APPLICATION OF NEW METHODS TO THE BOUNDARY ELEMENT METHOD ANALYSIS OF STRESS INTENSITY FACTORS

H. Kisu*, R. Yuuki and H. Kitagawa*****

Faculty of Engineering, Nagasaki University, Nagasaki, Japan
**Institute of Industrial Science, University of Tokyo, Tokyo, Japan*
***Faculty of Engineering, Yokohama National University, Yokohama, Japan*

ABSTRACT

It is necessary to develop the method suitable for Boundary Element Method (BEM) to determine the stress intensity factors (K) simply and accurately. New methods are proposed, using the solutions of the stress or the displacement near a crack tip obtained by BEM analyses. The methods can determine the accurate values of the stress intensity factors without any modifications of the given BEM programs. These methods proposed are applied to the BEM analyses for various two dimensional as well as three dimensional crack problems. From these numerical results, it is concluded that the present methods can be successfully applied to BEM analyses of K and can satisfy both demands of simplicity and high accuracy.

KEYWORDS

Boundary Element Method; Numerical Analysis; Stress Intensity Factor; Surface Crack; Fracture Mechanics.

INTRODUCTION

Evaluation of the stress intensity factors, K, for cracks is necessary for practical applications of linear fracture mechanics analysis. It becomes especially important to analyze the stress intensity factors for three dimensional cracks such as surface crack, because cracks observed in structural components are mostly surface cracks. Two prominent numerical methods which are used to analyze the three dimensional crack problems are the finite element method (FEM) and the boundary element method (BEM). It is well known that FEM has been successfully applied to the analyses of crack problems. Recently the boundary element method has attracted special interest as a powerful method to calculate the K values of three-dimensional cracks (Kitagawa and co-workers, 1984; Palusamy and Shaw, 1981; Tan and Fenner, 1979), since the BEM requires only the discretization of the boundary of the domain considered. To deal with the stress singularity at a crack tip in these numerical methods, some special devices or numerical techniques are necessary for obtaining accurate solutions of K. Various techniques were developed and used in FEM analyses (Murakami, 1976; Yagawa and co-workers, 1978; Yamamoto and Tokuda, 1973), while only a few convensional methods were used in BEM analyses. This is an obstacle to get the accurate solution of K by BEM. It should be useful if a new method is introduced into the BEM analysis to determine the value of K accurately and simply.

From this point of view, new methods suitable for the BEM analysis are proposed to calculate the accurate values of K. The methods can determine the values of K simply, using the direct solutions of the stress or the displacement near a crack tip in BEM analyses and they can be applied to any programs of BEM without any modifications of the programs.

The present paper describes the formulations of new methods and how to apply these methods to the BEM analysis. And we show the numerical results of K for various two-dimensional cracks and also surface cracks obtained by the present methods. Attention is confined to the opening mode (Mode I) cracks in the present paper. The usefulness and the accuracy of these methods will be discussed in comparison with the conventional methods.

NEW METHODS TO DETERMINE THE STRESS INTENSITY FACTORS

Conventional Methods (Extrapolation Methods)

For comparison with our methods, the conventional methods to determine the K values are described in the following. In a polar coordinates as shown in Fig.1, the mode I stress intensity factor, K_I, can be defined as follows,

$$K_I = \lim_{r\to 0} \sqrt{2\pi r}\, \sigma_y|_{\theta=0} = \lim_{r\to 0} \sqrt{2\pi r}\, \sigma^* \tag{1}$$

$$K_I = \lim_{r\to 0} \sqrt{\frac{2\pi}{r}}\, \frac{2G}{\kappa+1}\, v|_{\theta=\pi} = \lim_{r\to 0} \sqrt{\frac{2\pi}{r}}\, v^* \tag{2}$$

where σ^* is the stress component in y-direction on the line of $\theta = 0$, v^* is the displacement of crack surface in y-direction on the line of $\theta = \pi$, G is the shear modulus, $\kappa = (3 - \upsilon)/(1 + \upsilon)$ (plane stress), $\kappa = 3 - 4\upsilon$ (plane strain) and υ is Poisson's ratio.

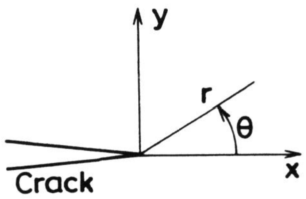

Fig. 1. Coordinate system at the crack tip.

In the conventional stress method and displacement method usually used in the BEM to determine the K values, the values of $\sqrt{2\pi r}\,\sigma_y^*$ and $\sqrt{2\pi/r}\,v^*$ obtained are plotted against the distance r from the crack tip and the value of K_I is determined by extrapolating these values to the limit $r \to 0$. However, these conventional methods have some problems on the accuracy of solutions, the use of large number of fine elements near a crack tip and so on. This is the reason why a new method is needed. To improve these disadvantanges, the authors have proposed the hybrid extrapolation method in the previous paper (Kitagawa and co-workers, 1984) which combines the displacement method with the stress method. It is found that this method is useful to some extent particularly when the division of elements near the crack tip is coarse. However it has a limit in the accuracy, because it is no more than one of extrapolation methods. Therefore, new powerful methods will be introduced in the following.

Method Using Stress Near the Crack Tip (Proportional Stress Method)

Equation (1) can be written near the crack tip in the follwing, since K_I is given in general as $K_I = \sigma_a \sqrt{\pi a}\, F_I$, where σ_a is an applied stress, a is crack length and F_I is correction factor of K_I.

$$\sigma_y^* = K_I/\sqrt{2\pi r} = \sigma_a\sqrt{\pi a}\, F_I/\sqrt{2\pi r} \tag{3}$$

Equation (3) suggests the method to calculate the K values. Equation (3) means that the ratio σ_y^*/F_I is constant for any crack problems if the relative distance r/a and the applied stress σ_a are taken as same values, respectively. Based on the fact, the value of F_I for an unknown problem can be easily calculated from the ratio σ_y^*/F_I for a known problem (a standard problem).

This method is essentially the same as the method which was proposed by Murakami (1976) and has been successfully applied to FEM analyses. He used the stress $\sigma_{tip} - \sigma_g$ for σ_y^* in Eq.(3) to improve the accuracy of the solution based on the concept of superposition in elasticity, where σ_{tip} is the solution of stress near the crack tip and σ_g is that at the same point in the field with no crack. He proposed this method in a slightly different way from the present study. The authors showed a general expression of his method in Eq.(3).

Paying attention to Eq.(3) again, it is found that the ratio σ_y^*/K_I is constant if the absolute distance r takes the same values. Therefore the values K_I can be directly calculated from the ratio σ_y^*/K_I for a standard problem in the similar manner to the method mentioned above.

Method Using Displacement Near Crack Tip (Propotional Displacement Method)

Similar methods can be derived from equation of the displacement near the crack tip. Equation (2) is written as follows,

$$v^*/r = K_I/\sqrt{2\pi r} = \sigma_a\sqrt{\pi a}\, F_I/\sqrt{2\pi r} \qquad (4)$$

In this case, the ratio $(v^*/r)/F_I$ is constant for any crack problems if the r/a and the σ_a are the same, and the ratio $(v^*/r)/K_I$ is constant if r is the same. The F_I or the K_I is calculated from those values of a standard problem, in the similar manner as mentioned above. In this case, there is no need to subtract the displacement component in the field with no crack, since the displacement v* for the field with no crack is zero. This is the point different from the proportional stress method. The method in this section is originally developed in the present study.

Although the stress or the displacement near the crack tip easily contains some numerical errors in the individual numerical analysis, these errors can be excluded by use of the ratio of the stress or the displacement to those in a standard problem. So it is expected that the proportional stress method and the proportional displacement method can give the accurate solutions. In fact, body force method (Nisitani, 1967; Nisitani and Murakami, 1974), which is well known as a high accurate numerical method to calculate K, used a concept similar to these methods. That is, body force method determines the K by use of the ratio of the density of body forces distributed on the given crack surface to that of a standard problem.

The present methods can be applied to any analyses, numerical or experimental, if the stress or the displacement closely near the crack tip are obtained. These methods are also applicable to the analyses of not only two-dimensional crack problems of the mode I but also three-dimensional crack problems and mixed mode crack problems of the mode I and II.

Procedures in the Application of the Present Methods to BEM Analysis

The present methods can determine the K values by use of the direct solutions of the stress or the displacement near the crack tip in BEM analysis and can be applied to any program of BEM. It seems that these methods are suitable and useful for the BEM analysis. These methods are applied to the BEM analysis as follows.

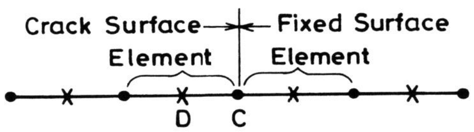

Fig. 2. Elements near the crack tip.

Figure 2 shows an example of the division of boundary elements near the crack tip, where the quadrilateral isoparametric elemtnts are used. In the present analyses, the stress σ_{yC} at the crack tip node C in Fig.2 and the displacement v_D at the middle node D are employed as the stress σ_y* in Eq.(3) and the displacement v* in Eq.(4). Although of course these values are different from the theoretical values due to the singularity at the crack tip, they give a kind of average values in the element and represent the values at a point with a certain distance r*. If the sizes of the element near the crack tip and the crack lengthes are the same in two crack problems, it is evident that these values give the accurate values at the point with same distance r* Now we assume that the problem A is unknown and the problem B is known or a standard problem. The values of F_I for an unknown problem A, F_{IA}, can be obtained as follows, if F_{IB} for a standard problem is given and the values of σ_{yC} or v_D for a standard problem are obtained beforehand.

In the case that the relative sizes of elements r^*/a are the same in both problems,

$$F_{IA} = \frac{(\sigma_{yC} - \sigma_{gC})_A}{(\sigma_{yC} - \sigma_{gC})_B} F_{IB} \qquad \text{(Proportional Stress)} \qquad (5)$$

$$F_{IA} = \frac{(v_D)_A\, r^*_B}{(v_D)_B\, r^*_A} F_{IB} = \frac{(v_D)_A\, a_B}{(v_D)_B\, a_A} F_{IB} \qquad \text{(Proportional Displacement)} \qquad (6)$$

where a_A and a_B are the crack length of the problem A, B, respectively. In the case that the absolute sizes of elements r* are the same in both problems,

$$F_{IA} = \frac{(\sigma_{yC} - \sigma_{gC})_A}{(\sigma_{yC} - \sigma_{gC})_B}\sqrt{\frac{a_B}{a_A}}\, F_{IB} \qquad \text{(Proportional Stress)} \qquad (7)$$

$$F_{IA} = \frac{(v_D)_A}{(v_D)_B}\sqrt{\frac{a_B}{a_A}}\, F_{IB} \qquad \text{(Proportional Displacement)} \qquad (8)$$

It must be noted to subtract σ_g from σ_y in the proportional stress method in the same manner as Murakami's method. In these methods, it is important that the elements not only at the crack tip but also near the crack tip must be same in both problems.

NUMERICAL RESULTS

To confirm the usefulness of these methods, the BEM analyses are carried out on two or three dimensional crack problems. The BEM programs are developed essentially after Lachat and Watson (1976). The quadrilateral isoparametric elements are used as boundary elements and the quarter point (singular) elements are placed at the crack tip at need. The method developed by Lachat and Watson (1976) to improve the integration scheme is introduced to our programs. The details of the BEM analyses are omitted in the present paper, because the present methods can be applied to any BEM programs.

Two Dimensional Cracks Problems

The BEM analyses were carried out on three kinds of plate specimens under uniform tension, such as, center cracked (CCT), single edge cracked (SEC) and double edge cracked (DEC) specimens and the F_I values are determined by the present methods. The models analyzed are shown in Fig.3. The analyses

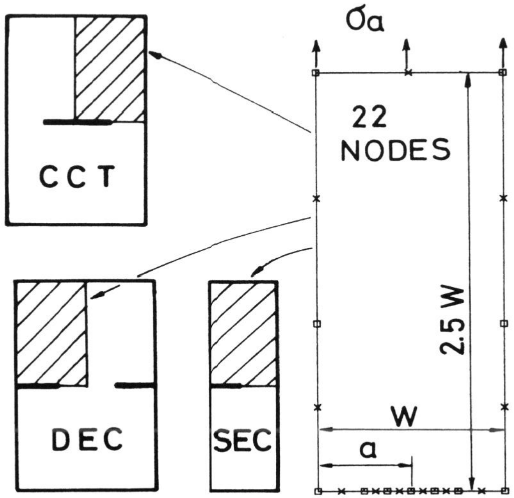

Fig. 3. Analysis models of plate specimens.

TABLE 1 F_I Values Obtained by Use of the Present Methods

Method		Use Of Displacement Near The Crack Tip						Use Of Stress Near The Crack Tip						Ref. Values of F
No. of Nodes		404 ~ 424		154 ~ 170		22		404 ~424		154 ~170		22		
Elm.Size r/a		7.81×10^{-3}		2.08×10^{-2}		2.50×10^{-1}		7.18×10^{-3}		2.08×10^{-2}		2.50×10^{-1}		
ε		1.0×10^{-6}		1.0×10^{-4}		1.0×10^{-2}		1.0×10^{-6}		1.0×10^{-4}		1.0×10^{-2}		
cpu-time* sec		667		96		17		667		96		17		
$\xi = a/W$		V +	F	V	F	V	F	$\sigma_{tip}\cdot\sigma_g$	F	$\sigma_{tip}\cdot\sigma_g$	F	$\sigma_{tip}\cdot\sigma_g$	F	F **
Element		Singular Element												
CENTER CRACK CCT	0.25	6.25721	1.0393	10.1916	1.0395	34.2444	1.0288	1169.83	1.0382	711.416	1.0376	195.999	1.0217	1.03916
	0.3333	6.45868	1.0728	10.5237	1.0734	35.5856	1.0690	1207.82	1.0719	734.922	1.0719	204.085	1.0639	1.07263
	0.5	7.14418	base	11.6346	base	39.5005	base	1337.09	base	813.583	base	227.634	base	1.18666
	0.625	8.07341	1.3410	13.1289	1.3391	44.3743	1.3331	1512.33	1.3422	919.406	1.3410	257.066	1.3401	1.34142
SINGLE EDGE CRACK SEC	0.25	9.01137	1.4968	14.6769	1.4970	50.2745	1.5103	1688.37	1.4984	1027.66	1.4689	287.846	1.5005	1.4941
	0.3333	10.7718	1.7892	17.5418	1.7892	59.6943	1.7933	2019.75	1.7925	1229.55	1.7934	341.642	1.7810	1.7843
	0.5	17.0110	2.8256	27.6909	2.8243	92.4484	2.7773	3193.04	2.8338	1942.83	2.8337	522.692	2.7248	2.8266
	0.625	26.8880	4.4661	43.5251	4.4393	137.208	4.1220	5047.18	4.4793	3050.45	4.4493	756.215	3.9422	4.4809
DOUBLE EDGE CRACK DEC	0.25	6.69606	1.1122	10.9104	1.1128	37.3392	1.1217	1252.26	1.1114	761.783	1.1111	212.668	1.1086	1.11
	0.3333	6.73822	1.1192	10.9786	1.1198	37.5642	1.1285	1260.23	1.1184	766.654	1.1182	214.258	1.1169	1.12
	0.5	7.04147	1.1696	11.4739	1.1703	39.2817	1.1801	1317.50	1.1693	801.855	1.1696	225.145	1.1737	1.163
	0.625	7.58417	1.2597	12.3597	1.2606	42.2607	1.2696	1419.96	1.2602	864.771	1.2613	243.868	1.2713	1.26
Element		Normal Element												F **
CCT	0.25	8.35370	1.0396	13.5883	1.0400	44.7412	1.0385	600.034	1.0374	363.211	1.0374	96.4710	1.0282	1.03916
	0.3333	8.62224	1.0730	14.0294	1.0738	46.3621	1.0661	619.647	1.0713	375.324	1.0720	100.339	1.0694	1.07263
	0.5	9.53561	base	15.5046	base	51.1265	base	686.363	base	415.463	base	111.340	base	1.18666
	0.625	10.7730	1.3406	17.4824	1.3380	56.8413	1.3193	776.753	1.3429	470.232	1.3431	125.431	1.3368	1.34142
SEC	0.5	22.7011	2.8250					1640.54	2.8363					2.8266
DEC	0.5	9.40044	1.1698					676.279	1.1692					1.163

* HITAC M200H SYSTEM (University Of Tokyo)

** CCT ; ISIDA, SEC ; Approximate Equation***
DEC ; NISITANI (Interpolation), + E=1, $\nu=0.3$

*** $F = \sqrt{\frac{2}{\pi\xi}\tan\frac{\pi\xi}{2}}\;\frac{0.752+2.02\xi+0.37\,[1-\sin(\pi\xi/2)]^3}{\cos(\pi\xi/2)}$

were carried out under the three conditions; 1) fine division of elements to obtain the highly accurate solution (404~424 nodes, parameter of integration $\varepsilon = 10^{-6}$), 2) coarse division to obtain the solutions most simply (22 nodes, $\varepsilon = 10^{-2}$), 3) intermediate division (154~170 nodes, $\varepsilon = 10^{-4}$). The exact solution for the CCT specimen of a/w = 0.5 given by Isida (1973) was employed as the standard solution in all cases.

Table 1 shows the results obtained by the present methods with comparison of the reference values of F_I (Isida, 1973; Nisitani, 1974; Tada and co-workers, 1973). It is found that the ratio calculated in the standard problem can be applyed to all cases with different crack lengthes and different types of specimen. The highly accurate solutions of F_I are obtained in the case of fine division and the satisfactory solutions can be obtained even in the case of coarse division (The errors are within the level of 1% except for the case of a/w = 0.625 in SEC and the cpu-time is less than 17 seconds). It seems to be not so important to employ the quarter point element (singular element) at the crack tip in the present methods, since there is little difference in the accuracy of solutions between the results with and without singular element (Normal element). In either case, the method using the displacement is superior to the method using the stress as for the accuracy.

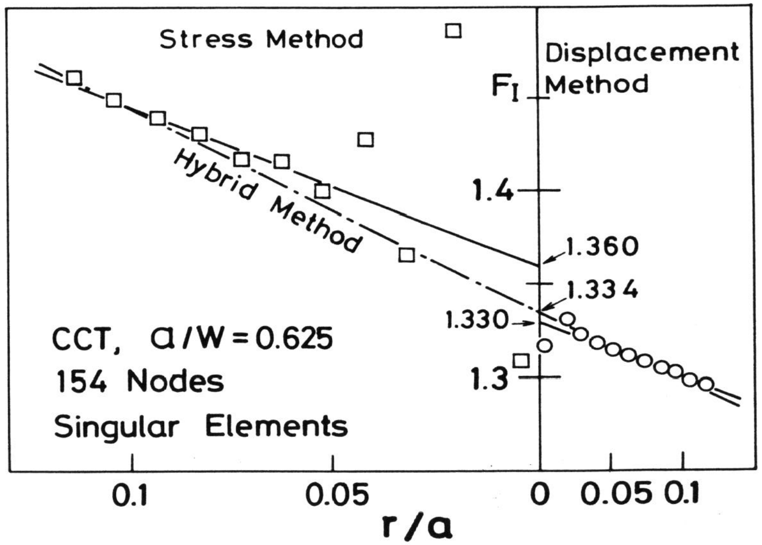

Fig. 4. F_I values by the extrapolation methods.

Figure 4 shows the results obtained by the conventional (extrapolation) methods in the case of a/w = 0.625 in CCT. Analyses are carried out under the intermediate division condition. In this case, the displacement extrapolation method and the hybrid method (Kitagawa and co-workers, 1984) give relatively the accurate values of F_I. However the accuracy is still lower by one order than that of the present methods as shown in Table 1. In the case that the number of elements is not so large, the accuracy of the solutions by the conventional methods falls down largely. It must also be emphasized that the extrapolated values can vary, depending on which data we use for extrapolation.

From these results, it is clarified that the present methods can be successfully applied to the analysis of K in BEM and can satisfy the demand of simplicity as well as that of high accuracy.

Surface Crack Problems

Based on the successful results in two dimensional cracks as mentioned above, the present methods are also applied to a three dimensional crack problem. The models analyzed are the plate specimen with a semi-circular surface crack as shown in Fig.5. The plate size parameters t/b and h/b are 1 and 3, respectively and the ratio of crack depth to plate thickness a/t is varied. The solution for embedded circular crack of a/t = 0.1 is employed as the standard solution, which can be considered as the crack in an infinite body. Pattern of division of elements on the cracked plane are shown in Fig.6.

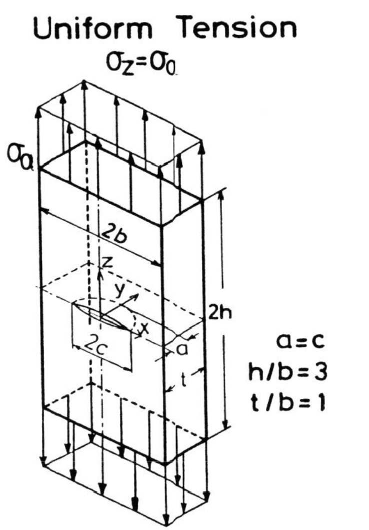

Fig. 5. Analysis models of plate specimen with a surface crack.

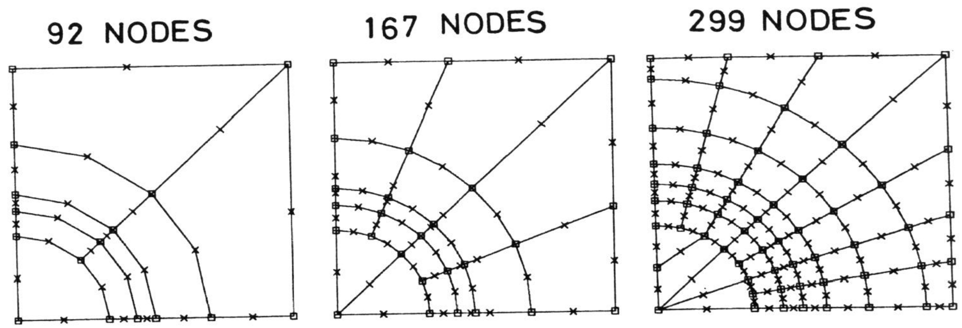

Fig. 6. Pattern of division of elements on the cracked plane.

Typical results of K_I values along the crack front obtained by the present displacement method are shown in Fig.7 in comparison with the results in references (Kitagawa and co-workers, 1984; Nisioka and co-workers, 1979; Raju and Newman, 1979; Yagawa and co-workers, 1977). The solutions obtained by the present methods agree well with those by FEM (Nisioka and co-workers, 1979; Raju and Newman, 1979: Yagawa and co-workers, 1977) and they are more accurate, in spite of only 92 nodes, than those by the extraporation methods in the analysis of 308 or 299 nodes (Kitagawa and co-workers, 1984). In the present methods, the values of the middle node on the crack front can be used to determine the K value at the point and the flexible division of elements near crack front can be introduced, which is useful for the analyses of a semi-elliptical surface crack (Kitagawa and co-workers, 1984). While in the extrapolation methods, it is difficult to use the values of middle node because of the lack of data for extrapolation and the division pattern of elements near the crack front is subjected to the restriction on the extrapolation.

In this study, it is found that the present methods are also useful for the analysis of three dimensional crack and it is expected that the BEM analysis combined with the present methods can be extended to more wide use for the analyses of three dimensional crack problems.

CONCLUSIONS

1). New methods to calculate the stress intensity factors are proposed. Those are, a) the method using stress near the crack tip (Proportional Stress Method) and b) the method using displacement near the crack tip (Proportional Displacement Method).
2). These methods can determine the stress intensity factor simply and accurately. These methods are suitable and useful especially for BEM analyses.
3). It has been assured that these methods can be applied to the BEM analyses for various two dimensional crack and also three dimensional crack problems. It is found that these methods give the accurate solutions even if the elements near the crack tip are fairly coarse.
4). It is expected that these methods are powerful paticularly for the analyses of three dimensional crack problems by BEM.

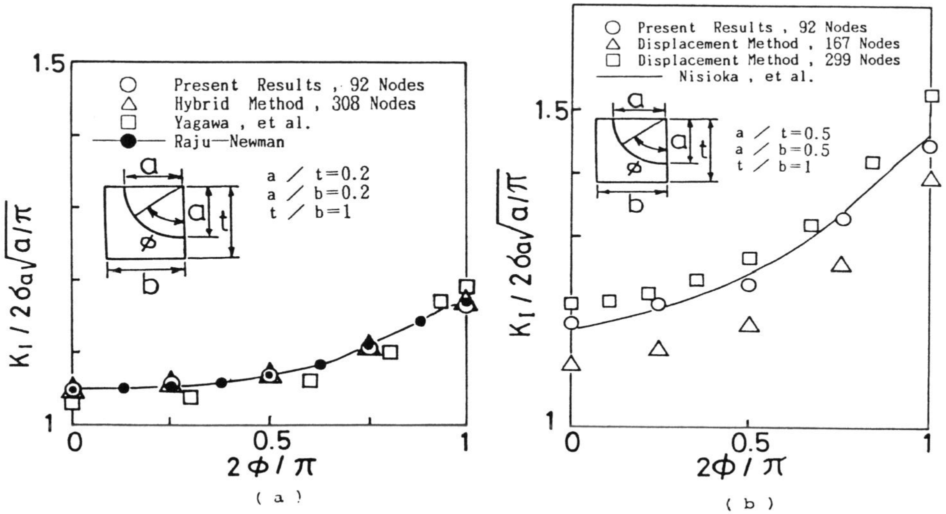

Fig. 7. Solutions of K_I along the crack front.

ACKNOWLEDGEMENT

The authors wish to extend their grateful thanks to Dr. H. Nisitani, professor of Kyushu University, for the valuable suggestions and helpful discussions.

REFFERENCES

Isida, M. (1973). Eng. Frac. Mech., 5, 647.
Kitagawa, H. et al. (1984). Trans. JSME, 50-450, to be published.
Lachat, J. G. and Watson, J. O. (1976). Int. J. for Num. Methods in Eng. 10, 991.
Murakami, Y. (1976). Trans. JSME, 42-360, 2305.
Nisioka, T. et al. (1979). Trans. JSME, 45-395, 717.
Nisitani, H. (1967). J. JSME, 70-580, 627.
Nisitani, H. (1974-10). Prelim. Proc. Japan-U.S. Seminor.
Nisitani, H. and Murakami, Y. (1974). Int. J. Frac., 10-3, 353.
Palusamy, S. S. and Shaw, K. G. (1981). ICF-5 in Cannes, 1, 131.
Raju, I. S. and Newman, J. C., Jr. (1979). Eng. Frac. Mech., 11-4, 817.
Tada, H. et al. (1973). "The Stress Analysis of Cracks Handbook", Del Res. Corp.
Tan, C. L. and Fenner, R. T. (1979). Proc. Roy. Soc., Ser.A, 369, 243.
Yagawa, M. et al. (1977). Pre-Pri. JSME, No. 770-11, 4.
Yagawa, M. et al. (1978). Trans. JSME, 44-379, 743.
Yamamoto, Y. and Tokuda, N. (1973). Int. J. Num. Meth. Eng., 6, 427.