atluri, Kobayashi and Nakazaki (1975) and Barsom (1976) used special crack tip elements with the correct stress singularities. Excellent accuracy was obtained by using substantially smaller number of elements.
Recently, Mazumdar and Murthy (1979) have presented highly accurate results of the application of mixed elements to the determination of Mode I stress intensity factors using relatively coarse meshes without resorting to any singular crack tip elements. Use of mixed elements was motivated by the fact that mixed elements are considerably more accurate than other comparable elements and are highly effective in capturing steep stress and displacement gradients. In the present paper, the results of the application of mixed elements to the determination of Mode I and Mode II stress intensity factors for combined mode crack problems are discussed.

MIXED ELEMENTS

The formulation of the regular mixed elements used in the present study has been described in detail in Mazumdar and Murthy (1979) and will not be repeated here. The elements are based on Reissner's variational principle utilizing both stresses and displacements as the unknown variables. Compatible mixtures of mostly linear elements and a few quadratic elements near the crack tips have been employed. Such mixtures of compatible elements have been conveniently generated by using isoparametric formulations. Same interpolation functions have been used for both the stresses and the displacements within an element. Triangular elements used at the crack tips for grading the meshes have been obtained by appropriately degenerating the quadrilateral elements (Zienkiewicz, 1973).

SEPARATION OF $K_I$ AND $K_{II}$

The mixed element solutions for the displacement components $U$ and $V$ (Fig. 1) for two nodes $i$ and $p$ on the two crack surfaces at the same radius were first transformed to directions parallel and perpendicular to the crack line by the standard operations $u = U_{cos} + V_{sin}$ and $v = -U_{sin} + V_{cos}$. The crack opening displacement (COD) and the crack sliding displacement (CSD) were taken as $\frac{1}{2}(V_i - V_p)$ and $\frac{1}{2}(U_i - U_p)$ respectively. The stress intensity factors for the two modes were then calculated by the least square fitting of the two modes of crack surface displacements for nodes close to the crack tip with the analytical distributions. The stress intensity factors were also estimated by fitting the solutions for the stresses on the crack extension line, but were found to be less accurate.

COMPUTATIONS

A FORTRAN IV computer program based on the analysis described above was written. The program is extremely short and simple because of the simplicity of the mixed isoparametric formulation. The program is fully automatic including the computation of Mode I and Mode II stress intensity factors. The element matrices for the mixture of elements are generated through a general interpolation function routine. For the solution of the system equations, an efficient Gauss elimination routine that exploits the prominent 'skyline' feature of the system matrix caused due to the mixing of linear and quadratic elements has been developed.

NUMERICAL RESULTS

The results of the analysis presented above for three combined mode crack problems with known solutions are given below. In each problem only two layers of quadratic elements were used near the crack tip. The size of the elements at the crack tip was taken as one-twentyfifth of the crack length.

Oblique Edge Cracked Tension Plate

The geometry and finite element breakdown of a tension plate with an oblique edge crack is shown in Fig. 2. Because of asymmetry, the entire plate had to be used in the analysis. For $a/b = 0.4$, the mesh composed of 63 elements and 111 nodes. Figure 3 shows a comparison of the computed values of $K_I$ and $K_{II}$ with the modified mapping-collocation solution by Bowie (1973). An excellent agreement between the two solutions for $K_I$ is noted for short cracks, but slightly higher values of the stress intensity factor are predicted for deep cracks by the present analysis. Attempts were not made at the refinement of finite element grids for the deep cracks to study the convergence of the solution. The agreement of the two solutions for $K_{II}$ is excellent for short as well as deep cracks. For $a/b = 0.4$, an error of 1.74% percent in $K_I$, and an error of 1.79 percent in $K_{II}$ were noticed in the present solution.

Central Curved Cracked Tension Plate

Figure 4 shows the geometry and finite element representation of a plate with a central quarter-circle crack. Due to symmetry, only one half of the plate was needed in the analysis. A total

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Fig. 1 Separation of Mode I and Mode II displacements
Pl. 2: Elliptical crack on a tension plate with crack tip stress intensity factors

Pl. 3: Stress intensity factors for an oblique crack on a tension plate

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<tr>
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<tbody>
<tr>
<td>$K_I/a^{1/2}$</td>
<td>0.81</td>
<td>0.81</td>
<td>0.00</td>
</tr>
<tr>
<td>$K_{II}/a^{1/2}$</td>
<td>0.94</td>
<td>0.98</td>
<td>0.04</td>
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</table>

$\mu = 2.0$
Fig. 4  Element breakdown of a plate with a central quarter-circle crack (biaxial and uniaxial tension)

Centre Cracked Tension Plate with Adjacent Edge Cracks

The geometry and finite element model of a tension plate with a centre crack and four adjacent edge cracks is shown in Fig. 5. Exploiting the double symmetry, one quarter of the plate was subdivided into 67 elements and 132 nodes. Table 2 shows a comparison of the computed values of $K_I$ and $K_{II}$ for the edge cracks with the crack tip hybrid singular element solution of Pian, Tong and Luk (1971). The agreement between the two finite element solutions for the edge cracks is good. The value of $K_I$ for the centre crack is not reported in Pian, Tong and Luk (1971).

**Table 2  Stress Intensity Factors for a Centre Crack and adjacent edge cracks in a tension plate**

<table>
<thead>
<tr>
<th></th>
<th>Present analysis</th>
<th>Pian, Tong and Luk (1971)</th>
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<tr>
<td></td>
<td>$K_I$</td>
<td>$K_{II}$</td>
</tr>
<tr>
<td>Centre crack</td>
<td>4.723</td>
<td>-</td>
</tr>
<tr>
<td>Edge cracks</td>
<td>4.804</td>
<td>-0.920</td>
</tr>
</tbody>
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*Finite element solution*
CONCLUSION

A mixed finite element procedure for the determination of combined mode stress intensity factors for general crack problems has been presented. The numerical results demonstrate that fairly accurate estimation of combined mode stress intensity factors can be obtained by using relatively coarse meshes of mixed elements without resorting to any crack tip singular elements. Due to the generality and accuracy, the mixed elements can be usefully employed in fundamental studies on fracture mechanics.

REFERENCES


